Instituto Tecnológico y de Estudios Superiores de Monterrey

Statistical Analysis of Bitcoin in a Multivariate Framework

A dissertation presented by

Mario Iván Contreras Valdez

Submitted to

EGADE Business School

in partial fulfillment of the requirements for the degree of

Doctor of Financial Science

Santa Fe, Ciudad de México, July 17th 2020

EGADE Business School
Tecnológico de Monterrey

Hacemos constar que, en la Ciudad de México, el día 17 de julio de 2020, el alumno:

Mario Iván Contreras Valdez

Sustentó el examen de grado en defensa de la tesis titulada:

"Statistical Analysis of Bitcoin in a Multivariate Framework"

Presentada como requisito final para la obtención del grado de

DOCTOR EN CIENCIAS FINANCIERAS

Ante la evidencia presentada en el trabajo de tesis y en este examen, el comité examinador presidido por el Dr. José Antonio Núñez Mora ha tomado la siguiente resolución:

Aprobado por unanimidad con tesis sobresaliente

Dr. Daniel Cerecedo Hernández
Asesor principal

Dr. José Antonio Núñez Mora
Miembro del comité

Dr. Guillermo Benavides Perales
Miembro del comité

Dr. José Ernesto Amorós Espinosa
Director de Programas de Doctorado

Declaration of authorship

I, Mario Iván Contreras Valdez, declare that this dissertation titled, Statistical Analysis of Bitcoin in a Multivariate Framework and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this dissertation has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this dissertation is entirely my own work.
- I have acknowledged all main sources of help.
- Where the dissertation is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Mario Iván

Mario Iván Contreras Valdez
07 17,20

@2020 by Mario Iván Contreras Valdez
All rights reserved
EGADE BUSINESS SCHOOL
Ciudad de México

Dedication

Para Alma Valdez, por ser mi maestra de vida, introducirme a la magia de los libros y mostrarme la belleza del mundo. Todo lo que alcanzo es debido a que estoy sobre tus hombros.

Y para Natalia Santiago, por ser mi segunda madre y enseñarme a ver el mundo con ojo crítico.

Acknowledgements

A mi madre, Alma Valdez, por ser mi apoyo incondicional a lo largo del camino.

A mi padre, Mario Contreras, por enseñarme lo que significa la diciplina y tener en mente objetivos claros.

A mis profesores del doctorado, José Antonio Núñez Mora, Daniel Cerecedo Hernández, Guillermo Benavides Perales, José Juan Chávez Gudiño y Carlos Viguri Bretón, por sus conocimientos, guía y consejo, y por convertirse en mis modelos a seguir.

A mis amigos del doctorado, Carlos Franco Ruíz, Alfredo Ramírez García y Eduardo Sánchez Ruenes, por convertirse en parte de mi familia.

A mis amigos, Erick Emmanuel Márquez Castillo y Sergio Hernández Weber, por creer en mí y darme tantas oportunidades para seguir aprendiendo.

Al Tecnológico de Monterrey, por ser mi segunda casa desde la preparatoria y tener siempre a los mejores maestros y compañeros dispuestos a ayudar y siempre dar todo de sí.

Statistical Analysis of Bitcoin in a Multivariate Framework

By

Mario Iván Contreras Valdez

Abstract

This work elaborates on the statistical study of the first cryptocurrency made: Bitcoin. It presents a brief introduction to the basic concepts behind the functioning of the entity, as well as some studies regarding technical, ethical, and legal aspects. Regarding the Economic and Financial themes, the issue to approach relates with the implementation into markets. The introduction of new assets into the basket available for investors may cause certain risks if it is now fully understood and inadequate assumptions are used to assess the exposure or asset allocation. To address this topic, this document is divided in 5 chapters regarding the analysis of certain properties through financial models. First, the stylized facts on the diversity of cryptocurrencies is studied through descriptive statistic and qualitative techniques. Second, a bubble detection algorithm is deployed over the Bitcoin series detecting 11 episodes. It is then analyzed the reasons behind such events. The results indicate the existence of three stages in the series: the oldest related with government intervention, second a speculative bubble and third a stabilization period related with the evolution of the market. Third, with these results a Value at Risk and Expected Shortfall methodology with the Normal Inverse Gaussian (NIG) distribution is presented as an argument to use this specification for further developments. Fourth, determined the capability of NIG to fit data (even above the general distribution) a multivariate rolling window estimation is used in trivariate baskets of financial assets. With the parameters adjusted to the statistical properties, the asset allocation problem is set to find the optimal weights that reduce risk. The results show the transition of Bitcoin from being a speculative asset with almost zero weight, to develop a hedging capability in the commodity portfolio.

Contents

List of tables

List of figures

Chapter 1: Introduction

In economic history, the writings of Adam Smith have proven to transcend the centuries into contemporaneous days. Demsetz (1990) exposes that one of the main tasks for economists is the formalization and gap filing of the system presented by Adam Smith (p 145). Expanding such idea, Coase (1992) shows that, even as the *Wealth of Nations* praise for the decentralization of economic activities, it is necessary a regulatory framework for the *invisible hand* to function properly. This necessity became visible during the economic crisis started in 2008 and lived through the next eight years; the lack of regulation and proper controls over financial institution´s decisions lead to a sub estimation of risk and the eventual collapse of economy. Nevertheless, another event occurred during the turmoil: these were the days that a new financial entity, whose properties and definition remain a debate theme in financial, economic and law forums, was born under the name of Bitcoin.

A presumably group of computer scientists and hackers under the pseudonym of Satoshi Nakamoto (2009) published the article titled “Bitcoin: A Peer-to-Peer Electronic Cash System”. In this work the entity was defined as a payment system in which electronic transactions relies on a cryptographic proof instead of trust in financial institutions (p 1). This authentication intends to avoid the double-spending problem through digital signatures – later known as blockchain – that records all the transactions ever made. This mechanism is possible because all the transactions realized are publicly announce, then it is responsibility of the network to determine which one arrived first and omit the second one. This allows the participants to access the public and unique history of transactions. The complete steps for such mechanism are expressed as follows: 1) The transaction is send to all the nodes of the network, 2) each node collects the transaction into a new block, 3) the nodes performs a computational intensive algorithm that works as a proof-of-work to generate a new block, 4) when one of them finds the answer, it broadcast the answer to the rest of the nodes, 5) the nodes will accept the new block if an only if all transactions in it, along with the solution, are valid, 6) the final acceptance is due via the nodes using the chain into the cryptographic problem and starting a new block for the chain (p 3). The final statement in the paper is that Bitcoin is a new system for electronic transactions that works without relying in trust (p 8).

In the basic aspect, Bitcoin is a service provided by node operators to validate electronic transactions. This service is payed to the so-called *miners* with a fee and the possibility to earn a Bitcoins by the completion of a block in the chain which works as the main incentive to participate in it (Nakamoto 2009, p 4). The algorithm is based on Elliptic Curve Cryptography and Elliptic Curve Digital Signature Algorithm (ECDSA) that allows for the authentication system which guarantee the chain to avoid hacking, backup information and provide a complete track of the Bitcoin units (Taleb, 2019). Rus et al. (2018) present a detailed description on the mining process; they expose the procedure to be computationally intensive. The expected time needed to find a solution is 10 minutes; nevertheless, if the computational power of the net is exceeded or decreases, this time may change. To avoid these movements in the expected time, the algorithm is programed to recalibrate the difficulty based on the processing power of the network. This calibration is made every 2,016 blocks equivalent to 14 days, approximately. The reward gained to find a solution is a fixed number of Bitcoins. But this amount suffers the so called “halving” which means that every 210,000 blocks – approximately every 4 years – the Bitcoins mined reduce in half. The first miners received a reward of 50 Bitcoins for every block; quantity that has suffered three half reductions. The last halving process up until the writing of this document occurred in 11th May 2020, leaving the reward in 6.25 Bitcoins per block. The result in such mechanism is that, eventually, approximately 21 million units would have been created; moment from which the miners will be paid only with transactional fees.

The final component of Bitcoin corresponds with the mean of storage of information also called *wallet*. This repository may be any memory gadget: from a USB flash, a hard disc in a CPU or even a cloud service. This wallet works as a bank account for Bitcoins to be transfer from one to another with an almost complete anonymity (Juhász et al., 2018) as the wallet is attach to a nickname (Nakamoto, 2009, p 6).

Although the algorithm and the miner system protect the blockchain to avoid double spending of falsification, the wallets are vulnerable to hackers and other kinds of destruction such as loose of the gadget (Raymaekers, 2015). Such attacks have caused the downfall in Bitcoin price due to vulnerability issues. But natural destruction works in an inverse way. As the number of Bitcoins is limited, then this reduction in quantity represents a shift in supply curve, causing an appreciation of the assets among users.

This last statement leads to consider the Bitcoin as money in the economic perspective. The Central Authorities that regulate the monetary policy for countries are the Central Banks who have the power to emit money and employ diverse tools to move the money market in order to control the inflation rate. Rogojanu and Badea (2015) present a model of this market for Bitcoin conditions. In this work, they observe the cryptocurrency to has a deflationary nature as the supply is almost deterministic and eventually constant with only possible negative shifts. The scarcity of the currency leads to an appreciation in relation to the goods and services in the hypothetical economy. The decentralized model then shows that the lack of a Central Authority to response to shifts in money demand could be detrimental to the economic stability.

In a theoretical perspective Mises (1912) presents the idea that money may be value through the latter known as Regression Theorem (cap 2, p 1 – 20). In this theory, the value of money is related with the marginal utility of agents who are willing to give up goods and services for money. The utility of money is then given by the expectation of the purchasing power in the future, so by tracing back the value of money one would get to a stage in which a commodity with intrinsic value was traded for another. Davidson and Block (2015) use this point to discuss on weather the Bitcoin may be classify as a new medium of exchange that violates the Regression Theorem. In this sense, they conclude that as Bitcoin did not emerge in a barter economy and it may be exchange for goods and services – and even other currencies – then it could be interpreted as a next step in the chain value. For the concept of money Menger (1892) goes beyond and defines it as a universally accepted medium of exchange (p 239-255). This immediately excludes Bitcoin and other cryptocurrencies as money but keeps the medium of exchange definition for certain markets. So, for the complete acceptance of Bitcoin in real economy, further development and regulation is necessary.

The intended use of Bitcoin is then a mere system for electronic transactions to purchase the goods and services sold in the various internet markets – legal and illegal ones –. One of the main characteristics of Bitcoin is the anonymity it provides; as any user may be created at any time, it works as a tool to hide the identity of criminals. Trautman (2014) presents the notion that virtual currencies, and especially Bitcoin, have been used for black markets that includes drugs, firearms, assassination contracts and child pornography (p 3-19). Activities that take place in the so-called *deepweb* for specialized agents that access through the TOR network to hide their IP. In such scenario, in conjunction with virtual currencies, technology provides them with the resources to realize anonymous transactions. The most famous case for such activities is the *Silk Road* website, marketplace that worked as the meeting point to reunite suppliers and consumers of drugs and controlled pharmaceutical through anonymous profiles. The infrastructure of the website was characterized for the efficiency and security for their users, while the agents involved where defined by themselves as “intelligent and responsible” consumers of drugs with a risk aversion profile (Van Hout and Ningham, 2013, p 3), making the Bitcoin a valuable asset to perform their activities.

Regardless the regulation as a black-market currency, Bitcoin have captured the spotlight in many disciplines. In a legal perspective, cryptocurrencies main issue is related to the tax policy. Kurek (2015) presents a complete study in the international treatment regarding taxation on cryptocurrencies. In Europe, Great Britain viewed Bitcoin as a voucher subject to Value Added Tax (VAT); however, initiatives raised to be reclassified under “private money” exempt from it. Germany opted for a mean of payment definition in order to tax the revenues sight in less than a year retention of a Bitcoin, while Russia classified it a huge speculative risk and decided to ban it. The legalization of Bitcoin in the USA began in a Texas tribunal during 2013, where the court considered it a mean of payment, like US dollar, so a pyramid fraud could be sentence as such. Roman (2015) expose the necessity of a clear and universal definition of Bitcoin either as a property – such as a stock – or a currency in order to provide a correct tax treatment. This problem is extended by Lambert (2016) and Segal (2018) who focus their attention in the lack of a valuation scheme for Bitcoin users to report the gains and looses of the asset due to the high volatility witness in the market, as well as the presence of multiple price providers with different results as presented by Shi et al (2019).

The problem is then extended by Ryznar (2019) who refer the issue with the development of Future Contracts with Bitcoin as underlying asset presented by The Chicago Board Option Exchange (CBOE) and the Mercantile Exchange Group (MEG) in 2017. In her work, she presents the Futures Industry Association´s (FIA) complain on such decision arguing the potential affectation to the economy. This problem extends to the valuation of a derivate instrument with high volatility and presumably speculative underlying asset. Other considerations to take are the possible presence of bubbles and the abrupt downfalls in prices mainly due to hacking and government regulations.

O´Donnell (2018) presents an argument in which poor government policies lead to innovative technologies into failure. The Bitcoin´s ban policy taken by governments such as Russia, India, South Korea and China has created a loose in the value of the cryptocurrencies. Under the idea that the virtual currencies allow for criminal activities to proliferate, different proposals harmful to them have been taken into consideration. Nevertheless, this panic relies on the threat to State Institutions; O´Sullivan (2018) states that fear about the advent of such technologies relies on the direct confrontation with political powers and the tools such as fiscal and monetary policy. In this sense Angel and McCabe (2015) concludes that Bitcoin is neither good nor evil, it depends on the usage it is given (p 610). In concordance

O´Donnell (2018) also states that fiat money is also used, and even in greater relevance, for those criminal activities (p 31). Such ideas incentive for a greater study of Bitcoin fundamentals and ways for it to incorporate it into social and economic activities.

The economic and financial literature on Bitcoin share almost the same interests on how the cryptocurrencies may be defined and the financial characteristics witness on the markets. In a first glance, economic literature defines money through three characteristics: 1) accepted medium of exchange, 2) store of value and 3) unit of account (European Central Bank, 2015, p 23). In this regard, only a few markets accept Bitcoins as payment system, so once again the first condition – as well as the third one – is not fulfilled. Sahoo (2017) presents a paper in which consider the cryptocurrency as the next step in the evolution of money into a digital platform. To test the store of value property, a volatility study using ARCH and GARCH models was implemented for the prices of Bitcoin. The results show that, although it presents new features that could be implemented in the real economy, the high volatility in GARCH models matches with a speculative currency. Baek and Elbeck (2015) perform a cross-sectional study to examine the volatility in conjunction with S&P 500 index daily data. Their results show that the volatility observed in Bitcoin is not related with other sources, i.e. it is the result of the supply and demand interaction. In sum, the cryptocurrency may be partially considered as money, but the main issue is the lack of understanding about the price dynamics and sources of volatility.

To address this topic Polasik et al. (2016) perform a study to determine the principal drivers of price movement of the cryptocurrency. Their findings conclude that Google searches with “Bitcoin”, the number of transactions and news with positive sentiment related to this topic are the variables that best fit the movement in returns. These findings are consistent with the theoretical explanation that Bitcoin depends mainly on demand shifts. Their study also shows that Bitcoin works as a dual mechanism: an investment or speculative asset and an exchange system. Further research on this area was made by Mai et al. (2018) who examine the impact from social media in specialized forums and Twitter. Their findings reveal that positive and negative sentiment in both platforms explain the next day raise or downfall, respectively. These facts characterize Bitcoin market with a high sensibility to demand shocks, which may represent certain abnormalities.

Almudhaf (2018) explores the possibility of inefficiencies in the Bitcoin market. His conclusions determine that, even with high volatility, the small correlation with other assets turns out to be a diversifier for currencies of emergent economies and expose the necessity to find hedges for cryptocurrencies. Nevertheless, Dimitrova et al (2019) provide evidence of significant memory in time series and explain that such behavior could be caused because of the underlying distribution. Bouri et al. (2018) find that the price volatility process has long memory and prices does not present the mean reverting component. This means that shocks remain for prolonged periods. Another finding they expose is that series present four structural breaks related with the collapse of prices. These results are closely related with the findings of Cheung et al. (2015). However, further studies presented by Senarathne (2019), in which GARCH family models are implemented to model volatility and leverage effects, conclude that statistical tests to residuals does not fill the normality assumption. This relates to a highly speculative assets and the requirement to employ different techniques and assumptions.

Regarding the possibility to employ Bitcoin as diversifier due to the lack of relation with other classical assets Lintilhac and Tourin (2017) use a dynamic strategy with stochastic control techniques to create bivariate portfolios. On this topic, Qarni et al. (2019) determine that volatility spillover flows from Bitcoin to other financial assets are insignificant, allowing for certain hedging capability of the cryptocurrency with traditional portfolios. Poyser (2019) deepens into this topic and explores the relation with gold, indexes, and exchange rates. In this instance he reconciles with the idea of dynamic parameters and manage to conclude that Bitcoin has a negative relation with gold and Chinese Yuan/US Dollar (CYN-USD) while maintaining a positive one with market index and Euro/US Dollar (EUR-USD). However, this relationship may change because news and popularity of Bitcoin are the main drivers for the price fluctuation. Nevertheless, most of the models employed for such studies use the normality assumption in returns.

Given the increasing popularity and importance of Bitcoin in the financial framework, many questions and inconveniences may arise. There is no doubt that Bitcoin does not completely fulfill traditional definitions of financial assets; in essence it is a payment system, but the creation of a unit resembles a commodity, the usage and storage is similar to a virtual currency based on money and people are using it at a financial asset to perform investment and diversification in portfolios. On the other hand, valuation tools are not capable to determine the price movement because of the high volatility and traditional models are not able to capture all the properties witness in the market. These statements are also valid for traditional financial assets; nevertheless, Bitcoin and other cryptocurrencies have shown to have escalated behaviors. Under the name of *stylized facts* (Cont, 2001, p 224) several statistical properties are included; some of the most relevant are the heavy tails, non-unbiased distribution and volatility clustering.

The main problem with the stylized facts in financial series comes with the Gaussian distribution assumption. This approximation to the behavior of the distribution may be appropriate for some assets with low volatility or for low frequency observations like monthly, quarterly, or annual data. Nevertheless, the technological advantages that allows for high frequency data with weekly, daily and even intraday observations distance the normality assumption from empirical data. This phenomenon exacerbates with Bitcoin prices and returns, as the high volatility makes unfeasible to use most of the statistical models that require Gaussian distribution. Some of the most used techniques are the risk measure and portfolio selection. The first one depends on the correct computation of the probability to have higher looses than certain value, while the portfolio optimization require the volatility and covariance matrix to perform the algorithm. In both instances, the correct specification of a distribution that manages to capture the complete statistical components of the data is necessary.

Such scenario presents some questions about Bitcoin and cryptocurrencies as new financial assets:

1) Is there evidence for Bitcoin to be considered a speculative asset subject to bubble episodes?
    a. What are the main drivers of such behavior?
    b. Is it possible to model it?
2) Which is the adequate classification for cryptocurrencies in the traditional financial assets' definitions?
3) Is it possible to use Bitcoin a diversifier to improve portfolios exposure?

The aim of the present work is to provide a complete statistical analysis of Bitcoin and the extension to the financial models. To do so, the structure is as follows: Chapter 2 presents the statistical description of the data, with emphasis in the no-normality properties that leads to consider other distributions. Then, Chapter 3 performs a bubble detection technique for the main cryptocurrencies to analyze the duration of them and identify the intervals containing these episodes. In Chapter 4, a distribution that fits the empirical data of Bitcoin will be proposed. Particularly, the Normal Inverse Gaussian (NIG) distribution presented originally by Barndorff-Nielsen (1977) as a member of the Generalized Hyperbolic (GH) family. Among other laudable properties unique in the GH family, the NIG distribution resembles most of the Gaussian characteristics like mathematical tractability, close under convolution, close under affine transformations and computationally less intensive to obtain than the general case (Barndorff-Nielsen, 1997; Lillestöl, 2002). This distribution in the univariate case will be then employed to perform the Value at Risk (VaR) and Expected Shortfall (ES) comparison with different distributions to compute the sub estimation of risk in the Bitcoin returns with different exchange rates. Chapter 5 presents a multivariate framework to compute the optimal weights for portfolios including currencies, indexes and commodities along with Bitcoin. Finally, the conclusions are presented.

Chapter 2: Stylized Facts in Cryptocurrencies

2.1 Introduction

Diverse disciplines employ time series and subsequent models to analyze certain phenomenon through time. However, in natural sciences the normality assumption is most of the times fulfilled. The reason for such behavior comes from the Central Limit Theorem, which states: let $\boldsymbol{X_1, X_2, \dots, X_n}$ be an i.i.d. sequence of random variables with finite mean $\boldsymbol{\mu}$ and variance $\boldsymbol{\sigma^2}$, for $\boldsymbol{n = 1, 2, \dots}$, let $\boldsymbol{Y_n = \sum_{i=1}^{n} X_i}$, then the standardized random variable converges to a standard normal distribution; such that for all $\boldsymbol{t}$:

$$\boldsymbol{P\left(\frac{\frac{Y_n}{n} - \mu}{\frac{\sigma}{\sqrt{n}}} \leq t\right) \xrightarrow[n\to\infty]{} P(Z \leq t)} \qquad (1)$$

With $\boldsymbol{Z \sim N(0, 1)}$. This convergence is called convergence in distribution so that the random variables converges in distribution to the aggregate one (Dobrow, 2014, p 380). A direct implementation for such result is the so-called Random Walk, stochastic process with applications in physics, biology, ecology, and other fields. If the length of the steps of the process approximate to 0, then the Brownian Motion is reached (Calin, 2015, p 45). The first usage in finance of such mathematical tools is recorded in the work of Bachélier (1900), who proposed the Brownian Motion to model the stocks in the French market. His original approximation was made to prices; nevertheless, there are no restrictions to obtain negative values, so multiple refinements lead to the Exponential Brownian Motion, for which the variable to model are the logarithmic returns rather than the level series.

Additional properties may be announced in the usage of logarithmic – also called continuous – returns, Brooks (2014, p 8) mentions some of them: 1) returns are scale free, so the interpretation of a increase or a downfall may be interpreted directly and compared to other assets even when the prices have a wide spread. 2) Continuous returns assume the instantaneous change so the frequency does not matter, and assets may be compared. 3) Continuous returns are time-additive, meaning that the sum of $\boldsymbol{N}$ observations is equivalent to the return achieved from step $\mathbf{1}$ to step $\boldsymbol{N}$.

In this scenario, the study of continuous returns and their statistical properties is key to any valuation of derivative products (Hull, 1997) or to evaluate and measure the exposure of the position in a risk management scheme (Duffie and Pan, 1997). Mantegna et al. (1999) state that the comprehension and analysis of the series, either in levels or in differences, is a necessary starting point from which any subsequent model is settle (p 220).

Cont (2001) performs a complete treatment of the stylized facts in financial series – particularly in financial returns – in which describes them as *universal* properties shared by these variables. In his study, the principal tool to perform the evaluation of time series is the employment of non-parametric techniques. These models have the quality that allow for flexible models but are black boxes that give only qualitative information on general properties. Nevertheless, such analysis allows to detect certain discrepancies with traditional assumptions like Normality in returns. The main statistical abnormalities are the presence of heavy tails – meaning an excess in kurtosis statistic –, skewness – also translated as an asymmetry in gains and losses – and volatility clusters i.e. episodes of exacerbated volatility. Such properties require specifying a distribution with at least four parameters (p 226). Bouchaud and Potters (2001) also include the increase of correlation in high volatility periods (p 61).

Some studies that explain the presence of such phenomenon relates with changes in the canonical assumptions. Westerhoff and Franke (2009) develop a model that considers two types of traders: technical and behavioral. The responses of the agents follow different information and the proportion of them affects the presence of stylized facts. The main disruptions are caused by behavioral traders, conclusion that relates with the study made by Nguyen et al. (2018) who decompose the Bitcoin demand with the speculative and transactional components. As the speculative demand is the one that cause the greater volatility, it could be then explained that the extreme behavior of Bitcoin and other cryptocurrencies is due to the type of agents that compose most of the market. These agents are characterized to be highly influence by technical analysis and news, so the movement of such variables can cause such disruptive behavior (Polasik et al., 2016; Mai et al., 2018)

In sum, there is evidence and theoretical background to consider that stylized facts are present in Bitcoin series as it could be broadly considered a financial asset subject to news and sentiment. In consequence, the quick evolution of the cryptocurrency and the underlying conditions lead to think about the escalation of such phenomenon. The next sections will treat the statistical description of price and return series and test the presence of stylized facts.

2.2. Data and Methodology

The data to consider in the subsequent analysis was obtained from the website coinmarketcap.com and corresponds with the daily observations of the top ten cryptocurrencies under a market cap criterion. The data is composed of all the available data until 31st December 2019 in United States Dollars (USD); the total number of observations vary through the cryptocurrencies due to the subsequent development of blockchain technologies based on Bitcoin. The complete information of the data is presented in Table 1.

Table 1. Daily data for Cryptocurrencies

Cryptocurrency	**Observations**	**Starting date**	**Market cap***
Bitcoin (BTC)	1931	2014/09/18	66.63%
Etherum (ETH)	1608	2015/08/07	8.91%
XRP	1931	2014/09/18	3.39%
Bitcoin Cash (BCH)	892	2017/07/23	1.69%
Bitcoin SV (BSV)	418	2018/11/09	1.41%
Litecoin (LTC)	1931	2014/09/18	1.09%

Binance Coin (BNB)	890	2017/07/25	1.00%
EOS	914	2017/07/01	0.92%
Tezos (XTZ)	821	2017/10/02	0.76%
Stellar (XLM)	1931	2014/09/18	0.54%

Elaborated by author with information of coinmarketcap.com

*Percentage of the total market cap for cryptocurrencies

To compute the return series the continuous approach will be taken, i.e. the logarithmic differentiation. According to the Exponential Brownian Motion model, such returns must fit the Gaussian distribution. The formula to generate the new series is:

$$\boldsymbol{r_t = Ln\left(\frac{P_t}{P_{t-1}}\right)} \qquad (2)$$

Where $\boldsymbol{P_t}$ is the price in time $\boldsymbol{t}$.

As express by Cambell et al. (1996) this returns permit the scale free property and even more important is the ergodic property (p 9). This is interpreted as a *sine qua non* property to use statistic as it permits the usage of past information to perform inference about the possible outcomes. Additionally, prices are usually non-stationary, but returns are, so the statistical assumptions are fulfilled.

To test the stationarity of the series, four different statistical procedures will be deployed. The reason to do so is that the robustness of the models is subject to debate among econometricians (Malik and Rehman, 2015, p 1). To address this issue and be able to proceed with the statistical analysis of the series, the Augmented Dickey Fuller (ADF), Phillips-Perron (PP), Ng-Perron and Kwiatkowski Phillips Schmidt Shin (KPSS) tests will be applied to the prices and continuous returns. The first three are constructed under the null hypothesis of presence of unit root, which means that series must be differentiated at least once, on the other hand, the KPSS test is constructed on the null hypothesis that the series are stationary.

The reason to do so is to have the complementary result and increase the robustness of the series.

The ADF test assumes the data is driven by an ARMA(p,q) model with unknow order and is a generalization of the original Dickey-Fuller test (Dickey and Fuller 1979). The test is performed by estimating the regression:

$$\boldsymbol{Y_t = \phi Y_{t-1} + \sum_{j=1}^{p} \psi_j \Delta Y_{t-j} + \epsilon_t} \quad (3)$$

Where $\boldsymbol{p}$ is the lagged difference terms of $\boldsymbol{\Delta Y_{t-j}}$; $\boldsymbol{p}$ is selected so that the errors are serially uncorrelated. Under this specification the null hypothesis states that $\boldsymbol{Y_t}$ has unit root, i.e. $\boldsymbol{\phi = 1}$ so the t-statistic is defined as:

$$\boldsymbol{ADF = \frac{\widehat{\phi} - 1}{SE(\phi)}} \quad (4)$$

Where SE stands for the standard error. Other ways to select the optimal lag length is to use an information criterion.

The PP unit root test was proposed by Phillips and Perron (1988) and differs on the ADF test on how to treat the serial correlation and heteroskedasticity in the error term. Particularly, the ARMA structure is set aside and the model to estimate becomes:

$$\boldsymbol{\Delta Y_t = \pi Y_{t-1} + \epsilon_t} \quad (5)$$

The only condition to satisfy is that the error term be stationary, even when it may present heteroskedasticity. To correct the serial correlation, some modifications are proposed for the t-statistic:

(6)

$$Z_t = \left(\frac{\hat{\sigma}^2}{\hat{\lambda}^2}\right)^{\frac{1}{2}} \cdot t_{\pi=0} - \frac{1}{2}\left(\frac{\hat{\lambda}^2 - \hat{\sigma}^2}{\hat{\lambda}^2}\right) \cdot \left(\frac{TSE(\hat{\pi})}{\hat{\sigma}^2}\right)$$

Under the null hypothesis $\pi = 0$ and the Z_t statistic have the same asymptotic distribution as ADF test.

Ng and Perron (2003) develop new tests that according to their statistical criterions performs better than previous ones; specially when a negative moving average process takes part. This test relies on the spectral density estimation and propose to implement four tests, which are:

$$MZ_a = \frac{((T^{-1}Y_t)^2 - f(0))}{2k} \quad (7)$$

$$MZ_t = MZ_a \cdot MSB \quad (8)$$

$$MSB = \left(\frac{k}{f(0)}\right)^2 \quad (9)$$

$$MPT = \begin{cases} \dfrac{c^2k - cT^{-1}{Y_t}^2}{f(0)} \; if \;\; d_t^0 \\ \dfrac{c^2k - (1-c)T^{-1}{Y_t}^2}{f(0)} \;\; if \;\; d_t^1 \end{cases} \quad (10)$$

Where d_t^0 stands for the drift and d_t^1 for the drift and trend, $f(0)$ is the spectral density at frequency cero and is the representation of heteroskedasticity and autocorrelation of the standard error.

Finally, the KPSS tests, formulated by Kwiatkowsky et al. (1992), states under the null hypothesis that the series is stationary. The model to estimate for this test is:

$$\begin{aligned} Y_t &= \mu_t + \epsilon_t \quad (11) \\ \mu_t &= \mu_{t-1} + \epsilon_t \\ \epsilon_t &\sim N(0, \sigma_\epsilon^2) \end{aligned}$$

Where $\boldsymbol{\mu_t}$ is a random walk with constant variance, the null hypothesis is then $\boldsymbol{\sigma_\epsilon^2} = \mathbf{0}$, meaning the random walk is constant. So that the statistic is specified as follows:

$$\boldsymbol{KPSS} = \frac{\boldsymbol{T^{-2}\sum_{t=1}^{T}\sum_{j=1}^{t}\hat{\epsilon}_t}^{\mathbf{2}}}{\boldsymbol{\hat{\lambda}^2}} \tag{12}$$

With the corresponding data set, the descriptive statistics to employ are the location, dispersion, and third and fourth moments. For the location parameter the arithmetic mean is computed, according to the financial literature, one expects this value to be close to cero. The next formula corresponds to the punctual estimation for the mean for $\boldsymbol{n}$ observations:

$$\boldsymbol{\mu} = \frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} \boldsymbol{x_i} \tag{13}$$

The dispersion statistics to employ are the variance ($\boldsymbol{\sigma^2}$), standard deviation ($\boldsymbol{\sigma}$), the variation coefficient ($\boldsymbol{cv}$). Although the only restriction for the variance is to be finite and constant, the simple statistic related dispersion is a function that maps $\mathbb{R}^{\boldsymbol{n}} \to \mathbb{R}_+ \cup \mathbf{0}$ so the variability in the parameter through time is not fully observable. The formulae to compute the statistics are:

$$\boldsymbol{\sigma^2} = \frac{\mathbf{1}}{\boldsymbol{n-1}}\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} (\boldsymbol{x_i} - \boldsymbol{\mu})^{\mathbf{2}} \tag{14}$$

$$\boldsymbol{\sigma} = \sqrt{\frac{\mathbf{1}}{\boldsymbol{n-1}}\sum_{\boldsymbol{i=1}}^{\boldsymbol{n}} (\boldsymbol{x_i} - \boldsymbol{\mu})^{\mathbf{2}}} \tag{15}$$

$$\boldsymbol{cv} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\mu}} \quad \boldsymbol{for} \;\; \boldsymbol{\mu} \neq \mathbf{0} \tag{16}$$

The moments statistics are mainly two and represent some properties of the data that conventional models are unable to incorporate. The skewness ($\boldsymbol{s}$) and kurtosis ($\boldsymbol{k}$) refer to the third and fourth moment, respectively. The skewness may be interpreted as the mass concentration in comparison with the mean. This means that data tends to be higher (lower) than mean when the statistic is positive (negative). Meanwhile kurtosis takes positive values and is often interpreted as the tail concentration of the distribution, i.e. a higher statistic represents more data away from the mean. The formulae to compute them are shown below:

$$\boldsymbol{s} = \frac{\mathbf{1}}{\boldsymbol{\sigma}^{\mathbf{3}}}\left(\frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}}(\boldsymbol{x}_{\boldsymbol{i}} - \boldsymbol{\mu})^{\mathbf{3}}\right) \tag{17}$$

$$\boldsymbol{k} = \frac{\mathbf{1}}{\boldsymbol{\sigma}^{\mathbf{4}}}\left(\frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}}(\boldsymbol{x}_{\boldsymbol{i}} - \boldsymbol{\mu})^{\mathbf{4}}\right) \tag{18}$$

The kurtosis for a standard Gaussian distribution is $\mathbf{3}$, so the excess kurtosis is often preferred as a comparison with the distribution, so the formula turns to:

$$\boldsymbol{k}* = \frac{\mathbf{1}}{\boldsymbol{\sigma}^{\mathbf{4}}}\left(\frac{\mathbf{1}}{\boldsymbol{n}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}}(\boldsymbol{x}_{\boldsymbol{i}} - \boldsymbol{\mu})^{\mathbf{4}}\right) - \mathbf{3} \tag{19}$$

Nevertheless, the fixed estimation of the parameters does not show the movements and changes through time that are able to change the intrinsic behavior of the series. To address the issue, the rolling window procedure is employed with 120 days equivalent to approximately 4 months as data is available for every day of the week, so the possible changes and presence of heteroskedasticity in return series may be capture.

Furthermore, a non-parametric procedure will be applied to the return series that enable to compare the kernel density estimation with the estimated Gaussian distribution corresponding to each of the cryptocurrencies. As stated by Cont (2001) these techniques provide only the qualitative analysis; nonetheless, this enable the possibility to visually determine the behavior of the information. The first technique to use is the histogram to plot the general distribution of returns. This procedure permits to account for the quantity of observations that relies in certain interval. Next the Gaussian distribution is calibrated with the corresponding data and is over-plotted to show the difference in the standard procedure and the empirical data. The density function of normal density is defined as:

$$\boldsymbol{f}(\boldsymbol{x}) = \frac{\mathbf{1}}{\boldsymbol{\sigma}\sqrt{\mathbf{2}\boldsymbol{\pi}}}\boldsymbol{e}^{-\frac{(\boldsymbol{x}-\boldsymbol{\mu})^2}{2\boldsymbol{\sigma}^2}} \tag{20}$$

With domain $\boldsymbol{x} \in \mathbb{R}$. This distribution is then completely characterized with mean and variance estimators, which ignores the higher moments of skewness and kurtosis.

Another useful non-parametric technique is the kernel density estimation. It assumes the unknown probability density function of a random variable and then considers a weighted sum of functions. The most popular approach is to use the normal density such that the Kernel Gaussian density is defined as:

$$\hat{\boldsymbol{f}}(\boldsymbol{x}) = \frac{\mathbf{1}}{\boldsymbol{N}}\sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{N}}\frac{\mathbf{1}}{\boldsymbol{\tau}\sqrt{\mathbf{2}\boldsymbol{\pi}}}\boldsymbol{e}^{-\frac{(\boldsymbol{x}-\boldsymbol{\mu})^2}{2\boldsymbol{\tau}^2}} \tag{21}$$

Where $\boldsymbol{N}$ is the number of independent realizations of the density and $\boldsymbol{\tau}$ is the scale parameter, also referred as the bandwidth. The choose of this number is crucial and can change the complete estimation of the density. Nevertheless, it provides a smoothing technique to capture the qualitative shape of the distribution.

2.3. Results

The first analysis to perform is a visual representation of the series. Figure 1 contains the information of the available data for Bitcoin, the graphs for the rest of the cryptocurrencies are displayed in Annex 1. The first thing to notice in the series is the range of the values for price series. The minimum value reported is 178.103 USD per Bitcoin, occurred in 2015/01/14; while the maximum value was observed in 2017/12/16 with a price of 19,497.4 USD per Bitcoin. This translates to a range of 19,319.3 or a percentual change of 10,847.26% in 1067 days. In the daily return series, the minimum value was in the same day of the minimum value for price and corresponded to a fall of -23.75%, while the maximum increase was 22.51% in 2017/12/07.

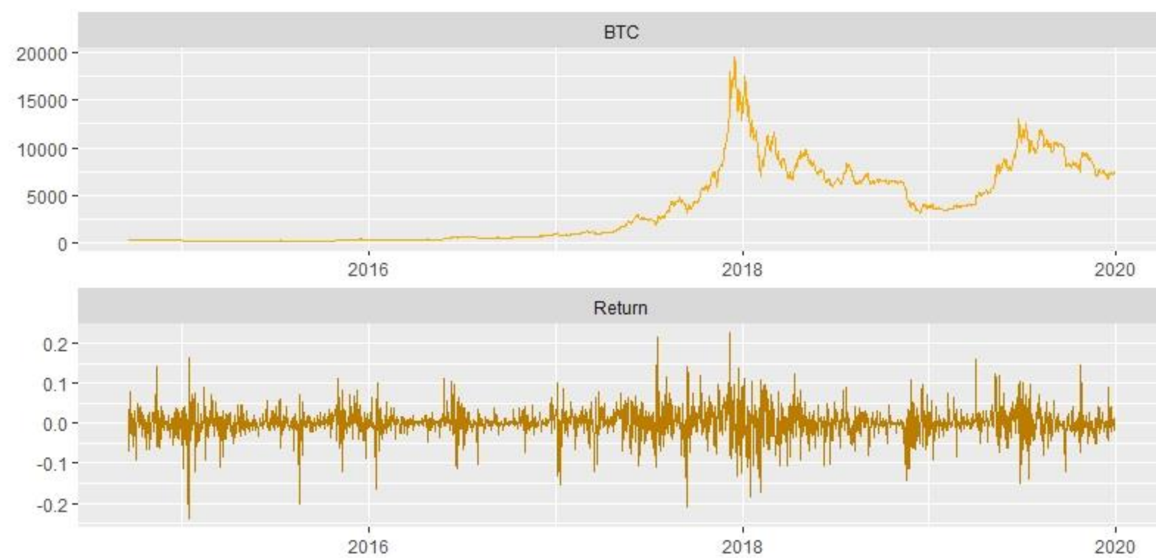


Fig.1 Bitcoin prices and returns

As mentioned before, the stationary tests must be performed over the prices and the returns of the series. To accomplish the robustness, the four statistical models are applied to the series. Table 2 and 3 presents the p-values for them.

Table 2. Stationary tests for prices

Cryptocurrency	ADF	PP	Ng-Perron	KPSS
Bitcoin (BTC)	0.3644	0.5018	0.1777	0.0000
Etherum (ETH)	0.2411	0.3696	0.4057	0.0000
XRP	0.0103	0.0070	0.0098	0.0000
Bitcoin Cash (BCH)	0.1562	0.2287	0.0990	0.0000
Bitcoin SV (BSV)	0.1315	0.1490	0.7420	0.0000
Litecoin (LTC)	0.0877	0.1418	0.1088	0.0000
Binance Coin (BNB)	0.3913	0.3700	0.8058	0.0000
EOS	0.1338	0.1825	0.3979	0.0000
Tezos (XTZ)	0.3568	0.1344	0.2621	0.0000
Stellar (XLM)	0.0346	0.0884	0.0564	0.0000

p-values for each of the statistical tests where the null hypothesis for ADF, PP and Ng-Perron is unit root and for KPSS is stationary series.
Elaborated by author with information of coinmarketcap.com

In the tests over the price series the KPSS under the null hypothesis of stationarity is rejected over the ten cryptocurrencies series. The other three criteria are consistent with this result over the complementary hypothesis for eight of them at the 90% confidence degree. The data that presents certain stationary behavior is the XRP – previously known as Ripple – and the Stellar. Both have the same length as Bitcoin, meaning that the same explosive behavior episodes are measured over the series. So, it shows that although some studies suggest a strong correlation among cryptocurrencies (Coreli, 2018), each one has its own statistical properties to address. If the confidence degree is raised over 99%, then Stellar exhibits the same non-stationary behavior as the rest of the data, while the XRP rejects the unitary root hypothesis with the PP and Ng-Perron tests. This leads to consider that the first difference is a necessary procedure to obtain the desired statistical properties.

Table 3. Stationary tests for returns

Cryptocurrency	ADF	PP	Ng-Perron	KPSS
Bitcoin (BTC)	0.0000	0.0001	0.0001	0.1043
Etherum (ETH)	0.0000	0.0000	0.0001	0.1793
XRP	0.0000	0.0001	0.0001	0.2193
Bitcoin Cash (BCH)	0.0000	0.0000	0.0001	0.7669
Bitcoin SV (BSV)	0.0000	0.0000	0.1573	0.8513
Litecoin (LTC)	0.0000	0.0000	0.0001	0.4005
Binance Coin (BNB)	0.0000	0.0000	0.0001	0.0383
EOS	0.0000	0.0000	0.0001	0.7068
Tezos (XTZ)	0.0000	0.0000	0.0001	0.8836
Stellar (XLM)	0.0000	0.0000	0.0001	0.4348

p-values for each of the statistical tests where the null hypothesis for ADF, PP and Ng-Perron is unit root and for KPSS is stationary series.
Elaborated by author with information of coinmarketcap.com

For the return series, the KPSS statistical test does not reject the stationarity hypothesis for nine of the series, while the Binance Coin does not reject with a 99% confidence degree. The ADF and PP tests rejects the null hypothesis of explosive behavior for all the series; nevertheless Ng-Perron does not reject to the Bitcoin SV. In sum, this procedure provides evidence that logarithmic returns must be the series employed for the statistical analysis as the ergodic and stationary test are fulfilled. Furthermore, the diffusion models are expressed as an exponential random variable, which guarantees the no-negativity in prices.

Table 4 contains the descriptive statistics for the return series for each of the cryptocurrencies. The location parameter may be interpreted as close to cero, which coincides with the Exponential Brownian Motion assumption. For the Standard Deviation it is notable that Bitcoin has the lowest one; meanwhile, the series with the higher ones corresponds with the younger cryptocurrencies (Bitcoin SV, EOS, Bitcoin Cash and Tezos). These facts highlight that the starting dates of the observations coincides with the peak in value for Bitcoin and according with Polasik et al. (2016) are also the dates in which the word “Bitcoin” was search the most. This strengthens the reasoning behind that the presence of exacerbated stylized facts in cryptocurrencies is due to the popularity and incoming to the market of behavioral traders who are heavily influence by news and technical analysis.

The moment statistics represented in the skewness and excess kurtosis statistics shows disparities among cryptocurrencies. In a first glance the positive skewness in certain series such as EOS and Stellar, which are some of the younger ones, provides a distinct result than expected by the stylized facts in financial series that present negative values. Furthermore, the series that resemble conventional statistical property are Bitcoin and Etherum, corresponding with the most popular and traded ones. Once again, these dissimilarities could be the consequence of certain maturity in the Bitcoin and Etherum markets, where the assets are starting to be used by financial institutions. Nevertheless, the excess kurtosis shows that Bitcoin is the one with lightest tails, being Etherum the one with the highest.

Table 4. Descriptive statistics for returns

Cryptocurrency	Mean	Standard Deviation	Skewness	Kurtosis*
Bitcoin (BTC)	0.00146641	0.03856164	-0.2772822	5.27664771
Etherum (ETH)	0.00239261	0.07139593	-3.4344526	71.8112011
XRP	0.00185185	0.06622767	2.9588184	43.5398353
Bitcoin Cash (BCH)	-0.0007884	0.07935709	0.6179772	7.4096881
Bitcoin SV (BSV)	0.00082748	0.09011341	0.86428111	16.9132264
Litecoin (LTC)	0.00108791	0.0568552	0.71163071	13.9008154
Binance Coin (BNB)	0.00547239	0.0786623	1.3856036	12.1835314
EOS	0.00103109	0.08278968	2.23417506	24.8183026
Tezos (XTZ)	-0.0003842	0.07512883	0.12554618	7.53955864
Stellar (XLM)	0.00133239	0.07493491	2.11009299	17.3195915

Descriptive statistics for return series of Cryptocurrencyes

Elaborated by author with information of coinmarketcap.com

*The statistic is the excess kurtosis

As mention earlier, the problem with these punctual statistics is that the parameters are computed with the whole series. Nevertheless, those estimations may change through time, leading to a time dependent estimation. One way to obtain the variability is with a rolling window procedure. With this technique a constant amount of data is used to calculate the statistics. Each new observation lead to a drop of the last one, permitting to capture the change in the value of the parameters. In a strict way, it would be expected that the variability of such parameters to be small. However, with the structural changes in the series, the population parameter may change, leading to a shift in the mean value of the estimator. Furthermore, in financial series the volatility clustering is common, so the rolling window procedure these sudden increases in volatility values could be captured; changes that may be interpreted as a change in volatility or heteroskedasticity. Figures 2 – 5 present the graphical representation of the mean, standard deviation, skewness and kurtosis, respectively with a rolling window of length 120, representing 4 months of information. Skewness and kurtosis plot a horizontal line with value of 0 to represent the normality benchmark.

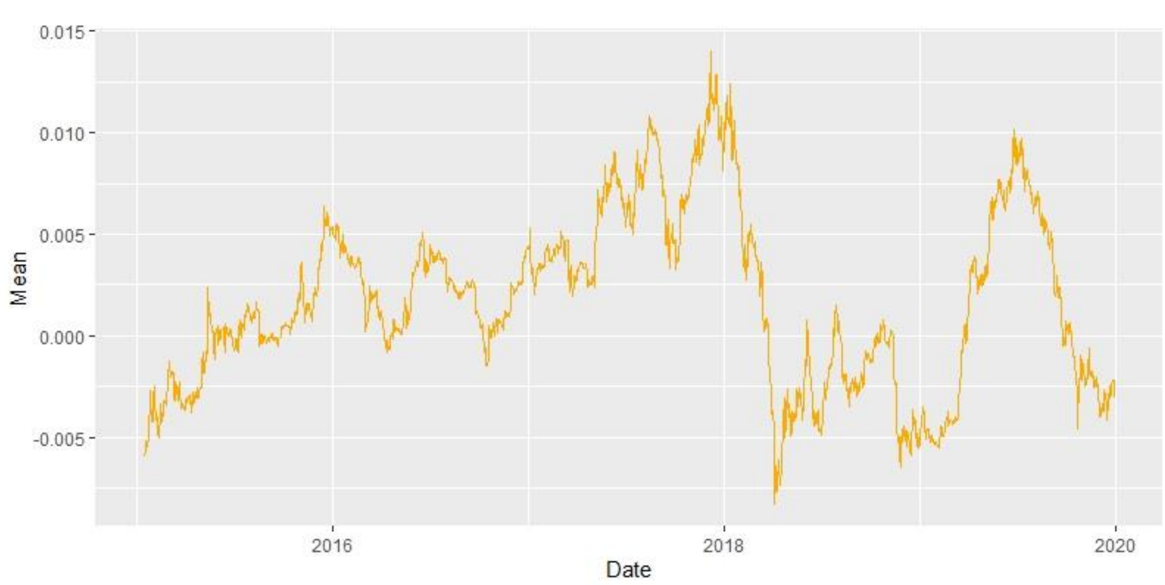


Fig.2 Bitcoin return´s mean 120 days

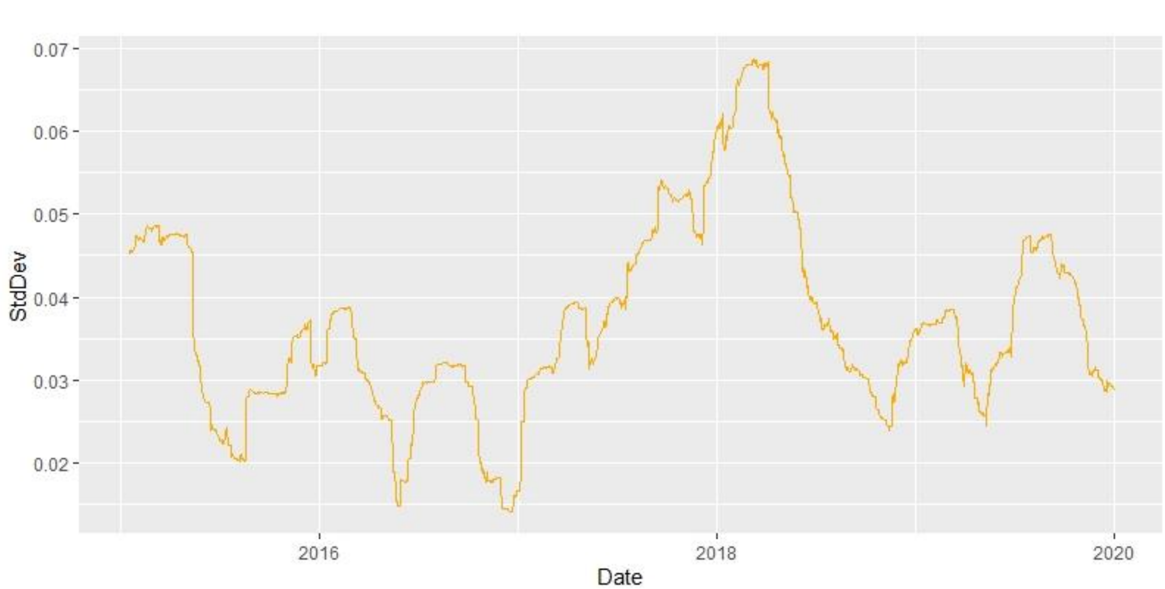


Fig.3 Bitcoin return´s Standard Deviation 120 days

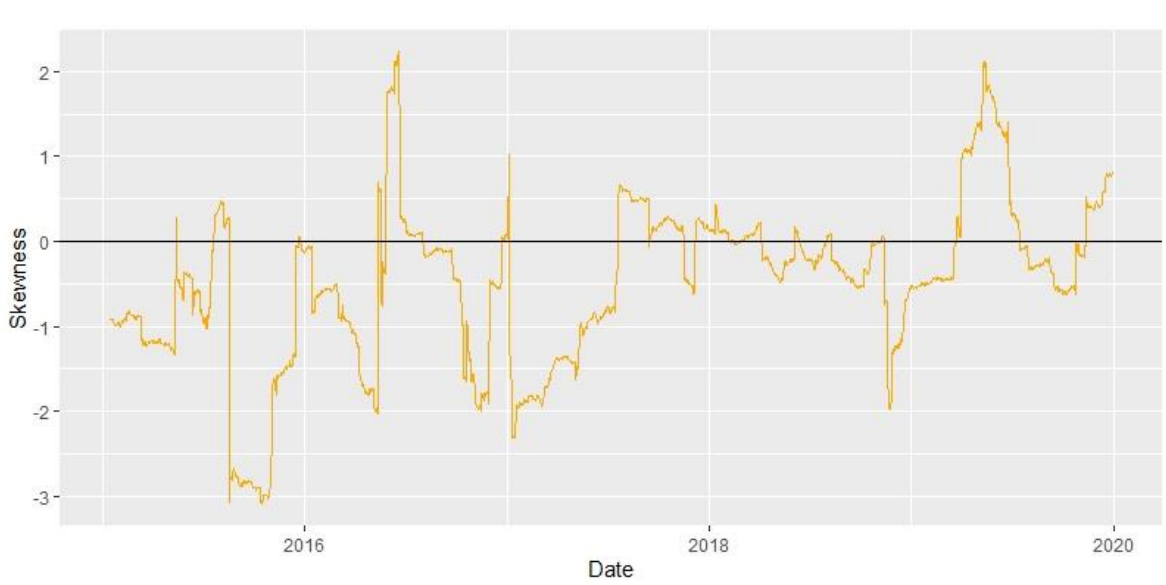


Fig.4 Bitcoin return´s skewness 120 days

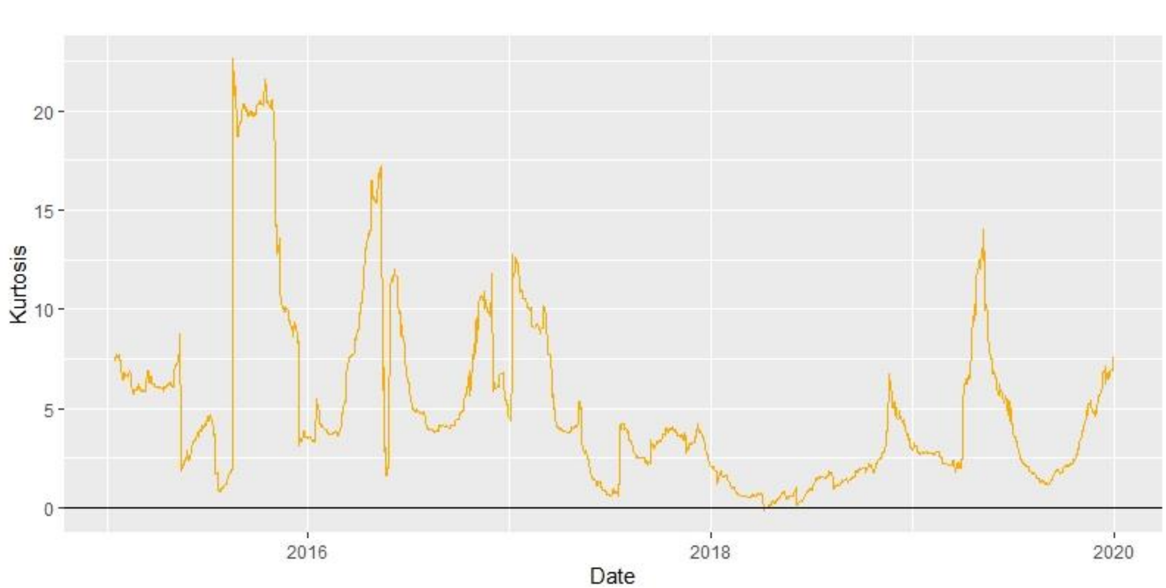


Fig.5 Bitcoin return´s kurtosis 120 days

The mean parameter has an oscillation movement around zero; nonetheless, the year 2017 presented an increase in the values of the mean parameter, coinciding with the maximum values in prices and returns. With the abrupt fall for the same period as the collapse, leading to consider a bubble episode for at least this period. This topic will be address in Chapter 3. In the standard deviation parameter, it is notable that the increase and highest values are present after the collapse of the bubble, showing the effect of that event moving the expectations of the market. Furthermore, there are other episodes in which the volatility, as well as the mean, has similar behaviors. Such performances may have been caused by other bubble episodes, characterizing the Bitcoin series as a heavily speculative asset.

The moment statistics show the parameter variability in the same periods as described before. For the skewness it is notable that, for most of the series, the data presents a negative value, while there are major increases prior to the collapses in prices. Meanwhile, the excess kurtosis shows big spreads from the normal estimation unless the periods later the collapse of the bubble. The usage of the rolling window permits to identify the change in parameters; however, in a risk and asset allocation perspective, the main issue relies in the density estimation.

Figure 6 depicts the over plot of three functions: 1) histogram (dark orange), 2) kernel density (orange) and 3) Gaussian density estimation (gray) with the mean and variance from Bitcoin´s return. The equivalent plots for the rest cryptocurrencies are shown in Annex 1. As mention before, these analyses correspond to qualitative ones; nevertheless, it permits to identify certain characteristics of the data. The plot shows the discrepancy between the standard procedure and the empirical data. The first thing to notice is the sharpness of the density in the location parameter, leading to interpret visually the excess kurtosis. Likewise, in the left tail of the bell shape, it is notable the mass concentration, meaning the negative skewness detected before. Figure 7 presents a zoom over this interval. It highlights the fact that Gaussian distribution is not able to capture the presence of heavy tails, leading to a sub-estimation in the probability to observe such downfalls. These plots show the necessity exposed by Dimitrova et al. (2019) to study the distribution of this new financial asset. As well, the conclusions made by Senarathne (2019) who shows that GARCH family models with the Gaussian distribution are unable to model the volatility in the series.

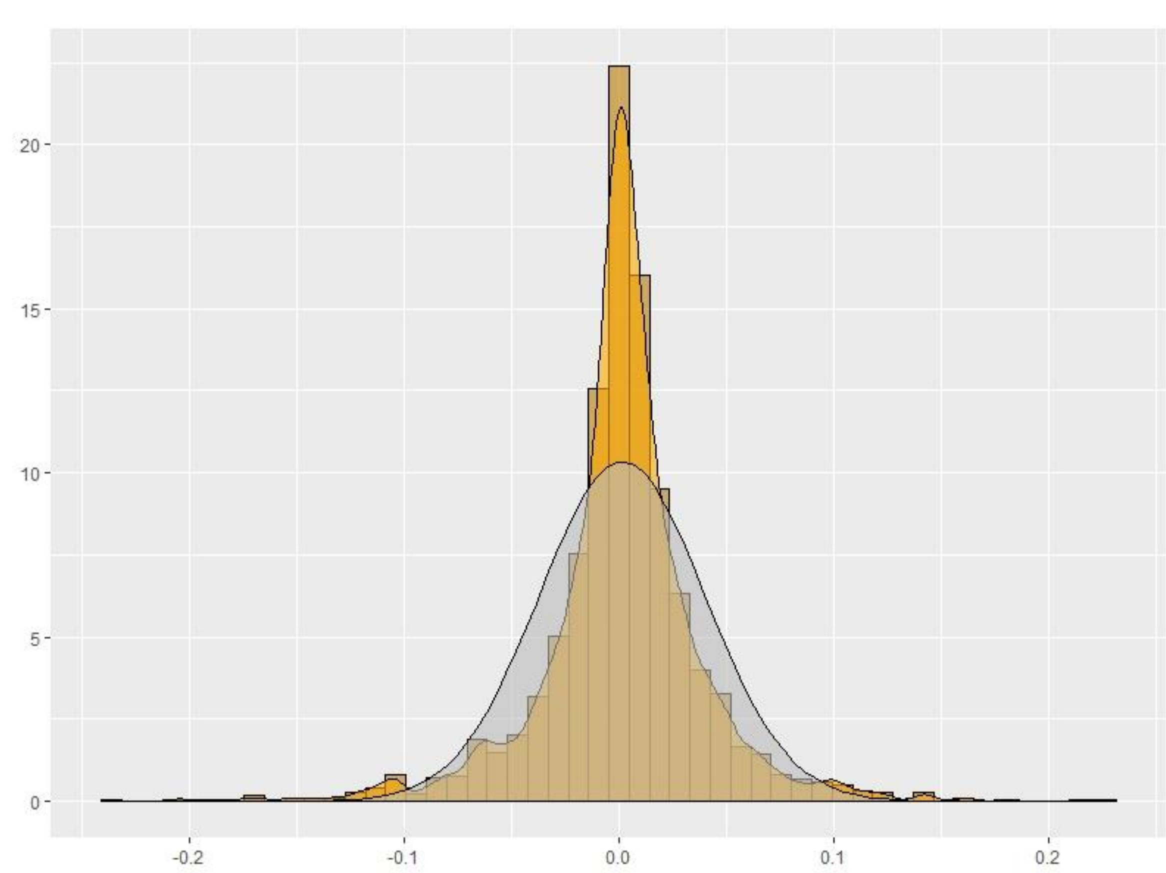


Fig.6 Bitcoin return density

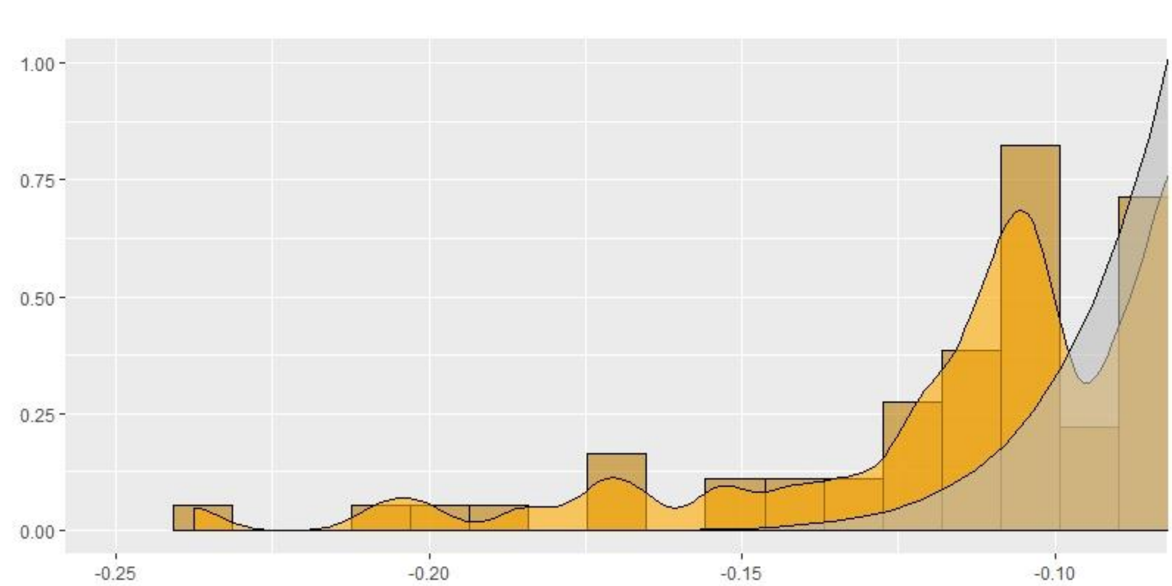


Fig.7 Bitcoin return density left tail

2.4. Conclusions

Unlike other disciplines that use time series to study certain phenomenon, the social sciences have the extra component of human factor. Natural world is governed by constant and well-behaved distributions that allow for certain kind of predictability. Nevertheless, human interactions create random variables that are difficult to measure with the same methodologies as nature. The so-called behavioral biases permit that people act with their own logic and maximize utility functions with different variables. In the financial markets this is translated to abnormalities in the behavior of prices and returns.

Under the nomenclature of stylized facts, the properties of financial series have gathered certain statistical properties shared by most of the series in greater or lesser measure. In this chapter the statistical analysis of the series from the ten most capitalized cryptocurrencies was deployed. The results show that, although they were not conceptualized as such, they behave as any other financial asset. The important part is that those properties have an escalated factor in a relatively young market. Furthermore, the almost exponential value growth in Bitcoin shows that this new asset, whose intrinsic value is cero (Rus et al., 2018, p 45), have manage to become a study object like any before.

Previous studies show that price is linked with news and social media information, and its mere fundamentals make that the demand side of the market be the one that determines it. In conjunction with the democratized technology that allows almost every person to have access to Bitcoin and to trade with it in financial markets, leads to a great dependence to behavioral biases. Such scenario makes Bitcoin market the perfect place to observe abrupt movements in prices, translated to extreme values in returns, positive and negative. This speculative component is also a reason to consider the presence of bubble episodes. Although they are difficult to determine, bubbles are characterized by an unusual increase in price, in most cases a value far away from its intrinsic value, followed by a collapse. Chapter 3 will discuss this topic and study through a statistic test the appearance of such episodes in cryptocurrencies markets.

Chapter 3: Explosion in Cryptocurrencies

3.1. Introduction

The cryptocurrency phenomenon that started Bitcoin caused a revolution in cryptography and the possibility of new fiat money under a decentralized scheme. According with Zimmer (2017) this event is possible because of the conjunction of two main elements: the globalization and the computational development. Additionally, Kloter et al. (2010, p 27-33) mention that this is a cooperation era, allowing people to participate in the co-creation and expansion of new technologies. Also, Zimmer (2017) states that the value of these new assets is given because of high tech scarcity. This statement corresponds with the fundamental value of Bitcoin that responds to Mises´ concept for value of money, where expectation from people moves the universal agreement of the value. However, the fact that there is no universal consensus on what cryptocurrencies are may cause the fundamental reason behind the speculation and possible appearance of bubbles.

This possibility is explored by Fry and Cheah (2016) who present that the volatility and exuberant price increases followed by an abrupt decrease are caused by the realization of self-fulfilling expectations. Another fact to include in the possible changes in prices refer to the lack of regulations and standardization from Central Banks and Financial Institutions. The impossibility to transform cryptocurrencies into goods and services in real economy, as occur with the low transaction costs associated with fiat money, promotes the conceptualization of these assets as a mere speculation tool sensible to the capability to store value of another currency, or in the extreme scenario as a speculation vehicle, creating the conditions for the appearance of bubbles.

One of the definitions for bubble episodes is given by Campbell et al (1996). They conceptualize it as the divergence of the price from the fundamental value; also, bubbles may cause damage to economic activities if there is a contagion to other markets. Nevertheless, bubbles are difficult to detect until it burst as stated by

Greenspan (2002) who points out the issue detecting if the diverge is due to a change in fundamental value or just because speculation. Joining the definition with the statement of Rus et al. (2018, p 45) that fundamental value of Bitcoin is zero, then any movement could be interpreted as a bubble episode. Nevertheless, such extreme conclusion does not work outside the utilitarianism perspective as cryptocurrencies main functionality is to work as a mean of payment with low transaction costs (Frisby, 2014).

Another reason to consider the possible presence of bubbles in cryptocurrencies markets is the reasoning presented by Brunnermeier and Oehmke (2012). They state that technological changes, that may be interpreted as innovations to previous mechanism, have the capability to create disturbances and then the conditions for bubble episodes until the shock is diluted. Furthermore, Caginalp et al. (2001) state that accessibility to data, information and media does nothing to reduce the risk of divergencies in prices. This context is what is witness in Bitcoin market: it is the first decentralized mean of payment working in a public network and relying in cryptography to provide value. Additionally, the relative democratization of technology and information flow allow people to use and operate these assets with ease, promoting the high volatility clustering, extreme movements and in some instances bubble episodes.

One of the first works on this behalf is presented by Gringerg (2011). His paper develops on the relationship between trust and the emerge of bubble episodes. This relates with the impact generated by government policies to prohibit the usage of cryptocurrencies, hacking, privacy issues and the fundamental deflationary characteristic leads to shifts in demand for the asset. Godsiff (2015) follows this idea and compares the movements in Bitcoin price with the tulip crisis, this conducts him to the conclusion that bubbles in Bitcoin have been created because of social interaction in forums and Google searches. Likewise, Thum (2018) points out that historically unusual behavior creates uncertainty within the agents, causing the apparition volatility and speculative bubbles.

Yermack (2015) provides a study regarding Bitcoin´s volatility to determine if it is possible to store value. The conclusion in this topic determines that because of the extreme changes, it resembles a speculative asset subject to bubble episodes. Bouoiyour and Selmi (2015) provide a GARCH approach and gets that news provide additional volatility to the markets. This reasoning is taken by Cheah and Fry (2015) to develop an econophysic methodology to detect bubbles in the cryptocurrency market. Their results show that the speculative element is part of the natural movement in the price; furthermore, they provide evidence that the fundamental value of Bitcoin is cero.

Zheng-Zheng et al. (2018) provide a weekly analysis of the Bitcoin series in USD and CNY to detect multiple bubble episodes using the technique proposed by Phillips et al. (2015). Their premise is that there exist significant discrepancies in the Chinese and American exchange rates regarding Bitcoin. Afterwards they use the so-called Generalized Augmented Dickey Fuller (GSADF) criteria under the original formulation to detect the intervals with this extreme behavior. Their results show the presence of six bubbles in Chinese market and five in the American one. Also, their conclusions state that cryptocurrencies are susceptible to exogenous shocks; the international economic events cause long term bubbles, while local events are the reason behind short term episodes.

In this chapter the statistical tests for single and multiple bubble episodes proposed by Phillips et al. (2011, 2015) with the corrections made by Harvey et al. (2016) are presented for Bitcoin. The results differentiate the previous ones in the actualization of the series, the use of daily data – that cause an increase in computational requirements – and the improvements suggested in later works.

3.2. Methodology

The techniques to use corresponds with the proposals presented by Phillips et al. (2011), also known as PWY, and Phillips et al. (2015) or PSY. The seminal work related with the time series analysis to detect the exuberant behavior described for bubbles traces back to Diba and Grossman (1988). They present the idea that stationary tests – like the ones presented in chapter 1 – may be used over stock prices to detect periods with a greater explosive behavior than others. The issue with this procedure was the burst episode. The problem was addressed by Phillips et al. (2011), who presented the first functional procedure to detect bubble episodes. The idea is to perform a recursive and rolling right-tail ADF test to detect the start and end of the episode. The null hypothesis is constructed under the unit root presence.

The premise of the model is the notion of the so-called *rational bubble*; the name is due to the rational expectation assumption that permit the existence of a bubble episode. This phenomenon occurs because a divergence in price from its fundamental value cause agents to expect a greater compensation or profit from this event.

The general procedure is based in the ADF autoregressive model:

$$\boldsymbol{Y_t = \phi Y_{t-1} + \sum_{j=1}^{p} \psi_j \Delta Y_{t-j} + \epsilon_t} \tag{22}$$

The original hypothesis remains as exposed in chapter 1. The difference starts with the recursive application of the test over a rolling window of constant length $\boldsymbol{r_w}$, the second difference is the change in the critical values as they are now constructed over the right-tail of the statistic distribution. This model receives the name of RADF and the statistic as Supremum ADF (SADF). This methodology is presented as PWY and it provides a recursive calculation of RADF over the time series of prices. The output of the model is then the SADF statistic defined as:

$$\boldsymbol{SADF(r_w) = \sup_{r_2 \in [r_w, 1]} \{ADF_{r_2}\}} \tag{23}$$

Where $\boldsymbol{r_w}$ is the size of the window, $\boldsymbol{r_0}$ is the starting value in the normalized interval $[\boldsymbol{0,1}]$ and $\boldsymbol{r_2}$ is the end value of the fist iteration. This means that from a starting point, the sample is increased one observation at a time until $\boldsymbol{r_2 = 1}$.

Nevertheless, Phillips et al. (2015) present the generalized SADF (GSADF), also known as PSY model. This presents a recursive estimation moving the starting point i.e. a rolling SADF procedure. According to the authors the PSY is superior on detecting multiple bubble episodes over a price series. The GSADF statistic is defined as:

$$\boldsymbol{GSADF(r_w) = \sup_{\substack{r_2 \in [r_w, 1] \\ r_1 \in [0, r_2 - r_w]}} \{ADF_{r_1}^{r_2}\}} \tag{24}$$

The backward sup ADF (BSADF) is the used statistic to detect the bubble episodes and is related with the GSADF as follows:

$$\boldsymbol{GSADF(r_w) = \sup_{r_2 \in [r_w, 1]} \{BSADF_{r_2}(r_w)\}} \tag{25}$$

Now that the statistic series is generated, it is possible to introduce a date stamping procedure to determine the start and end of a bubble episode. For the PWY methodology, each of the SADF values is compared with the corresponding right-tailed critical values for the ADF statistic. The starting value of the episode is characterized as the moment the SADF statistic crosses from below the critical values; while the end is the moment it crosses from above. Expressed in an analytical way:

$$\boldsymbol{\hat{r}_s = \inf_{r_2 \in [r_w, 1]} \{r_2 : ADF_{r_2} > cv_{r_2}^{\beta_T}\}} \tag{26}$$

$$\hat{r}_e = \inf_{r_2 \in [\hat{r}_s, 1]} \left\{ r_2 : ADF_{r_2} < cv_{r_2}^{\beta_T} \right\} \quad (27)$$

Where the $cv_{r_2}^{\beta_T}$ is the critical value of ADF test based on the T observations.

The GSADF has the same interpretation for the initial and final date of the episode, described as:

$$\hat{r}_s = \inf_{r_2 \in [r_w, 1]} \left\{ r_2 : BSADF_{r_2}(r_w) > cv_{r_2}^{\beta_{T_{r_2}}} \right\} \quad (28)$$

$$\hat{r}_e = \inf_{r_2 \in [\hat{r}_s, 1]} \left\{ r_2 : BSADF_{r_2}(r_w) < cv_{r_2}^{\beta_{T_{r_2}}} \right\} \quad (29)$$

Where $cv_{r_2}^{\beta_{T_{r_2}}}$ is the critical value based on the T_{r_2} observations.

Harvey et al. (2016) provides a critique on the procedure made by Phillips et al. (2011). They state that structural changes in time series´ unconditional variance may lead to misinterpretation. The changes in unconditional volatility may be caused by bubbles and change in the speculative component of a series. Therefore, Harvey et al. (2015) propose a Bootstrap rather than Monte Carlo procedure to get the critical values, they state that provides robustness to the eventual existence of no-stationary volatility. In the Bitcoin scenario, this assumption fit the empirical evidence reported for the volatility studies discussed earlier, so the final procedure will include the PSY methodology with the Harvey correction proposed.

3.3. Results

The methodology of PWY and PSY with Monte Carlo and Bootstrap simulation for critical values is applied with 2,000 simulations. The first result is the statistical test for the presence of at least one exuberant episode in the series. The null hypothesis is that there is no presence of bubbles in the series. Results are shown in table 5 and 6.

Table 5. Statistical test for exuberant behavior Monte Carlo Critical Values

	SADF	**GSADF**
t-statistic	14.3***	14.3***
Critical values		
90%	1.29	2.20
95%	1.55	2.42
99%	2.04	2.91

Critical Values for SADF and GSADF statistical tests to detect bubble episodes under the Null Hypothesis of no presence of bubble episodes (Monte Carlo Simulation).

*Rejects null hypothesis under 90% confidence degree

**Rejects null hypothesis under 95% confidence degree

***Rejects null hypothesis under 99% confidence degree

Elaborated by authors

Table 6. Statistical test for exuberant behavior Bootstrap Critical Values

	SADF	**GSADF**
t-statistic	14.3**	14.3**
Critical values		
90%	10.5	10.5
95%	12.2	12.2
99%	15.7	15.7

Critical Values for SADF and GSADF statistical tests to detect bubble episodes under the Null Hypothesis of no presence of bubble episodes (Bootstrap Simulation).

*Rejects null hypothesis under 90% confidence degree

**Rejects null hypothesis under 95% confidence degree

***Rejects null hypothesis under 99% confidence degree

Elaborated by authors

These results suggest the presence of at least one episode with a 95% confidence degree. So, the next step is to identify them with the simulated values. Tables 7 and 8 present the number of bubbles detected, the initial and final date and the duration of each episode measured in days. To avoid spontaneous movements in prices, the minimal bubble duration was established to have at least 5 days. This number was chosen because Pankaj et al. (2019) provide evidence that weekdays have greater transactional volume than weekends, so the minimum life of a bubble is set to resemble one week of speculative behavior in agents. To improve the visualization of the test a graphical approach was taken. Figures 8 and 9 plots the BSADF test series (blue) and simulated critical values (red) with Monte Carlo and Bootstrap, respectively. In addition, a reescalation of prices and logarithmic returns are included to compare the behavior of those series with the statistics. Finally, the episodes are highlighted with a shadow to ease the identification.

Table 7. Dates for bubble episodes PSY

Bubble	**Begin**	**End**	**Duration (days)**
1	03/06/2016	09/06/2016	6
2	11/06/2016	21/06/2016	10
3	26/12/2016	05/01/2017	10
4	27/02/2017	08/03/2017	9
5	01/05/2017	14/07/2017	74
6	20/07/2017	26/07/2017	6
7	27/07/2017	14/09/2017	49
8	27/09/2017	11/01/2018	106
9	11/05/2019	17/05/2019	6
10	21/06/2019	27/06/2019	6

Starting and end period for bubble episodes with PSY methodology and Monte Carlo Simulation for Critical Values.

Elaborated by author

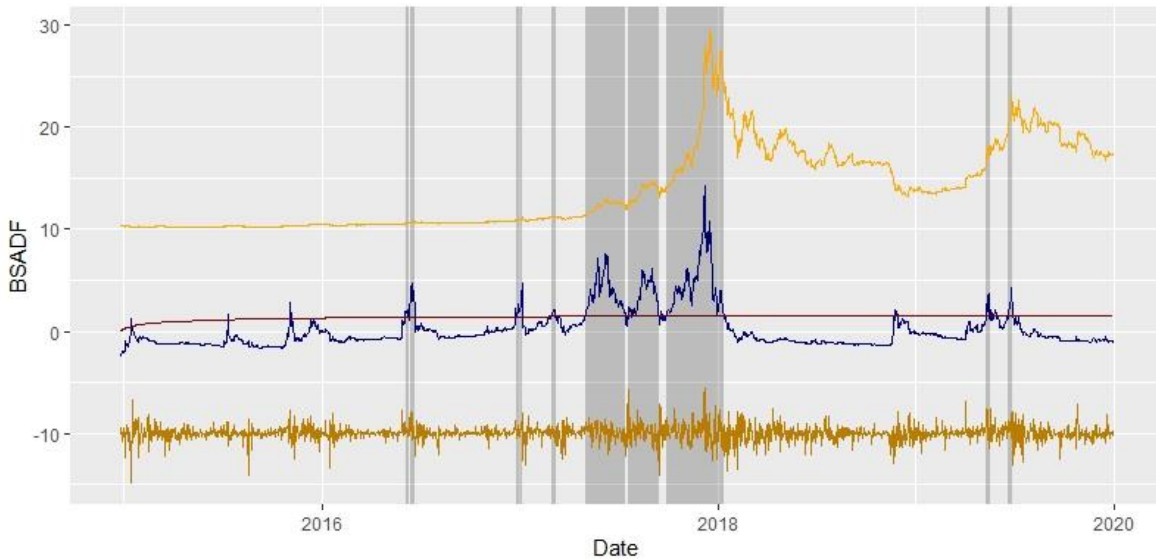


Fig.8 BSADF Monte Carlo Critical Values 95%

Table 8. Dates for bubble episodes PSY-Harvey

Bubble	Begin	End	Duration (days)
1	03/06/2016	21/06/2016	18
2	22/12/2016	05/01/2017	14
3	26/02/2017	08/03/2017	10
4	01/05/2017	14/06/2017	44
5	11/08/2017	08/09/2017	28
6	12/10/2017	24/10/2017	12
7	29/10/2017	11/11/2017	13
8	16/11/2017	09/12/2017	23
9	10/12/2017	20/12/2017	10
10	08/05/2019	17/05/2019	9
11	20/06/2019	27/06/2019	7

Starting and end period for bubble episodes with PSY methodology and Harvey correction with Bootstrap Simulated Critical Values.

Elaborated by authors

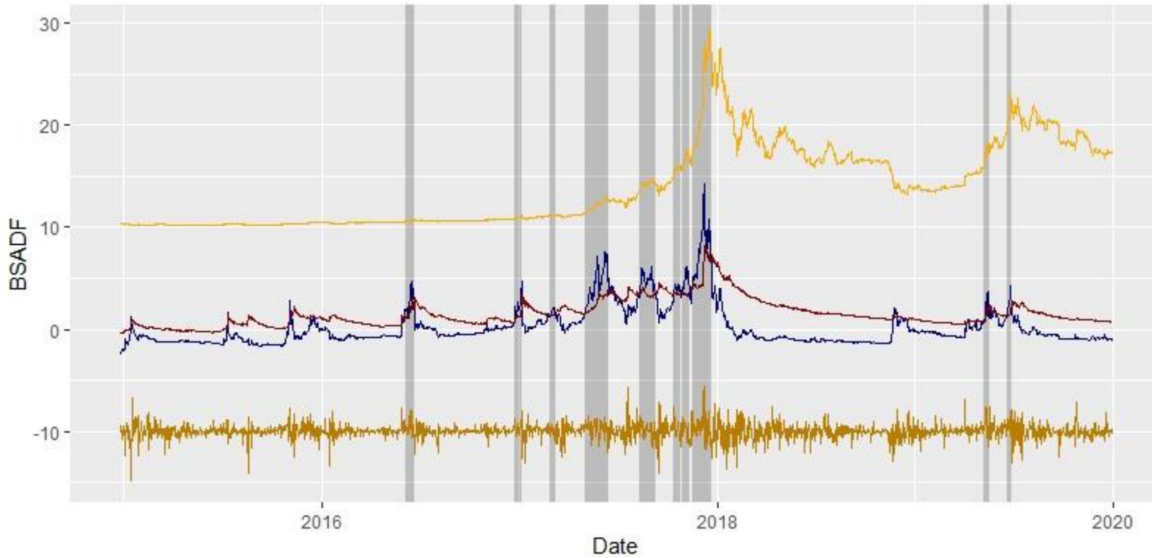


Fig.9 BSADF Bootstrap Critical Values 95%

With the periods identify, some transcendental news are listed below:

1. June 2016 Hong-Kong Bitcoin exchange Bitflex closed for few hours because network issues.
2. January 2017 China started investigations over Bitcoin for money laundering, market manipulation and unauthorized financing.
3. March 2017 China´s Central Bank announced that regulations on Bitcoin are permanent and Securities and Exchange Commission (SEC) denies the creation of a Bitcoin ETF
4. June 2017 Reports suggested that via a coordination with the cryptocurrency Tether there was price manipulation to artificially raise the price of Bitcoin.
5. September 2017 Chinese authorities order the closure of Bitcoin exchanges.
6. October and November 2017 The Chicago Board Options Exchange (CBOE) and Chicago Mercantile Exchange (CME) made the announcement of the imminent emission of Bitcoin Futures.
7. December 2017 CBOE and CME listed the Future Contracts and investors moved away from actual cryptocurrency, collapsing the demand for the asset
8. May 2019 A great euphoria reached Bitcoin investors and a bull period started; nevertheless, closure of big future contracts collapsed the price.

9. June 2019 The end of the bearish period lead price to increase rapidly among users; however, such drastic increase eventually drops the value.

3.4. Conclusions

In this chapter the bubble detection procedure proposed by Phillips et al. (2015) was applied to Bitcoin price series. This methodology managed to correctly identify some of the most important events regarding the cryptocurrency. With the listed results it is possible to identify 10 and 11 bubbles for the available data with the original Monte Carlo and the Bootstrap simulation, respectively. Also, for some episodes the dates of the burst are the same, specially in the last two, detected in mid-2019.

In the description of events that can create the exuberant episodes it is possible to detect a clear evolution. Fists events regard the government intervention and regulation – and in some instances the prohibition – of cryptocurrencies. The middle events, specially the mid-2017 are caused by exogenous price manipulation from privates, leading to the highest values reported. Nevertheless, the introduction of new derivative instruments in the form of Future Contracts lead to a shift of investor agents, willing to trade with the risk and returns of the asset but without using it. This led to a collapse in demand and an artificial over supply that dropped the price. Finally, the last two bubbles detected in 2019 relates with episodes of euphoria among traditional agents that create short bubble episodes.

Regarding the correction proposed by Harvey et al (2016), it is notable that with the Bootstrap simulations the procedure managed to detect one more bubble. Also, the statistical criterion shows that critical values under this methodology have higher robustness than Monte Carlo.

These results may be a motivation to further study the evolution of Bitcoin market regarding the structural changes and the perspectives of the agents that seems to have learned to identify abrupt movements in price as bubbles, promoting the burst in a short period of time. Also, this result gives robustness to consider that parameters and stylized facts have changed through time. Meaning the requirement to use actualized parameters to have certain flexibility at adapting to these shifts.

Additionally, the information of 2019 provides evidence that bubble episodes continue in Bitcoin markets. The sensibility to news has now changed to certain herd behavior that creates disruptions and eventually bubble episodes. It is important to point out that bubbles characterize for two components the abrupt increase in price, but the sudden downfall after certain period. In a risk management perspective this means the requirement to study the underlying distribution to have enough resources to face the collapse in value of a long position in cryptocurrencies.

Although the Bitcoin market have seen changes in volatility and certain extreme behaviors, it is evident that the statistical properties described in chapter 1 prevails. In such scenario, the traditional Gaussian distribution is characterized by only two parameters in location and dispersion, but the spectrum of statistical features showed in last chapters makes it clear that a more robust technique is required. Chapter 4 will address this topic with the implementation of Normal Inverse Gaussian (NIG) distribution into a Value at Risk (VaR) and Expected Shortfall (ES) procedure.

Chapter 4: Bitcoin´s Distribution

4.1. Introduction

In 2019 the cryptocurrencies achieved ten years in existence. Although their initial target were people who sought a release to the control of financial institutions and an alternative to conventional assets (Bouri et al., 2017) the popularity of this instruments in stock markets continuous to raise (Vo and Xu, 2017). The great achievement of Bitcoin during the economic turmoil created by the global crisis was to capitalize the general unrest and mistrust in government. This seek of different options lead certain people to use state of the art technologies (Buchholz et al., 2012).

One key element that captures the minds of people is the transparent mechanism behind the creation of new units. On the other side, 2009 proved that Central Authorities had the power to issue money in a virtually unlimited scheme. This sovereignty allows governments to implement monetary policy and provide the liquidity for the economy to function properly. Nonetheless, the financial rescues to the financial institutions, that in ultimate instance were the responsible for the crisis, lead people to mistrust these actions and the power of Central Authorities. Bitcoins maintain an algorithm that increases the number of units in a constant rate known by everyone. The algorithm cannot be change or altered by any user, providing certain consistency over the cryptocurrency. Nevertheless, this functionality carries the theme with the dependency over the demand side (Ciaian et al. 2016). On this behalf Bueno et al. (2017) state that assets with those characteristics' present high volatility.

Chapter 3 provides evidence that bubble episodes have occurred in Bitcoin series. However, by analyzing the chronology of events that caused the exuberant periods, it is notable that the market has witness certain evolution. It is possible to identify three main subsamples: 1) previous 2017 the series was affected by government communications and regulation or even prohibition of exchanges. 2) The 2017 expansion period in which, although manipulated, the price grew exponentially until the last day, when new instruments allowed the usage of Bitcoin without having to trade it directly. 3) Post 2017 subsample, at least in exuberant period, is characterized by certain stability, with divergent episodes that cause short period bubbles.

This transition in Bitcoin market maintains the general stylized facts described in Chapter 2. In addition, evidence from Chapter 3 confirms the requirement to analyze the distribution of returns in a risk management perspective. If this new short period bubbles will remain a constant phenomenon in the market, and the popularity of these instruments will continue among general population and financial institutions, then an appropriate model to quantify risk is needed. In such case, the heavy tail and skewness properties are key to model the risk.

On this regard, some authors have studied specifically the stylized facts in Bitcoin. Bariviera et al. (2017) perform a daily frequency analysis of Bitcoin from 2011 to 2017 and intraday for 2013 to 2016. Their findings conclude that volatility is ten times higher than currencies such as British Pound (GBP) and Euro (EUR), also they provide evidence that returns are not Gaussian. Similarly, Alvarez-Ramirez et al. (2018) studied series from 2013 to 2017, finding certain asymmetries in correlations for different periods. Their results also suggest the existence of fat tails, specially the left one. Zhang et al. (2018) expands the study to eight cryptocurrencies finding the presence of heavy tails, volatility clustering, long range dependence and no autocorrelation for returns.

These works, alongside the analysis performed in previous chapters, incentive the return´s distribution study. The aim of this Chapter is to propose, fit and compare theoretical distributions for the empirical data. Bubble episodes could be translated into return behavior as positive skewness for the exuberant period followed by a highly negative return, meaning heavy tail and skewness in empirical distributions.

According to Cont (2001) it is required a flexible distribution with at least four parameters that can capture these statistical properties.

4.2. Normal Inverse Gaussian Distribution

The seminal paper relating the Normal Inverse Gaussian (NIG) is presented by Barndorff-Nielsen (1977). This work is based on the observations provided by Bagnold (1954 and 1956) about the diameter of sand sediments that are carried by the wind. The data is then initially modelled trough a hyperbolic function, followed by the transformation into a density distribution, providing the name of Generalized Hyperbolic (GH) distribution. In this paper the author mentions that it is originally intended to model size distributions that uses Gaussian distribution as an approximation, but empirically shown other properties (p 401). The analytical expression is defined as:

$$\boldsymbol{f(x) = \frac{\left(\frac{\gamma}{\delta}\right)^{\lambda} \alpha^{\frac{1}{2}-\lambda}}{\sqrt{2\pi} K_{\lambda}(\delta\gamma)} (\delta^2 + (x-\mu)^2)^{\lambda-\frac{1}{2}} K_{\lambda-\frac{1}{2}}\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)} \qquad (30)$$

$$\boldsymbol{\gamma = \sqrt{\alpha^2 - \beta^2}}$$

With $\boldsymbol{K_v}$ the modified Bessel function of second kind order $\boldsymbol{v}$:

$$\boldsymbol{K_v(x) = \begin{cases} \frac{\pi \csc(\pi v)}{2}(I_{-v}(x) - I_v(x)) & if\ \ v \notin \mathbb{Z} \\ \lim_{\mu \to v} K_{\mu}(x) & if\ \ v \in \mathbb{Z} \end{cases}} \qquad (31)$$

And $\boldsymbol{I_v}$ the modified Bessel function of first kind order $\boldsymbol{v}$:

$$\boldsymbol{I_v = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+v+1)k!}\left(\frac{x}{2}\right)^{2k+v}} \qquad (32)$$

$$\Gamma(n) = \int_0^\infty u^{n-1} e^{-u} du \qquad (33)$$

For the domain $x \in \mathbb{R}$ and parameters defined as $\mu, \lambda \in \mathbb{R}$ and $\delta, \alpha > 0$ and $0 \leq |\beta| \leq \alpha$.

This means that GH distribution has five parameters, with the interpretation that α measures the flatness of the bell shape, i.e. the concentration of values around the location parameter μ. The skewness is measure with β, and δ works as a scale parameter. The fat tails or kurtosis level is controlled with the value of λ; nevertheless, trough the manipulation of this number, other distributions are defined. The NIG distribution is obtained when $\lambda = -\frac{1}{2}$, in this instance Jovan and Ahčan (2017 p 415) provide a parametrization for the NIG density function specially constructed for computational estimation:

$$f(x) = \frac{\alpha\delta}{\pi} e^{\delta\sqrt{\alpha^2-\beta^2}} \frac{K_1\left(\alpha\sqrt{\delta^2 + (x-\mu)^2}\right)}{\sqrt{\delta^2 + (x-\mu)^2}} e^{\beta(x-\mu)} \qquad (34)$$

Barndorff-Nielsen (1977) shows that a NIG random variable is a variance-mean mixture of Generalized Inverse Gaussian (GIG) distributions such that:

$$X = \mu + \beta Z + \sqrt{Z} Y \qquad (35)$$

Where $Z \sim N(0,1)$ and $Y \sim IG(\delta, \sqrt{\alpha^2 - \beta^2})$ are independent random variables.

In a subsequent work Barndorff-Nielsen (1997) present the moment generating function of NIG with:

$$M_{NIG}(u) = e^{\delta\left(\sqrt{\alpha^2-\beta^2} - \sqrt{\alpha^2-(\beta+u)^2}\right)} \qquad (36)$$

Then, the first four moments are:

$$\boldsymbol{M(X) = \mu + \delta \frac{\beta}{\sqrt{\alpha^2 - \beta^2}}} \tag{37}$$

$$\boldsymbol{V(X) = \delta \frac{\alpha^2}{\sqrt{\alpha^2 - \beta^2}^3}} \tag{38}$$

$$\boldsymbol{S(X) = 3 \frac{\beta}{\alpha\sqrt{\delta\sqrt{\alpha^2 - \beta^2}}}} \tag{39}$$

$$\boldsymbol{K(X) = 3 + 3\left(1 + 4\left(\frac{\beta}{\alpha}\right)^2\right)\frac{1}{\delta\sqrt{\alpha^2 - \beta^2}}} \tag{40}$$

Also, in this paper, it is proven an important property: it is close under convolution, i.e. $\boldsymbol{f(x;\alpha,\beta,\delta_1,\mu_1) * f(x;\alpha,\beta,\delta_2,\mu_2) = f(x;\alpha,\beta,\delta_1+\delta_2,\mu_1+\mu_2)}$. This provides the infinitely divisible property required for the Lévy processes, from which the Brownian Motion is a special case. This assures that the logarithmic increments for an exponential process maintain the same NIG distribution.

In finance, the original intention to use the GH family to model returns is the work of Eberlein and Keller (1995) who proposed a GH distribution to fit data of DAX index from 1989 to 1992. Their results provide evidence that such model outperforms the traditional Gaussian distribution. Future works like Eberlein and Prause (2000) and Rydberg (1999) confirms the capabilities for the GH distribution in capturing the stylized facts and improving risk estimations.

The studies regarding the capability of these distributions to fit empirical data of diverse assets are wide (Trejo et al. 2006; Nuñez et al., 2018; Shen et al. 2017). However, the first applications regarding cryptocurrencies is proposed by Joerg

(2017) who performs a statistical analysis of Bitcoin and other six series. The study uses data from 2013 to 2016 and confirms the presence of heavy tails proposing members of GH family are the best suited to model the data. A similar path is followed by Bueno et al. (2017) who used the complete series of Bitcoin (2013 to 2017) to fit different members of GH family and performs a Value at Risk analysis. Their conclusions state that GH distribution is the best due to the higher requirements achieved.

A deeper analysis is conducted by Chu et al. (2015) who use the Bitcoin USD exchange to deploy a series of statistical tests. Their study contains data from 2011 to 2014. The procedure consisted in the evaluation of 15 different distributions, from exponential and GH families, to fit the empirical observations. With a maximum likelihood criterion, they conclude that Gaussian distribution is the worst, while GH is the best. Similar results were obtained by Min et al. (2019) on the capability of this distribution to model cryptocurrencies´ returns. With these results it could seem apparent that the general case is the best option because of the extra parameter regarding the heavy tail component. However, additional considerations must be taken.

Barndorff-Nielsen (1997) exposes that NIG distribution manage to capture heavy-tailed behavior better than the GH, furthermore, it is the only member of the family to have the close under convolution and close under affine transformations property. From a computational perspective, the fixed λ permit to improve the calculation time because of the Bessel function. Additionally, Barndorff-Nielsen and Shepard (2001) provides the financial elements to conclude that the capabilities of NIG to model stock returns; likewise, the possible extension to risk management and portfolio constructions.

4.3. Data and Methodology

To provide a robust analysis of Bitcoin and the NIG capabilities to match the data a set of tests will be employed. First, the data to use is the exchange rate between Bitcoin and GBP, Japanese Yen (JPY), EUR, USD, Russian Ruble (RUB), CNY, Korean Won (KRW) and Canadian Dollar (CAD). The dates are from 18/09/2014 to 31/12/2019 with daily frequency, which gives 1931 observations. Second, the bubble episodes detected in Chapter 3 will be used as a classification regarding the type of speculative factor that caused the abnormalities in the series. Third, the in sample statistical test will consider goodness of fit criterions, but a final out-sample evaluation will the used in the bubble detected for the period of June 2019. To provide a complete analysis, three distributions will be tested: Gaussian as a benchmark, GH as the most flexible and NIG as the proposal.

To provide an adequate subsample selection, results from Chapter 3 will be used to divide the 1931 original observation into four samples. The criterion to use is that each one of them must have at least one bubble episode. Three of them works as in-sample periods and will be treated with a goodness of fit test. The fourth period contains the last observed bubble and is intended to be used as an out-sample test with risk management perspective. Table 9 contains the dates and number of observations.

Table 9. Sample specification

Sample	Start	End	Observations
1	18/09/2014	30/06/2016	652
2	01/07/2016	31/12/2017	549
3	01/01/2018	15/06/2019	531
Out	16/06/2019	31/12/2019	199

Starting and Ending dates for the subsamples to employ in NIG, Gaussian and GH empirical fit
Elaborated by author

The first part consists in evaluating the Gaussian assumption for the return distribution. The graphical approximation of Chapter 2 proves a visual representation of the mismatch between normal distribution and empirical data adjusted by a kernel density and histogram plot. Nevertheless, the goodness of fit statistical test will be applied to them. Thode (2002) provides the elements to perform such statistical evaluations for the Shapiro-Francia (SF), Lilliefors, Anderson Darling (AD), Cramer-von Mises (CVM) and Jarque-Bera (JB). All these statistical tests are constructed under the null hypothesis that data is distributed as Gaussian distributions. The reason for the quantity of tests is to improve the robustness of the results.

Shapiro and Francia (1972) propose an estimation for the Wilk-Shapiro $\boldsymbol{W}$ statistic in which the order statistics are considered independent for large sample sizes. In such case, they employ a least squares regression for sample statistics on expected values, so the modified statistic is defined as:

$$\boldsymbol{W} = \frac{(\boldsymbol{a} * \boldsymbol{x})^{2}}{\left((\boldsymbol{n}-\mathbf{1})\boldsymbol{\sigma}^{2}\right)} \tag{41}$$

$$\boldsymbol{a} *= \frac{\boldsymbol{w}^{t}}{\sqrt{\boldsymbol{w}^{t}\boldsymbol{w}}}$$

Where $\boldsymbol{w}$ is the vector of the order statistics. For the Gaussian density it is defined that the order statistics may be approximated by:

$$\boldsymbol{w}_{\boldsymbol{i}} = \boldsymbol{\Phi}^{-1}\left(\frac{\boldsymbol{i}-\mathbf{0.375}}{\boldsymbol{n}+\mathbf{0.25}}\right) \tag{42}$$

Where $\boldsymbol{\Phi}^{-1}(\boldsymbol{x})$ is the inverse Gaussian distribution.

The Kolmogorov-Smirnov tests are based on the differences between the empirical distribution function (EDF) and the empirical probability $\boldsymbol{p}_{\boldsymbol{i}}$. The statistics are defined as:

$$D^{+} = \max_{i=1,I,n}\left\{\frac{i}{n-p_i}\right\} \qquad (43)$$

$$D^{-} = \max_{i=1,\dots,n}\left\{\frac{p_i-(i-1)}{n}\right\} \qquad (44)$$

$$D = max\{D^{+}, D^{-}\} \qquad (45)$$

The Normality test is suggested by Lilliefors (1967) giving the table with the critical values of the D statistic.

Anderson and Darling (1952) define an EDF goodness of fit test defined by:

$$GF = n\int_{-\infty}^{\infty}[F_n(x)-F(x)]^2\psi\big(F(x)\big)dF(x) \qquad (46)$$

With $\psi\big(F(x)\big)$ a weight function, in their work they propose $\psi(p) = [p(1-p)]^{-1}$ resulting in the statistic:

$$A = -n - \frac{1}{n}\sum_{i=1}^{n}[2i-1][Ln(p_i)+Ln(1-p_{n-i+1})] \qquad (47)$$

A special case of the statistic when the weight function is defines as $\psi\big(F(x)\big) = 1$ gives the CVM test:

$$CVM = \frac{1}{12n} + \sum\left(p_i - \frac{2i-1}{2n}\right)^2 \qquad (48)$$

Finally, the JB test (Jarque and Bera, 1980) works with the skewness and kurtosis empirical statistic to contrast the Gaussian values:

$$JB = \frac{n}{6}\left(s^2 + \frac{1}{4}(k-3)^2\right) \tag{49}$$

With the Normality assumption testes, the next step is to adjust the parameters for NIG and GH distribution. This procedure is made with the maximization of the likelihood function with the Broyden–Fletcher–Goldfarb–Shanno (BFGS) algorithm. With it, the parameters for each of the data series may be obtained and then to simulate random variables with the specified distribution. Because of the specification of statistical goodness of fit tests, some modifications may be applied to Kolmogorov-Smirnov and Anderson-Darling, in order to accept two samples and compare them under the null hypothesis that both samples – empirical and simulated – have the same theoretical distribution. The additional test to include in this step is the Kruskall Wallis or QN criterion specified as:

$$QN = \frac{1}{\sigma^2}\sum_{i=1}^{k}\frac{(S_{iN} - n_i\mu)^2}{n_i} \tag{50}$$

Where S_{iN} is the sum of rank scores for the i^{th} sample.

Finally, for the out-sample series, the Value at Risk (VaR) is computed under the 95% and 99% confidence degree, as well as the Expected Shortfall (ES).

The VaR is defined as the potential loss given a level of probability α (J.P. Morgan, 1996 p 1), i.e. $P[X < VaR_\alpha] = \alpha$. This definition allows the possibility to use different techniques to evaluate the risk, such as Monte Carlo simulations with the three proposed distributions. Additionally, the Expected Shortfall specified by Basel Committee on Banking Supervision (2019, MAR33) as the expected loos given the data exceeded VaR threshold, i.e. $ES_\alpha = -E[X|X < q(\alpha)]$ where $q(\alpha)$ is the quantile function.

4.4. Results

The fist results are the parameters for the Gaussian distribution in the three periods and with the different exchange rates. Tables 10 – 12 contain the location and dispersion parameters. Also, the corresponding Normality tests are applied to the same information. The p-values under the null hypothesis of normality are displayed in Tables 13 – 15 in Annex 2. It is shown that the Gaussian assumption is rejected for all the data with 99% confidence degree. This result confirms with statistical basis the error of applying such distribution in models that require the underlying distribution as input.

The maximization algorithm for the likelihood function is deployed for the series and the resulting parameters for NIG distribution are depicted in Tables 16 – 18 in Annex 2. For the goodness of fit statistics, a random sample from the parameters obtained with the same length as the sample periods is generated. With those two inputs the test described in section 4.3 were applied. The results expressed in the p-values under the null-hypothesis that samples come from the same distribution are shown in Tables 19 – 21. For periods 1 and 3 the three tests confirm the goodness of fit for NIG distribution, nevertheless, period 3 for the CNY exchange with the KS test the hypothesis is rejected under a 95% confidence degree.

Period 2 of the whole sample consist of the information containing the major bubble episodes registered. This extreme behavior for the different exchange rates is capture with the NIG distribution, the only series that showed certain discrepancies was the CNY. However, this rejection of the null hypothesis happened in one single test. For the rest of the information it is proved that the stylized facts can be modeled for the different periods, containing a variety of speculation sources.

Table 19. NIG goodness of fit statistics for period 1

Series	AD	KW	KS
USD	0.84685	0.68436476	0.63104716
CNY	0.93706	0.91181682	0.94471554
GBP	0.16742	0.16354783	0.08956677
JPY	0.45261	0.50084513	0.41212565
EUR	0.087479	0.05418508	0.1938659
RUB	0.94117	0.80860046	0.88773993
KRW	0.32316	0.2099641	0.27334286
CAD	0.43186	0.45444714	0.45255079

p-values for the Goodness of fit statistical tests for the period 1 with NIG distribution

Elaborated by author

Table 20. NIG goodness of fit statistics for period 2

Series	AD	KW	KS
USD	0.39438	0.64746933	0.21450514
CNY	0.24647	0.25309317	0.03577386
GBP	0.35511	0.39113775	0.34557779
JPY	0.40504	0.85120945	0.42789906
EUR	0.47627	0.52266601	0.30859481
RUB	0.5078	0.73827868	0.72102119
KRW	0.57978	0.97684374	0.89743321
CAD	0.71295	0.85897583	0.38537977

p-values for the Goodness of fit statistical tests for the period 2 with NIG distribution

Elaborated by author

Table 21. NIG goodness of fit statistics for period 3

Series	AD	KW	KS
USD	0.5068	0.73622645	0.64999089
CNY	0.23414	0.337488	0.36496229
GBP	0.37247	0.95348833	0.45144738
JPY	0.4835	0.46451296	0.40684412
EUR	0.6392	0.56545096	0.88592293
RUB	0.094954	0.09779064	0.25670428
KRW	0.17124	0.12601609	0.19892882
CAD	0.79517	0.63242272	0.49851747

p-values for the Goodness of fit statistical tests for the period 3 with NIG distribution

Elaborated by author

The final step of the methodology proposed consist in a out-sample test. To do so, the parameters obtained from period three were used to generate a random sample of 1 million simulations for NIG and GH distribution. Establishing a criterion of 95%, 99% and 99.9% the VaR and ES were computed. The results for the exchanges are shown in Tables 23 and 24.

Table 23. VaR Values for NIG and GH distributions

Series	**NIG**			**GH**		
	95%	**99%**	**99.9%**	**95%**	**99%**	**99.9%**
USD	-0.06585	-0.14346	-0.29050	-0.068861	-0.125688	-0.21029
CNY	-0.06577	-0.14139	-0.28809	-0.068247	-0.125950	-0.210790
GBP	-0.06554	-0.14096	-0.28682	-0.068532	-0.125605	-0.211970
JPY	-0.06691	-0.14482	-0.28857	-0.069631	-0.128712	-0.2164
EUR	-0.06560	-0.14388	-0.29231	-0.067981	-0.12495	-0.210705
RUB	-0.06528	-0.13986	-0.28158	-0.068137	-0.124293	-0.207112
KRW	-0.06506	-0.13933	-0.28078	-0.06783	-0.125149	-0.210531

CAD	-0.06574	-0.13980	-0.27847	-0.067645	-0.122484	-0.202110

Value at Risk estimates for 95%, 99% and 99.9% confidence degree for NIG and GH distribution with period 3 data
Elaborated by author

Table 24. ES Values for NIG and GH distributions

Series	**NIG**			**GH**		
	95%	**99%**	**99.9%**	**95%**	**99%**	**99.9%**
USD	-0.11512	-0.20564	-0.36444	-0.103790	-0.161353	-0.245919
CNY	-0.11438	-0.20398	-0.36106	-0.10425	-0.163354	-0.250736
GBP	-0.11378	-0.20210	-0.35634	-0.104415	-0.163187	-0.249808
JPY	-0.11628	-0.20748	-0.36711	-0.106594	-0.167573	-0.25787
EUR	-0.11534	-0.20667	-0.36717	-0.103344	-0.161256	-0.2465
RUB	-0.11323	-0.20088	-0.35400	-0.103072	-0.160312	-0.24444
KRW	-0.11340	-0.20106	-0.35383	-0.102966	-0.161407	-0.24795
CAD	-0.11278	-0.19837	-0.34676	-0.101948	-0.157841	-0.239775

Expected Shortfall estimates for 95%, 99% and 99.9% confidence degree for NIG and GH distribution with period 3 data
Elaborated by author

The results of these evaluations show that for the period 3, NIG distribution managed to outperform GH distribution in terns of loss estimation. This empirical result confirms the statement of Barndorff-Nielsen (1997) in which the NIG can capture heavy tails with better precision.

As a visual example of these computations, the out-sample data is displayed in Figure 10. The blue lines correspond to the GH VaR estimations, while red ones are for the NIG. With the confidence degrees established, it is notable that although the VaR is recommended to be projected only 10 days (J.P. Morgan, 1997, p 35), the values are valid for all the 199 observations. Furthermore, the bubble episode detected in Chapter 3 is visible in the volatility cluster at the beginning of the series.

The VaR values established by NIG are closer to the empirical downfalls reported. However, for a precise comparison a VaR accuracy ration was computed. It is established as the percentage of observations smaller than the value specified. In such scenario it would be expected a close margin to the confidence degree. Table 25 contains the ratios and it is notable that with a 95% NIG constantly outperforms GH. The advantage for GH distribution is in the extreme values beyond 99%, as the NIG overestimates the risk, while the GH provides and almost perfect fit. This provides evidence that certain model flexibility may be establish in function to the desired exposure.

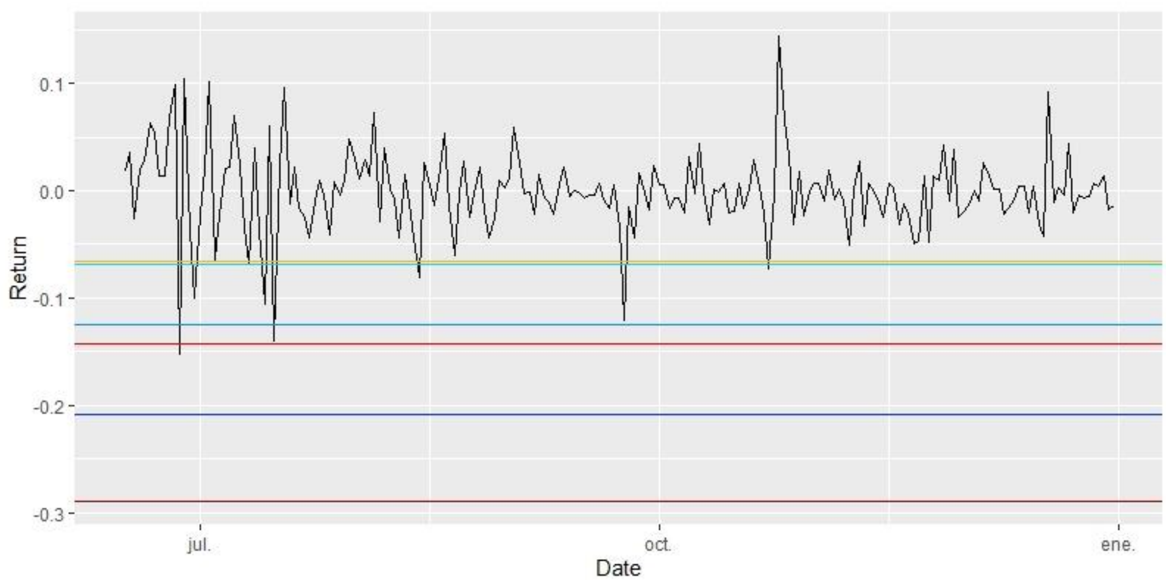


Fig.10 VaR comparison for NIG and GH distributions

Table 25. VaR accuracy ratio

Series	NIG			GH		
	95%	**99%**	**99.9%**	**95%**	**99%**	**99.9%**
USD	0.04	0.005	0	0.035	0.01	0
CNY	0.045	0.005	0	0.035	0.01	0
GBP	0.04	0.005	0	0.04	0.01	0
JPY	0.035	0.005	0	0.035	0.01	0

EUR	0.04	0.005	0	0.035	0.01	0
RUB	0.04	0.005	0	0.04	0.01	0
KRW	0.045	0.005	0	0.04	0.01	0
CAD	0.04	0.005	0	0.04	0.015	0

VaR accuracy ratios for 95%, 99% and 99.9% confidence degree for NIG and GH distributions; The value portrayed must be close to the complement of the confidence degree
Elaborated by author

To increase the robustness of the tests and portray the advantages of NIG versus GH distribution Table 26 presents the likelihood estimation, AIC criterion and iterations made until convergence for the series. It is possible to see that the computational efficiency in the number of iterations is almost a third in NIG against the maximum achieved for GH. Furthermore, the likelihood and AIC provide evidence of the opportunity cost in the extra parameter, as it increases the likelihood, but it tends to overfit the model. In conclusion, these results provide extra evidence on the advantages of NIG in Bitcoin´s returns modelling.

Table 26. Computational Evaluation

Series	**NIG**			**GH**		
	Log-Likelihood	**AIC**	**Iterations**	**Log-Likelihood**	**AIC**	**Iterations**
USD	1025	-2062	163	1036	-2042	502
CNY	1024	-2055	127	1032	-2040	414
GBP	1025	-2059	141	1034	-2041	502
JPY	1024	-2056	265	1033	-2040	494
EUR	2026	-2060	143	1035	-2045	502
RUB	1022	-2056	189	1033	-2036	502
KRW	1026	-2058	151	1034	-2044	502
CAD	1022	-2049	189	1029	-2036	502

Computational results for efficiency and adjustment of data. AIC: Akaike Information Criterion, Number of iterations is set with a maximum of 502 to avoid loop

Elaborated by author

4.5. Conclusion

This chapter treated the underlying distribution of Bitcoin returns for eight of the most important exchanges. The methodology included the subsampling of the information with the bubble episodes detected in previous chapters. Thus, the periods correspond with different stages in Bitcoin market: 1) The government impacts due to regulation and prohibition, 2) the speculative period that had certain price manipulation schemes and 3) the stabilization period post Future instrument emissions.

With these periods in mind, and the exuberant episodes, the NIG distribution was proposed to fit the empirical data. The complete procedure included the implementation of normality test to refute the traditional assumption made over financial data. The results suggest that none of the series nor the periods resemble a normality behavior. Later, the parametrization of NIG distribution and the goodness of fit statistics with two samples provided the necessary evidence to conclude that for all the period and exchanges, the theoretical proposal fits the empirical evidence. Finally, an out-sample evaluation under VaR and ES methodology compared the proposal of GH distribution as the best option to model Bitcoin returns. Nonetheless, the capital requirements provided by the NIG managed to provide a better hedge against the bubble episode in the out-sample data.

Although the risk management perspective is a crucial area in finance, one of the most addressed problems is asset allocation. Regarding this topic the optimal portfolio selection proposed by Markowitz (1952) started a revolution in the way investments are made. With the problem statement of variance minimization, he proposed a mechanism to obtain the optimal weights of a portfolio. Nevertheless, his proposal consisted in the mean and variance sample estimation. This approach indirectly assumes the multivariate Gaussian distribution for the returns. However, as it happens in univariate series, the stylized facts of skewness and kurtosis are not modeled in such assumption. The result of such is the misallocation of resources into risk positions. For this reason, the natural step forward becomes the multivariate approach of NIG to establish portfolios and even provide information to the nature behind Bitcoin and the way it interacts with different assets.

Chapter 5: Multivariate framework

5.1. Introduction

Previous chapters treated with the study of statistical properties regarding Bitcoin. It could be stablished that the market is in constant evolution; new regulations from governments, popularity growth, news and derivative instruments issuing cause agents to adapt their expectations. The result for the Bitcoin price is the appearance of bubbles and escalated stylized facts that makes the Gaussian assumption unable to handle those behaviors. Chapter 4 presented the NIG distribution as a better option to model return´s distributions. The properties presented by Barndorff-Nielsen (1997) makes NIG an adequate candidate to substitute Normal distribution – specially in Bitcoin exchange market –. The parameter flexibility allows this distribution to capture the extreme behavior witness in the market. It was possible to model a Monte Carlo VaR model that fit the data with a 95% confidence degree, and performed well with a 99% and 99.9%.

One of the major areas studied in finance regard the asset allocation problem. Several methodologies based on expertise were employed; however, the true revolution came with the seminal paper of Markowitz (1952) who managed to state the problem as an optimization. The idea behind such statement relies on the risk itself, variable that in a multivariate scheme turns to be represented with a covariance matrix. This statistic is constructed to model the relationship between variables and could be used to establish a minimization problem. However, this theoretical framework may have certain difficulties in empirical procedures. The reason is once again the stylized facts of the financial returns.

When the Bitcoin and other cryptocurrencies started to be traded in financial markets, the natural procedure was to include them in portfolios composed of different assets. However, the usage of an asset that is not fully understand may cause certain complication in risk management (Caginalp et al. 2001) and portfolio optimization problems (Haubo, 2015, p 85). The argument is reinforced with the results of Cian et al. (2016) and Bueno et al. (2017) regarding the high volatility and heavy tail behavior. These facts may lead to a misallocation of assets due to the underestimation of risk.

In a risk aversion agent perspective, the idea is to maximize returns at the minimum risk possible. The original framework proposes the construction of portfolios with uncorrelated assets. This procedure guarantees that maximum losses does not happened simultaneously and in average a positive gain is earned. However, because of volatility in cryptocurrencies markets, although the low correlation with traditional assets, could result in an over exposure and the impracticality to use them as diversifiers.

Chapter 4 provides evidence of the capabilities of NIG to capture stylized facts in univariate framework. Nonetheless, the natural step forward is the implementation of a multivariate distribution. The study of Bitcoin alongside other assets may be determinant in finding the real interaction of cryptocurrencies with other financial assets, as well as a better way to provide a portfolio optimization procedure (Ji et al. 2019). It is then the aim of this chapter to provide a risk management and portfolio optimization model with baskets of different financial assets such as currencies, indexes and commodities.

5.2. Multivariate Bitcoin Literature

The problem of incorporate Bitcoin into a practical portfolio model is not new in finance literature. The main efforts have been made with the implementation of GARCH type modelling. Katsiampa (2017) test a variety of these models to determine the best fit. His results show that the Autoregressive-Conditional GARCH model manage to explain most of the volatility in Bitcoin market. With this, it becomes relevant to consider a dynamic approach as well as a short- and long-term conditional variance. These results are consistent with the bubble episode explanation provided in Chapter 3, in which the source of speculative behavior showed changes through time, causing a variation in parameters as the market evolved.

Following this line, Cermak (2017) exposes with a GARCH(1,1) model the interaction of Bitcoin with real economy. His findings show the safe-heaven property in the Chinese market. Also, another important finding is the reduction in volatility for the studied period corresponding of years 2011 to 2017. With a forecast estimation, he concludes the periodical reduction of Bitcoin volatility until 2019 where it would reach similar levels than other currencies. Similarly, Bouoiyour and Refk (2016) coincide with the hypothesis that Bitcoin market is immature but will eventually normalize its behavior. Similar results are shown in Chapter 2 and 4, where the speculative components of the series have changed, and the return behavior have reduced in comparison with 2017.

The multivariate extension of GARCH models have been popularly used to study the interaction of Bitcoin with other assets. Corelli (2018) proposes a multivariate analysis of six cryptocurrencies with a Ganger causality test and a VECM model with exchange rates of Europe, Asia, Africa and Oceania. These results show evidence of strong correlation of Bitcoin and Etherum with Asian currencies (Thai Baht, Taiwan Dollar and Chinese Yuan). Further development of these techniques is deployed by Baur et al. (2018) who performs a univariate and multivariate GARCH modeling to Bitcoin with USD and Gold. Their conclusions show a low correlation between cryptocurrencies and other assets.

In their paper Al Mamun et al. (2019) present an indicator capable to model Bitcoin´s volatility and the correlation with other financial entities. The premise of the model is the effect that geopolitical events may affect the risk premium of the cryptocurrency. With a Dynamic Conditional Correlation (DCC) GARCH model with the additional asymmetric response, they conclude that gold is the only asset capable to hedge Bitcoin. Alongside, they find out that geopolitical events have a direct impact on volatility. Bouri et al. (2019) present other methodology regarding tail behavior in a cross-quantilogram model that shows that the interaction between cryptocurrencies and equities in the American stock market are heterogeneous. In this aspect, they have similar safe-heaven properties. Another proposal is made by Corbet et al. (2018) who develops a generalize variance decomposition. This procedure is made under the idea to find the sign and severity of the spillovers that shocks have from markets (MSC GSCI, the US Broad Exchange Rate, SP500, gold, VIX and Markit ITTR110 index). The results of such study conclude that cryptocurrencies are isolated from the traditional assets traded in different markets, the corollary is then that they could be used as diversifiers.

Following this line Hussain et al. (2019) use an Autoregressive Generalized Dynamic Conditional Correlation GARCH to capture the correlation between the cryptocurrency, gold and the stock indexes of G7 countries via a bivariate model. The results show that the commodity stills the safe-heaven for the indexes. Meanwhile, Bitcoin is able to hedge the Canadian index. In conclusion, they state that these assets may be used as diversifiers, however, the precious metal still the most consistent. Another paper is presented by Gil-Alana et al. (2020) who use fractional integration and cointegration with Bonds, Dollar, Gold, GSCI, S&P and VIX together with six major cryptocurrencies. In this bivariate approach, they conclude that there exists an isolation between cryptocurrencies and other financial assets. Their conclusions also state for a diversifier property of cryptocurrencies, as no cointegration is found among cryptocurrencies and the indexes. Charfeddine et al. (2020) use copula and Multivariate GARCH methodologies to create portfolios. Once more, they use bivariate portfolios and discover that there exists a weak relationship with Bitcoin Ethereum, S&P500, gold and crude oil. Finally, they show that weights achieved for the cryptocurrencies range from 11.62% and 24.25% in the portfolio with oil.

5.3. Data and Methodology

To perform the statistical analysis seven series will be used. The Bitcoin as the common component of the portfolios, for the currencies EUR and JPY will be used; Gold and Oil (price of West Texas Intermediate mixture) obtained from the Federal Reserve will be the commodities, while the indexes correspond to the US Dow Jones and Chinese SZSE. Because of the nature of Bitcoin market which operates every day, the weekly observations are used to homologate the series frequencies. The length period is the same as previous data, which provides a total of 275 observations. The boxplots of the data are presented in Figure 11.

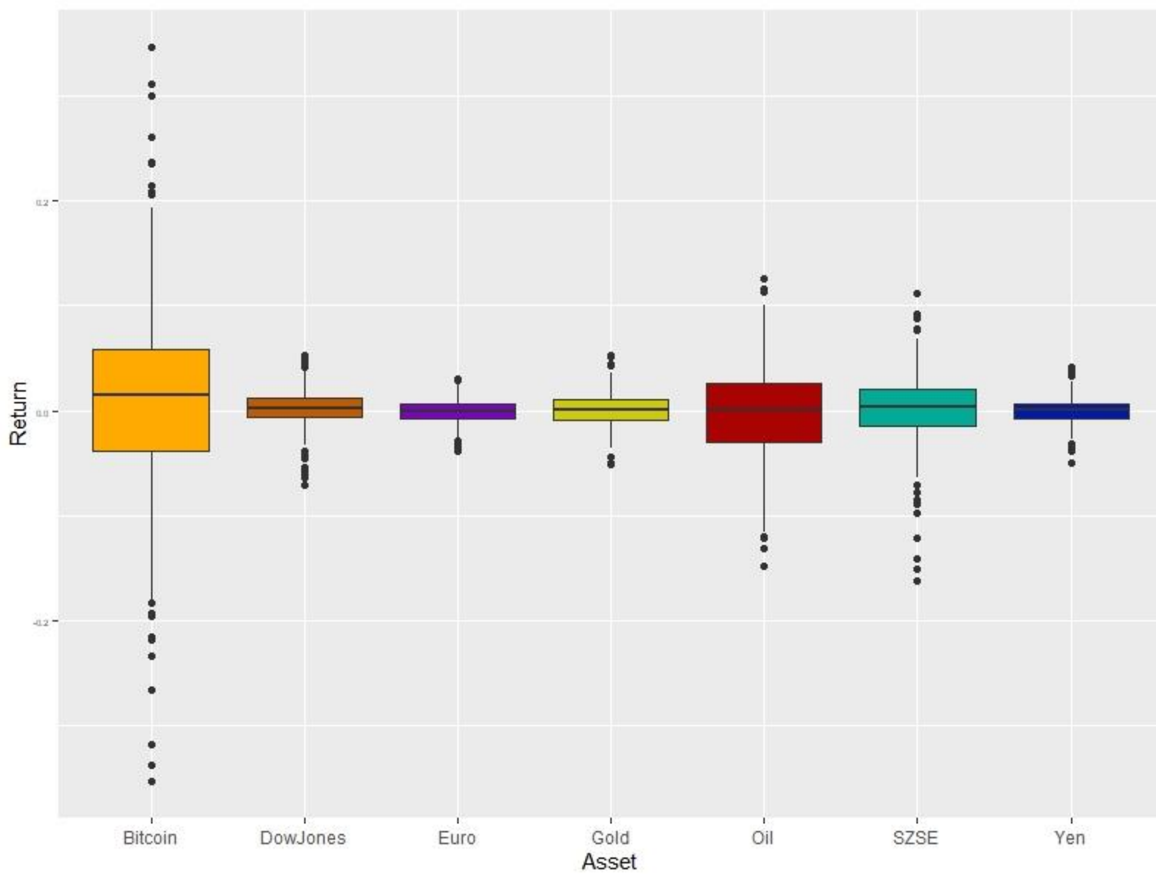


Fig. 11 Boxplots

This univariate graphics provide reference on the quantiles of the distribution. The dots above and below the lines are consider as extreme values. With it, is possible to compare the behavior of the data, in which the Bitcoin displays the heaviest tails and widest dispersion, followed by the Oil. Meanwhile, the dispersion in currencies (Euro and Yen) are the lowest.

The first thing to do is perform a multivariate normality test in to provide robust evidence to avoid the traditional assumption. The datasets are constructed as trivariate portfolios with Bitcoin as the common asset. Although one of the stylized facts mentioned by Cont (2001) is the convergence to normality with lower frequency data, it could be tested if the weekly periodicity holds to this property. To do so, the Cramer-von Mises (CVM) statistic for tow multivariate samples will be used, altogether with the Kernel density test (KDE) and Ball Divergence (BD)

The KDE test is originally presented by Anderson et at. (1994) as s kernel-based comparison of two multivariate samples. With the integrated squared errors of the kernel estimations:

$$\boldsymbol{KDE} = \int \left(\hat{\boldsymbol{f}}_{\mathbf{1}} - \hat{\boldsymbol{f}}_{\mathbf{2}}\right)^{\mathbf{2}} \tag{51}$$

Where $\hat{\boldsymbol{f}}_{\boldsymbol{i}}$ is the kernel density for the $\boldsymbol{i}$ sample this comparison leads to an estimation of the discrepancy between densities.

The BD statistic was proposed by Pan et al. (2018). It is based in the evaluation of densities considered a metric space. The test is performed the evaluation of the number of observations that are inside the hyperspheres of different radius. For the two samples the statistic is defines as:

$$\boldsymbol{BD} = \frac{\mathbf{1}}{\boldsymbol{n}_{\mathbf{1}}^{\mathbf{2}}} \sum_{\boldsymbol{i},\boldsymbol{j}=\mathbf{1}}^{\boldsymbol{n}_{\mathbf{1}}} \left(\boldsymbol{P}_{\boldsymbol{ij}}^{\boldsymbol{\mu}_{\mathbf{1}}\boldsymbol{\mu}_{\mathbf{1}}} - \boldsymbol{P}_{\boldsymbol{ij}}^{\boldsymbol{\mu}_{\mathbf{1}}\boldsymbol{\mu}_{\mathbf{2}}}\right)^{\mathbf{2}} + \frac{\mathbf{1}}{\boldsymbol{n}_{\mathbf{1}}^{\mathbf{2}}} \sum_{\boldsymbol{k},\boldsymbol{l}=\mathbf{1}}^{\boldsymbol{n}_{\mathbf{2}}} \left(\boldsymbol{P}_{\boldsymbol{kl}}^{\boldsymbol{\mu}_{\mathbf{2}}\boldsymbol{\mu}_{\mathbf{1}}} - \boldsymbol{P}_{\boldsymbol{ij}}^{\boldsymbol{\mu}_{\mathbf{2}}\boldsymbol{\mu}_{\mathbf{2}}}\right)^{\mathbf{2}} \tag{52}$$

$$P_{ij}^{\mu_1\mu_1} = \frac{1}{n_1}\sum_{t=1}^{n_1}\delta(X_{1i}, X_{1j}, X_{1t})$$

$$P_{ij}^{\mu_1\mu_2} = \frac{1}{n_1}\sum_{t=1}^{n_1}\delta(X_{1i}, X_{1j}, X_{2t})$$

$$P_{ij}^{\mu_2\mu_1} = \frac{1}{n_2}\sum_{t=1}^{n_1}\delta(X_{2i}, X_{2j}, X_{1t})$$

$$P_{ij}^{\mu_2\mu_2} = \frac{1}{n_2}\sum_{t=1}^{n_1}\delta(X_{2i}, X_{2j}, X_{2t})$$

And:

$$\delta(x, y, z) = \mathbf{1}_{B(x,y)} \tag{53}$$

Such that $B(x, y)$ is the closed ball.

Confirmed the no-normality of the series the Multivariate NIG (MNIG) is presented as a special case of the Multivariate GH (MGH) distribution. In this case, the generalized form is defined as a mixture of Gausian and Generalized Inverse Gaussian (GIG):

$$X = \mu + W\gamma + \sqrt{W}A\overline{Z} \tag{54}$$

Such that:

$$\overline{Z} \sim N(\overline{0} I_k)$$

$$A \in \mathbb{R}^{d\times k}$$

$$\mu, \gamma \in \mathbb{R}^{d}$$

With $W \geq 0$ independent of $\overline{Z}$ and $W \sim GIG(\lambda, \chi, \psi)$, which density is defined by:

$$f(w) = \left(\frac{\psi}{\chi}\right)^{\lambda/2} \frac{w^{\lambda-1}}{2K_\lambda(\sqrt{\chi\psi})} e^{\left\{-\frac{1}{2}\left(\frac{\chi}{w}+\psi w\right)\right\}} \quad (55)$$

With $\chi > 0, \psi \geq 0, \lambda < 0$ for the NIG case and K_λ is the modified Bessel function of the third kind.

The interpretation of the parameters can be completely defined as follows: λ, χ, ψ represent the shape parameters, μ is the location, $\Sigma = AA^t$ is the dispersion matrix and γ stands for the skewness parameter.

Isolating the GIG component, the first two moments corresponding to the Expected Value and the Variance of the multivariate distribution are:

$$E[W] = \frac{K_{\lambda+1}(\sqrt{\chi\psi})}{K_\lambda(\sqrt{\chi\psi})} \sqrt{\frac{\chi}{\psi}} \quad (56)$$

$$VAR[W] = \frac{\chi}{\psi}\left(\frac{K_{\lambda+2}(\sqrt{\chi\psi})}{K_\lambda(\sqrt{\chi\psi})} - \left(\frac{K_{\lambda+1}(\sqrt{\chi\psi})}{K_\lambda(\sqrt{\chi\psi})}\right)^2\right) \quad (57)$$

For the special case when $\lambda = -\frac{1}{2}$ then the distribution is MNIG. This constant term in the GH family collapses the Bessel function of the third kind (K_λ) to a number, meaning that the iterations to maximize the likelihood are considerably reduced so the computational iterations becomes faster. For the MGH distribution, the expected value and the variance matrix are given by:

$$E[X] = \mu + E[W]\gamma \quad (58)$$

$$VAR[X] = VAR[W]\gamma\gamma^t + E[W]\Sigma \quad (59)$$

It is important to emphasize that the Expected Value contains the location, shape and skewness parameters, while the Variance matrix does the same except for the location. The result of such definition becomes a more precise and flexible interpretation to model the expected returns and the covariance among assets. One more consideration to use is the shift of the parametrization to one that permits the algorithm to converge. Let $\bar{\boldsymbol{\alpha}} = \sqrt{\boldsymbol{\chi\psi}}$ then for the NIG $\left(\boldsymbol{\lambda} = -\frac{1}{2}\right)$, $\boldsymbol{\chi} = \boldsymbol{\psi} = \bar{\boldsymbol{\alpha}}$, such that $\bar{\boldsymbol{\alpha}} > \mathbf{0}$. With this new definition, the calculus becomes even faster from a computational perspective.

For the portfolio selection, the framework originally proposed by Markowitz (1952) is deployed via the minimization of Variance. In this case, as the assets present high volatility levels, and the intention is to use a rolling window, the probability to get a corner solution is increased. To address this issue, the no short-sale restriction will be avoided. This specification allows for further interpretation of the weights´ changes through time, allowing the free movement between positive and negative values. In this case, the no leverage restriction stills on as it works as a boundary to maintain plausible quantities of the assets. So, the final problem to solve may be interpreted as the quadratic problem of:

$$\min \boldsymbol{w^t}\mathbf{VAR(X)w} + \boldsymbol{w^t E}[\boldsymbol{X}] \tag{60}$$

$$\boldsymbol{subject\ to} \sum_{\boldsymbol{i=1}}^{\boldsymbol{N}} \boldsymbol{w_i} = \mathbf{1}$$

Where $\boldsymbol{w_i}$ stands for the weight of asset $\boldsymbol{i}$ and $\boldsymbol{w}$ is the weights vector.

With the employment of a rolling window, the procedure of fitting a multivariate distribution and then use its parameters as an input to obtain the optimal weights is repeated for every move in the window. The length of it is constant through all the periods and move week by week. The number of observations in each one corresponds to a total of 104. It was selected because it is required a quantity large enough to obtain the distribution parameters, as well as it represents the information of two years in history, a common practice in the industry. Furthermore the 275 observations available restricts the total number of observations to use. Longer periods would mean the loss of periods to analyze and shorter ones would cause a loss in the robustness of the algorithm to find the optimal parameters – increasing the standard deviation of them –. The results are intended to take the short- and long-term information into account.

5.4. Results

The parameters for Gaussian and NIG multivariate distribution are shown in Annex 3; with them, it is possible to generate random multivariate samples that have the same distribution as the estimates. Tables 27 and 28 display the p-values for the two multivariate samples goodness of fit test. Under the null hypothesis that both data series have the same theoretical distribution it is possible to reject the Multivariate Gaussian assumption for the trivariate data sets under a 95% confidence degree. Nevertheless, the only test that does not reject the null hypothesis is the Multivariate CVM. In the multivariate NIG the null hypothesis is not rejected under a 95% confidence degree for all the provided tests and series. These results show the capability of NIG and the equivalent Multivariate version to model the returns of financial series. Furthermore, it is proven that weekly information maintains the stylized facts described for high frequency data.

Table 27. Normality tests´ p-values

Portfolio	KDE	CVM	BD
Commodity	0.04618124	0.1528432	0.04
Index	1.24768×10^{-5}	0.046615	0.01
Currency	5.00244×10^{-8}	0.036504	0.01

p-values for multivariate Goodness of fit statistical tests under the null hypothesis that samples come from the multivariate Gaussian distribution

Elaborated by authors

Table 28. NIG goodness of fit tests´ p-values

Portfolio	KDE	CVM	BD
Commodity	0.4604625	0.092723	0.06
Index	0.8128640	0.973027	0.92
Currency	0.7655634	0.415574	0.34

p-values for multivariate Goodness of fit statistical tests under the null hypothesis that samples come from the multivariate NIG distribution

Elaborated by authors

With the statistical criteria that confirms the usage of NIG to model the data and obtain the location and dispersion parameters required for the portfolio optimization problem it possible to compute the rolling window estimation for the 104 bandwidth. The main result of the procedure is the dynamic covariance matrix for the portfolios. Figures 12 – 14 plots the off-diagonal elements of the symmetric matrix corresponding to covariances.

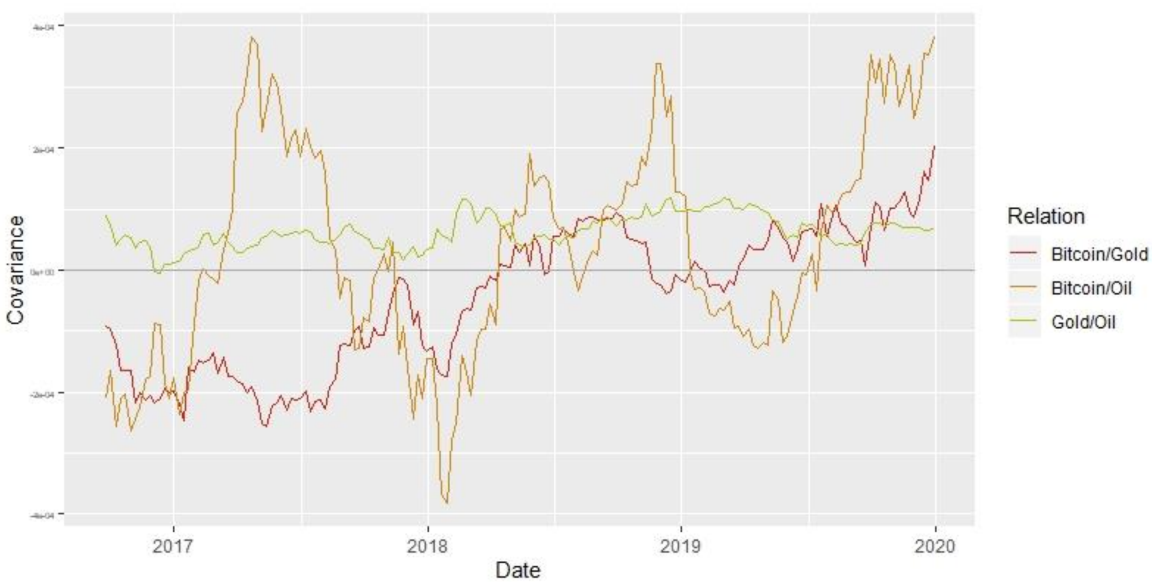


Fig. 12 Commodity portfolio Covariances

Fig. 13 Index portfolio Covariances

Fig. 14 Currency portfolio Covariances

In general, the covariance between Bitcoin and other assets has shown changes trough time. The post bubble episode of 2017 appears as the greatest shock for structural changes in Bitcoin interaction. The Commodity portfolio shows that previous the burst of the bubble, the relation with gold was negative, allowing for diversification with the asset; on the other hand the oil had a oscillations in negative and positive values; Likewise, the bubble episode started with a negative relationship between Bitcoin and these assets, but as the bubble raised, it did began to approximate the covariance to zero. In the same line, it is notable the negative relation after the burst. In 2019 the behavior remained almost the same as previously with oil; movements around zero; however, the negative covariance with gold change signs for the rest of that year even increasing the value at the end of the year. In the interaction between assets it can be noticed to be constant in sign and with little variation, showing the parameter stability for them and the accentuating the shifts with Bitcoin.

Index portfolio covariances show a similar behavior as the commodity one. The traditional assets maintain the sign of the relationship and small variations are reported. However, for the cryptocurrency and the Chinese index the covariance remains positive for most of the periods. Once more, the bubble episode of 2017 reduced this factor until it burst moment from which it reestablished the increasing tendency until 2019. For the last year, the relationship decreased, but for the last observations it seems to have reach certain stability. Meanwhile, the US index displayed a similar behavior, with the burst of the bubble the moment that incentive the covariance to cross the positive threshold. The same stabilization behavior can be seen for the year 2019.

Finally, currency portfolio displays the constant sign and stability for traditional assets, but movements with Bitcoin. For the interaction between the cryptocurrency and Yen, the bubble burst represented the moment that accentuated the positive relationship; meanwhile, the Euro interaction kept negative until 2019, period that presented a positive and increasing relationship. These results show the evolution of Bitcoin that resembles the phases detected prior with the PSY methodology. Moving to the optimal weights for portfolios, Figures 15 – 17 plot the evolution of the asset allocation assuming a weekly rebalancing.

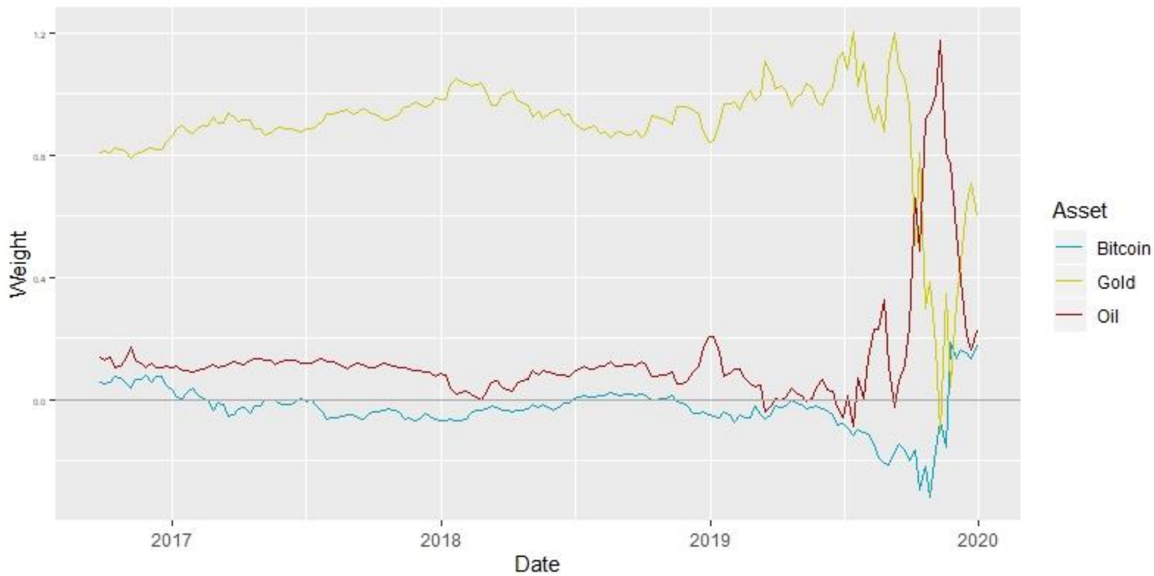


Fig. 15 Commodity portfolio Weights

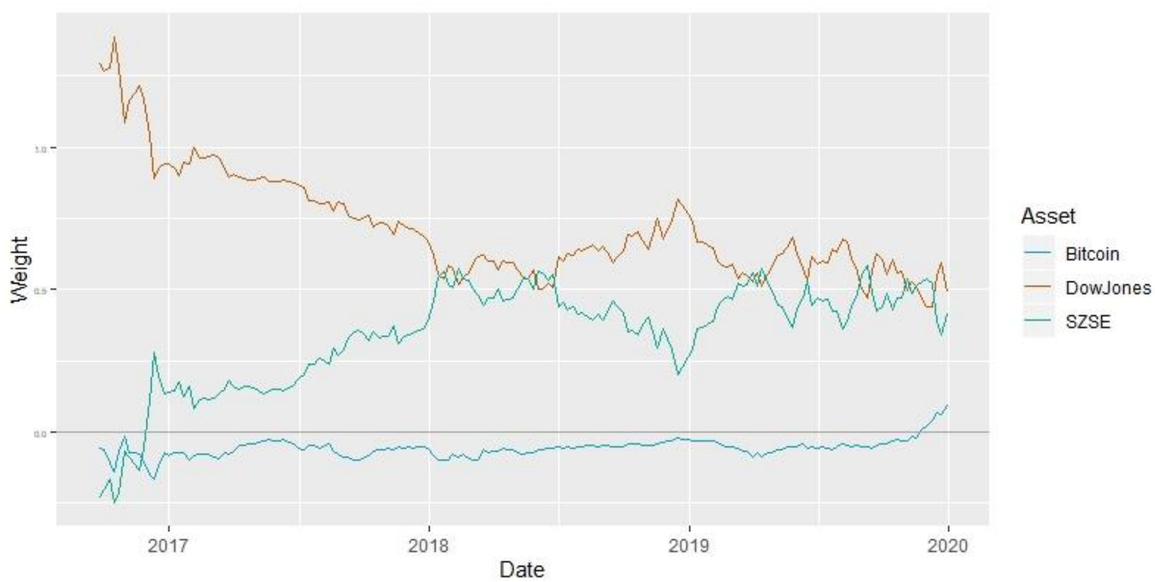


Fig. 16 Currency portfolio Weights

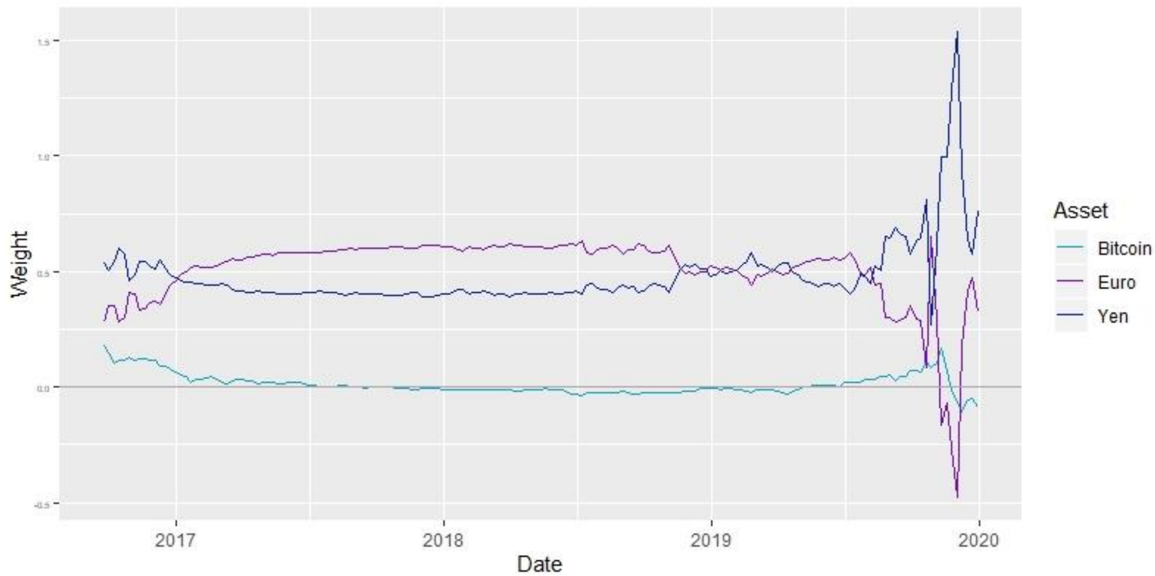


Fig. 17 Index portfolio Weights

The main result for the weight series is the almost null ponderation assigned to Bitcoin. In Currency and Index portfolio the weights for most of the series remain with low variation and close to zero; nevertheless, the movements reported in the commodity portfolio resembles a hedging strategy with gold and Bitcoin even with similar weights in Bitcoin and oil. If the comparison is made through the time series, the hedging capabilities of cryptocurrencies are present for most of the series and portfolios. However, the solution considers the risk generated by Bitcoin and reduce its participation. Nevertheless, the market of cryptocurrencies keeps in evolution. The dynamic approach is capable to create a portfolio for the last periods in which Bitcoin has a relevant weight in a commodity basket. In fewer extent, a similar behavior could be noted in the index portfolio, where, for the last periods, Bitcoin changed from a negative weight to a positive and increasing one.

5.5. Conclusions

This chapter began with the premise that Gaussian distribution is not able to model Bitcoin extreme behavior. Likewise, the evolution of the market makes and important factor to consider in the distribution analysis and the relationship with other assets. In such scenario, a dynamic and multivariate approach was necessary. To do so, a rolling window estimation for the MNIG distribution was proposed. However, the output of the methodology consisted in the clean covariance matrix and expected value vector. These variables permitted the specification and computation of an improvement to the original portfolio optimization problem stated by Markowitz (1952) in which the inputs were originally proposed as the Multivariate Gaussian.

The results of these methodologies were a dynamic covariances and optimal weight series for three trivariate portfolios each containing two traditional assets and Bitcoin. The results in covariance show the hedging capability of the cryptocurrency in short and long positions. However, translated to the optimal weights the results show that the risk of the asset for most of the period analyzed is too high to be used as a diversifier. However, for the last observations – period that in previous chapters presented certain stabilization in behavior – the weights assign to Bitcoin increased in a long position for commodities and in lower measure with indexes. Furthermore, the hedging capabilities described in literature are present for most of the series, to finally accentuate in the last dates recorded.

The relevance in this study is that it was possible to identify a substantial change in Bitcoin market for the last period of the series. Originally considered as a speculative asset with properties that incentive the avoidance of it in investment decisions, it may be in a transition stage to become an accepted financial asset. The market seems to have self-regulated characteristics in which the agents learned from past episodes and provide certain controls. It should not be misinterpreted as the reduction of volatility to similar levels of gold or Euro, but as a stabilization of the stylized facts that identifies Bitcoin as an innovative proposal.

Chapter 6: Final remarks

Technological development is a constant in human nature, innovative processes for millennial activities took our species to the digital and electronic era. Economics and finance are two of the social areas that had to manage with the creation of a new entity. Valuation techniques and theories were unable to determine the price fluctuations witness for the next years after its first emission. Relying in the cryptography and the revolutionary blockchain technology, the decentralized currency proved to be a plausible idea. However, multitude episodes of government regulations, prohibitions and imitations provided cryptocurrencies with high volatility periods. These days were also subject to social euphoria and hacking attempts to artificially manipulate prices, leading to a bubble that threatened to destroy the value of these technologies. However, the market was able to learn and adapt to new conditions, avoiding at certain degree the past errors of speculative herd behavior.

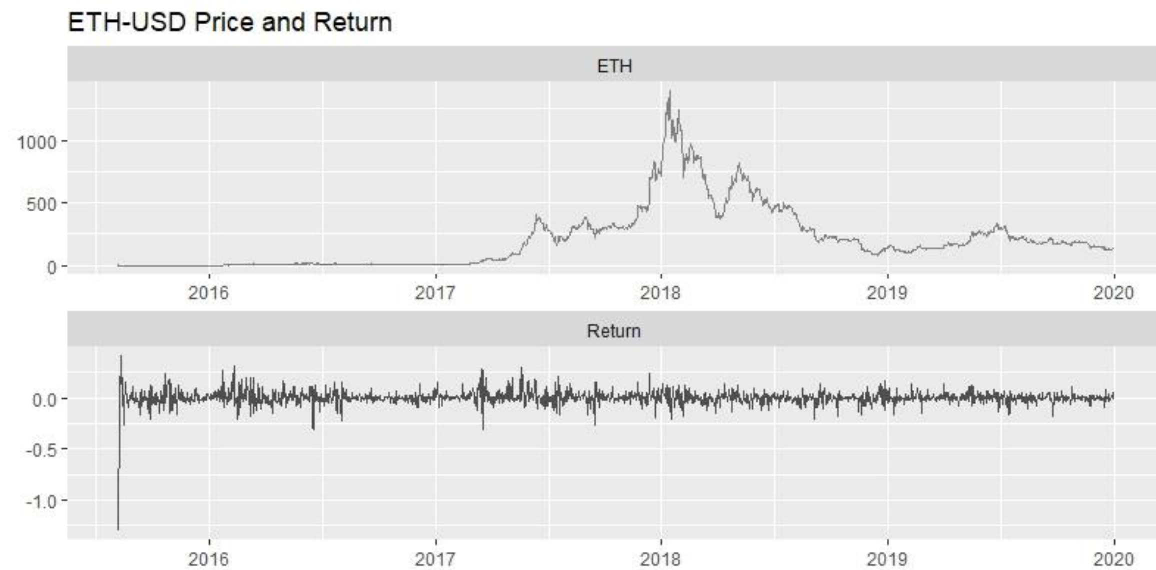
This work presented an analysis of Bitcoin from end 2014 to end 2019. In almost five years of daily information a set of tests where implemented with the idea to improve the understanding and possible usage of Bitcoin in financial practice. To treat this topic some questions were stated. The first one considered the possibility to consider Bitcoin as a speculative asset. Chapter 1 provided a statistical description of some of the most important cryptocurrencies, being able to detect escalated stylized facts, mainly the heavy tail behavior. Such results lead to the idea to monitor the series in search for bubble episodes. Chapter 2 deployed the PSY methodology with some corrections to identify the exuberant periods. The results gave almost the exact dates for some of the most important news regarding cryptocurrencies. Also, by analyzing the nature of them, it was possible to determine three phases in the series. The first one consisted in the government interventions with the regulation and prohibition of Bitcoin. Second one consists of the exponential growth for cryptocurrencies, social euphoria for these new assets and the implication of certain price manipulation lead to the greatest bubble episode in the series. The burst was then caused by the issuing of Future Contracts with Bitcoin as the underlying asset; leading agents to flee from the asset to the derivative instrument, so collapsing the price. From that period on, the price and returns presented certain stability; although the bubble episodes seem to be a regular variable in the market, agents seem to have learned to avoid this artificial increases in price, which cause the bubbles to be short period.

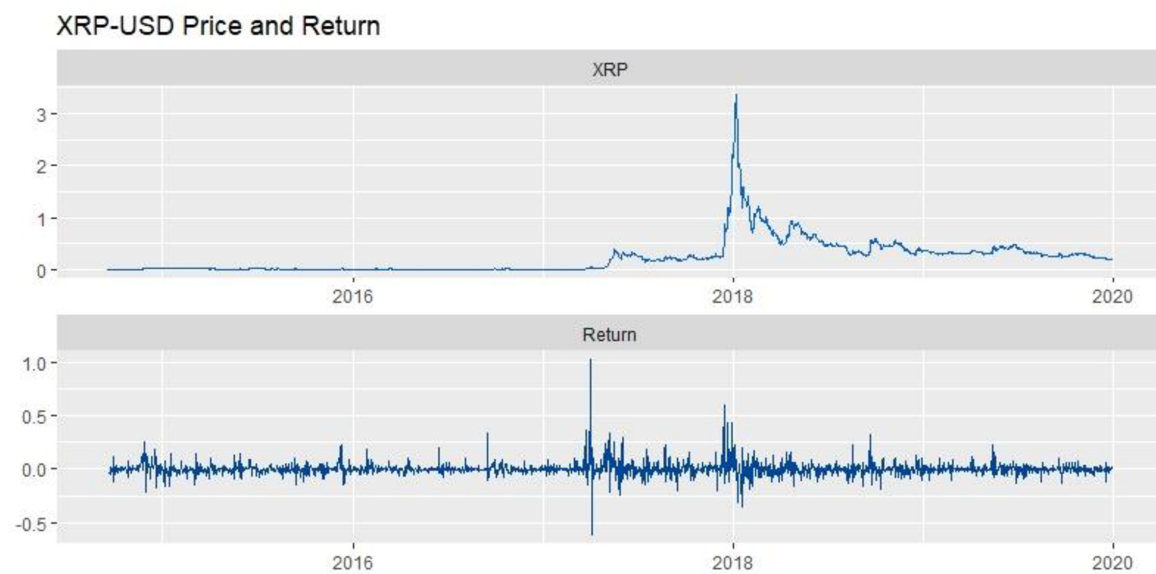
With the presence of bubbles, the major risk are the effects of the burst. Such extreme behaviors present the necessity to find a distribution able to capture the whole range of statistical properties in Bitcoin series. To address this issue, the NIG distribution was proposed as an alternative to traditional Gaussian assumption. With the phases identified with the bubble episodes subsamples of the series were created to contain the different types of bubbles. Statistical tests suggested this improvement was correct, however, to test the out-sample capability of the distribution a period with a bubble episode was tested with a VaR framework. The results confirmed the flexibility of NIG, and the ergodic property required to implement any statistical model. These properties lead to give one step forward into the multivariate framework.

With a univariate distribution the behavior of the series may be used in risk management and instrument valuation, but the other major area in finance is asset allocation and hedging. The requirements for those models consider a multivariate scheme, nevertheless, the Multivariate Gaussian assumption does not hold for series as Bitcoin, even in weekly frequency. The solution came with the computation of MNIG distribution. However, the changes seen in Bitcoin market lead to consider a dynamic approach that consisted in a rolling window estimation of the distribution. With this procedure, the dynamic covariance series were computes, which allowed for a dynamic Markowitz portfolio optimization. The results showed that although the relationship between Bitcoin and traditional assets incentive to use it as a diversifier, the variance minimization procedure weighted the volatility in Bitcoin and assigned a small proportion of the portfolio for most of the periods. However, for the last observations, that consisted mostly in post 2017 bubble, a hedging capability was observed in the commodities and indexes portfolio.

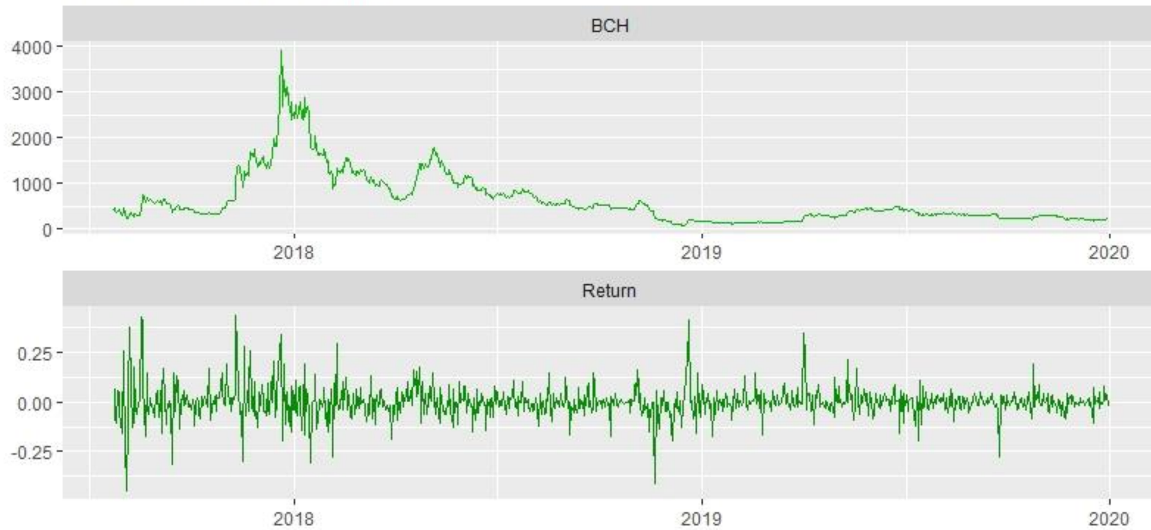
Such results provide evidence about the evolution of Bitcoin market and the transition to a useful asset to incorporate in portfolios. Furthermore, despite the name of cryptocurrencies, the lowest similarities were found between Bitcoin and Euro or Yen. In contrast, commodities seem to have a closer relationship with Bitcoin, not only in the fundamental *mining* process to obtain them, but on its place in the minds of agents who seem to use cryptocurrencies as an investment or safe-heaven in crisis events. Although, Bitcoin does not have an intrinsic value in real economy, it seems that blockchain technology have work as the fundamental and most valuable contribution of Bitcoin to human interactions.

Annex 1

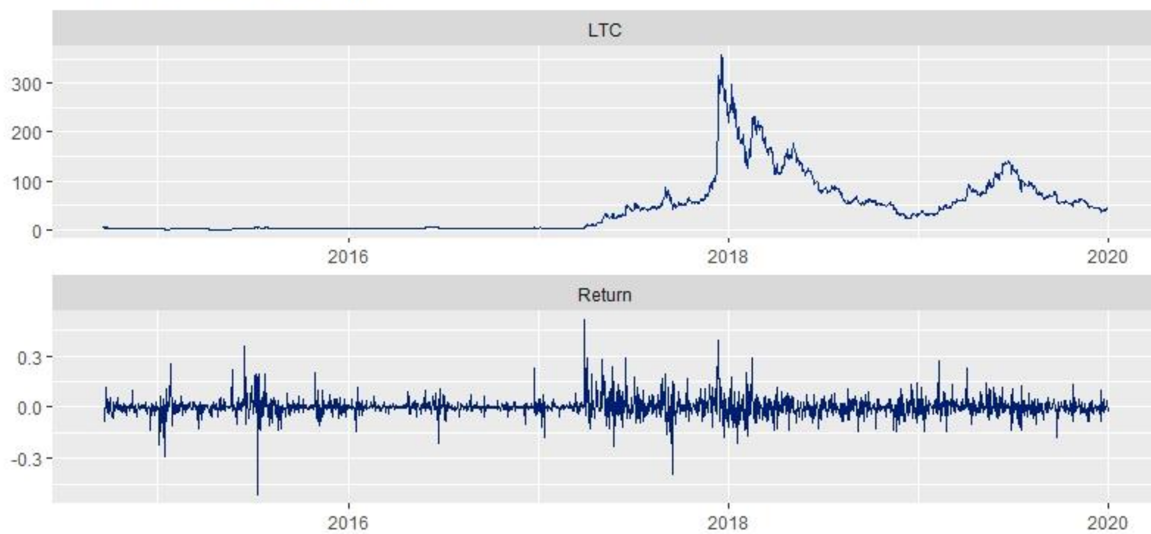

ETH-USD Price and Return
ETH
1000
500
0
2016
2017
2018
2019
2020
Return
0.0
-0.5
-1.0
2016
2017
2018
2019
2020


XRP-USD Price and Return
XRP
3
2
1
0
2016
2018
2020
Return
1.0
0.5
0.0
-0.5
2016
2018
2020

BCH-USD Price and Return

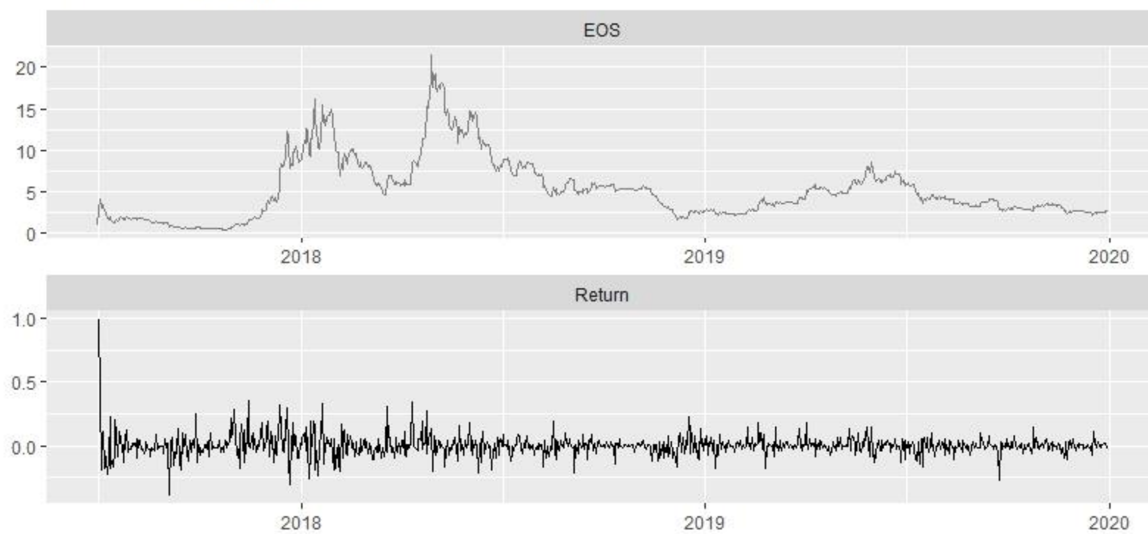


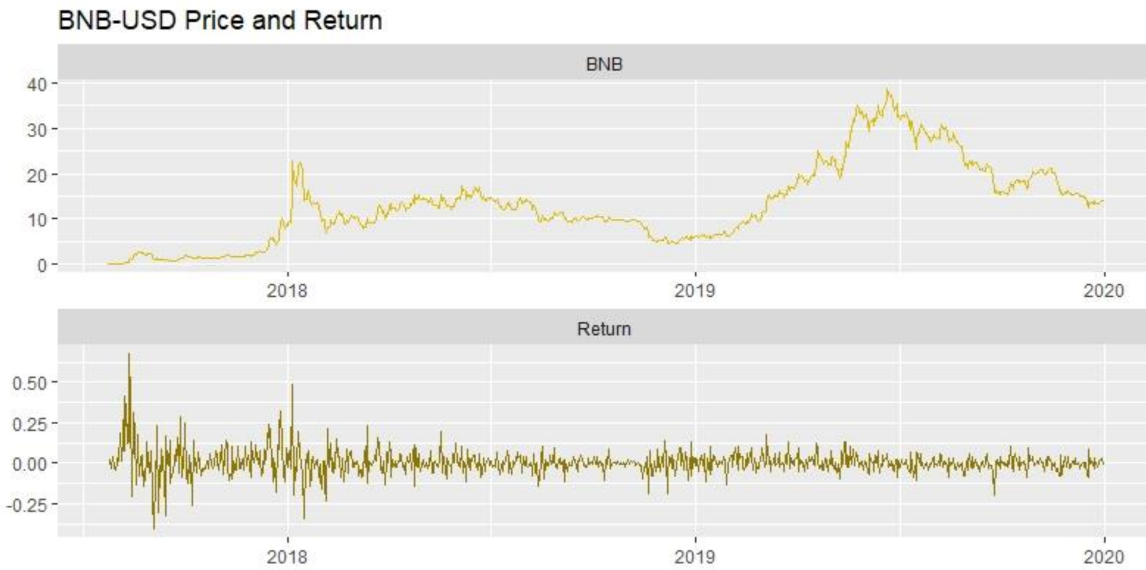
LTC-USD Price and Return

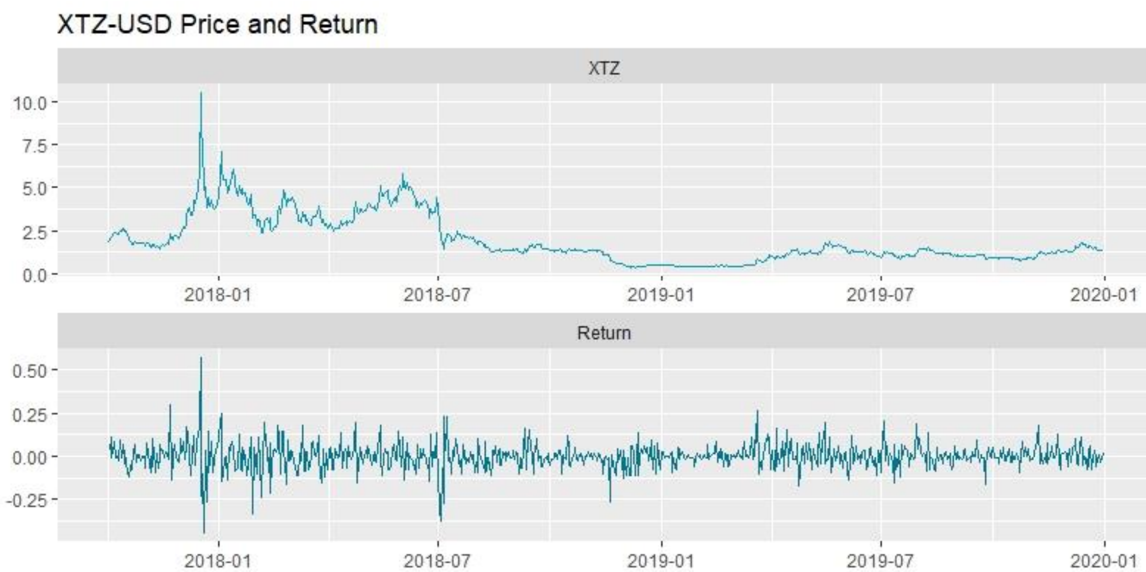


BSV-USD Price and Return

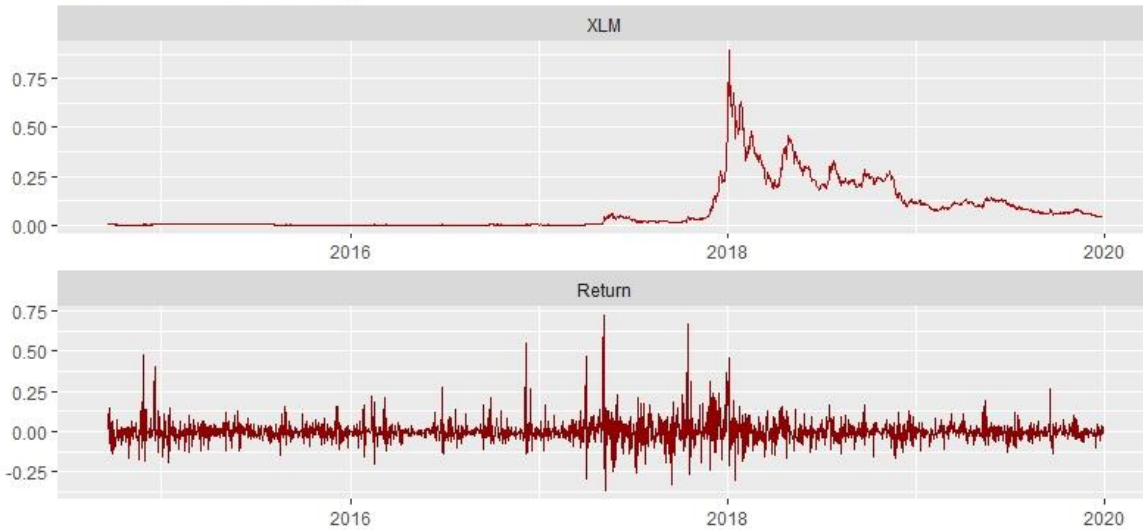
EOS-USD Price and Return



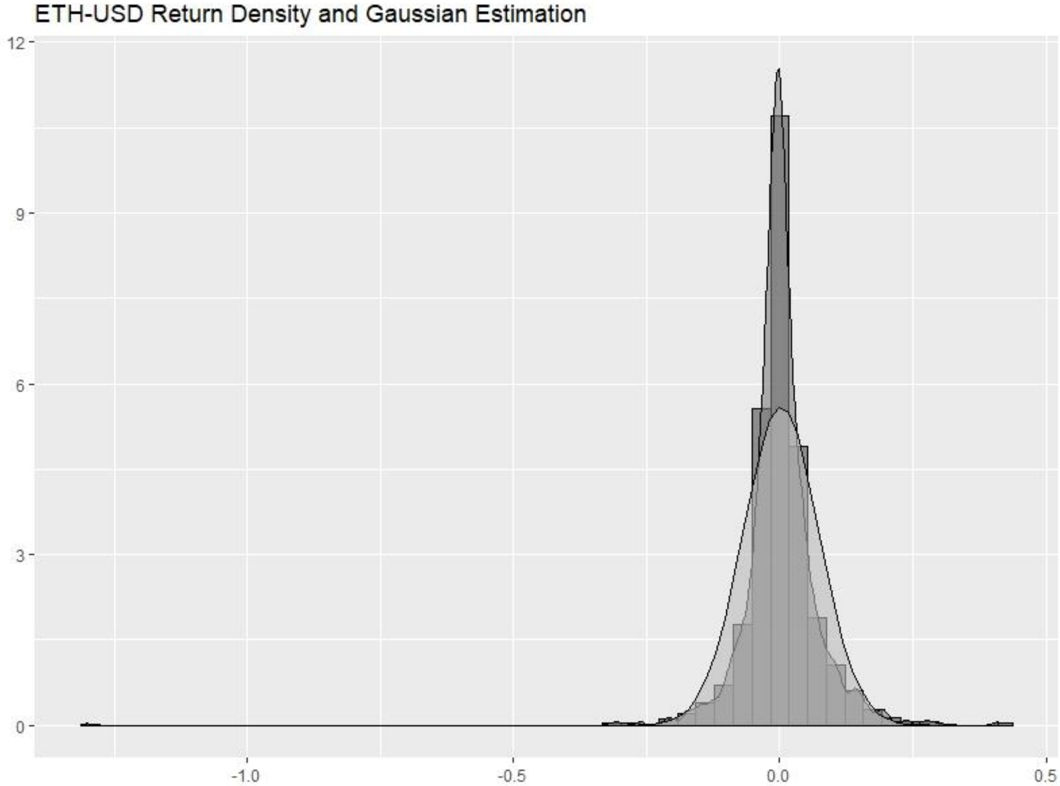

BNB-USD Price and Return
BNB
40
30
20
10
0
2018
2019
2020
Return
0.50
0.25
0.00
-0.25
2018
2019
2020


XTZ-USD Price and Return
XTZ
10.0
7.5
5.0
2.5
0.0
2018-01
2018-07
2019-01
2019-07
2020-01
Return
0.50
0.25
0.00
-0.25
2018-01
2018-07
2019-01
2019-07
2020-01

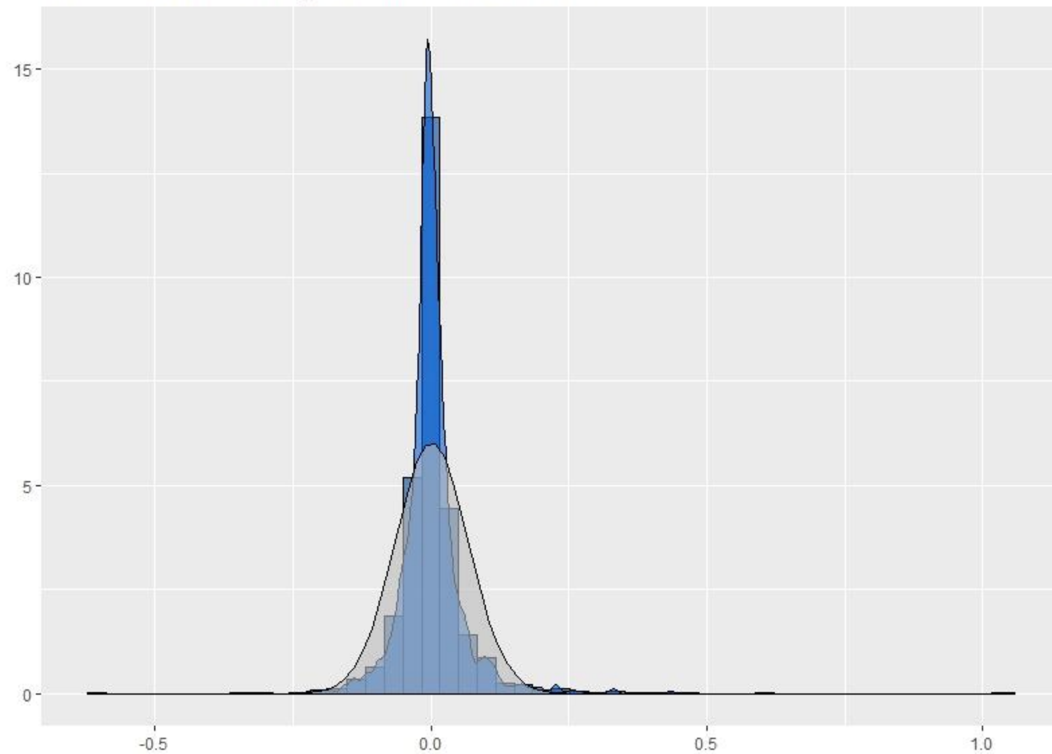
XLM-USD Price and Return



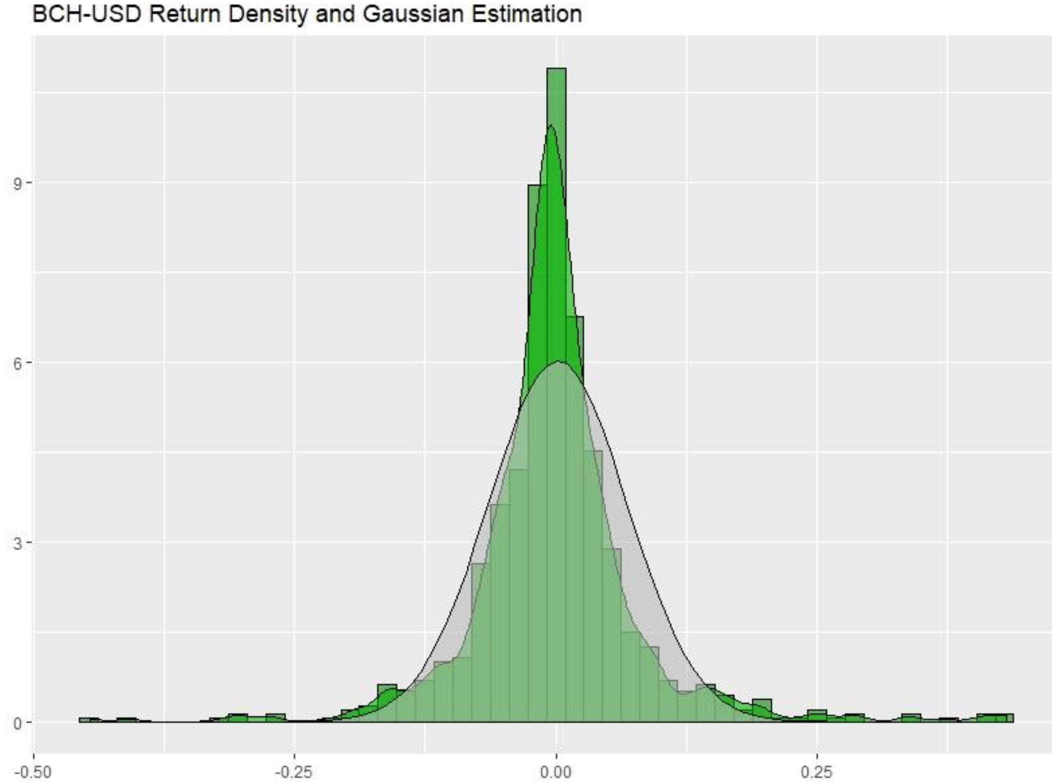
ETH-USD Return Density and Gaussian Estimation



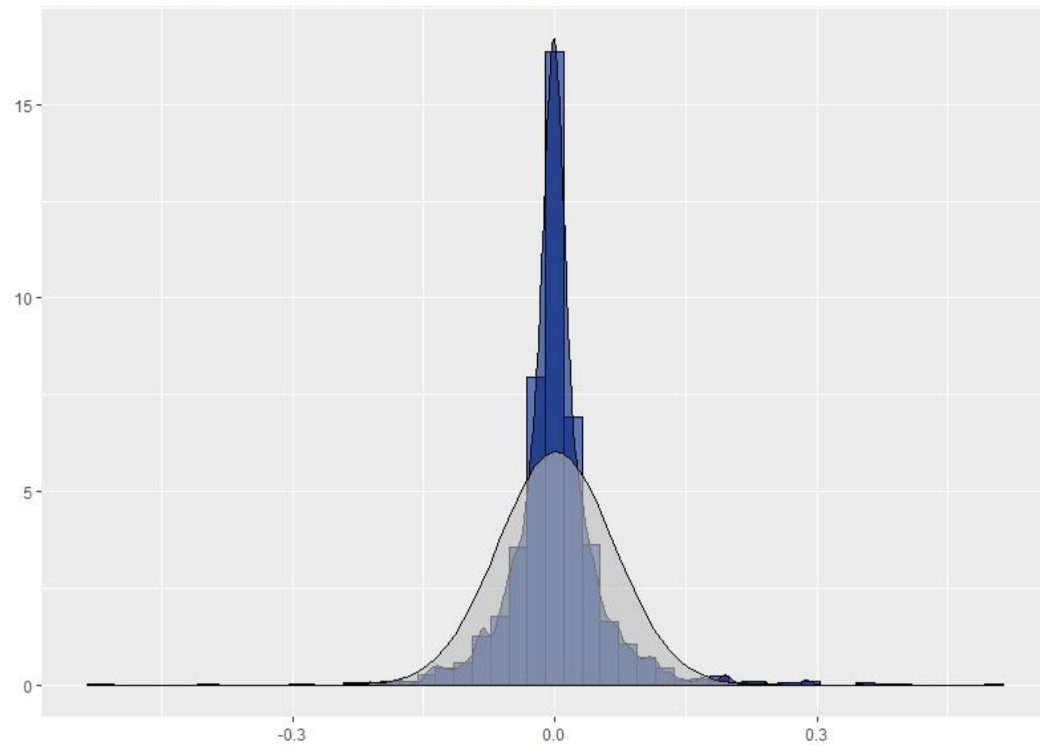
XRP-USD Return Density and Gaussian Estimation



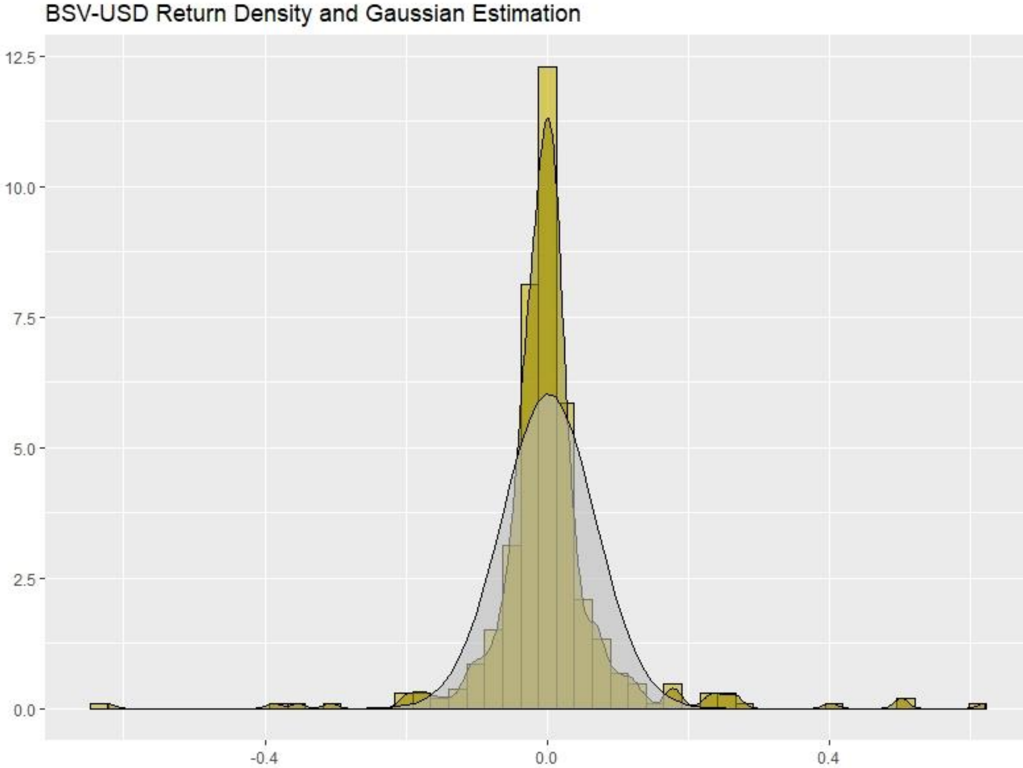
BCH-USD Return Density and Gaussian Estimation



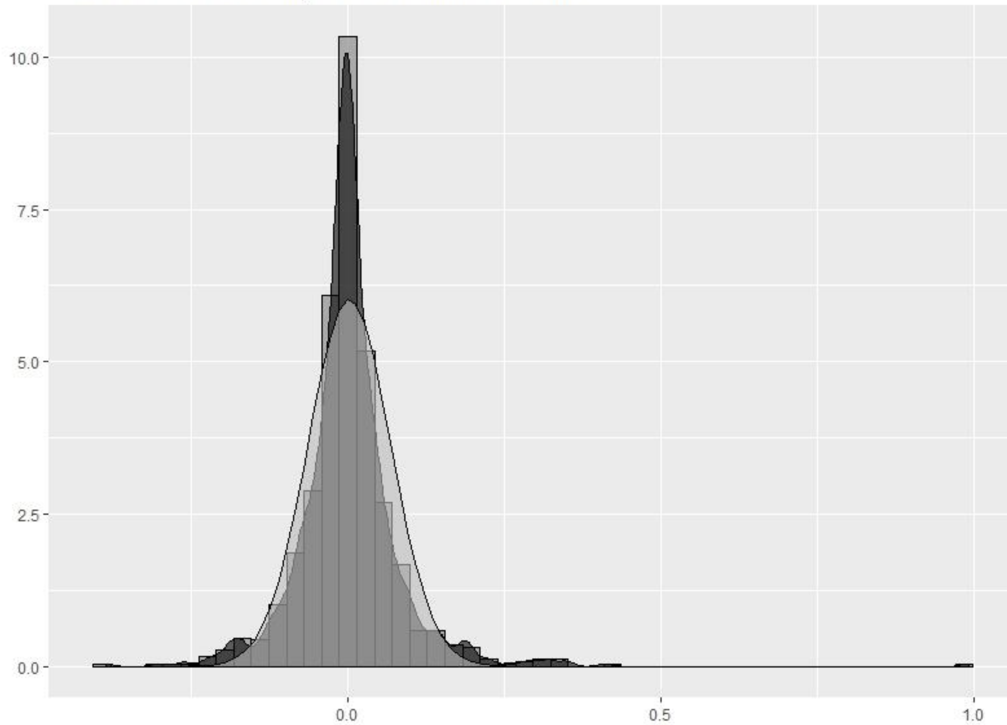
LTC-USD Return Density and Gaussian Estimation



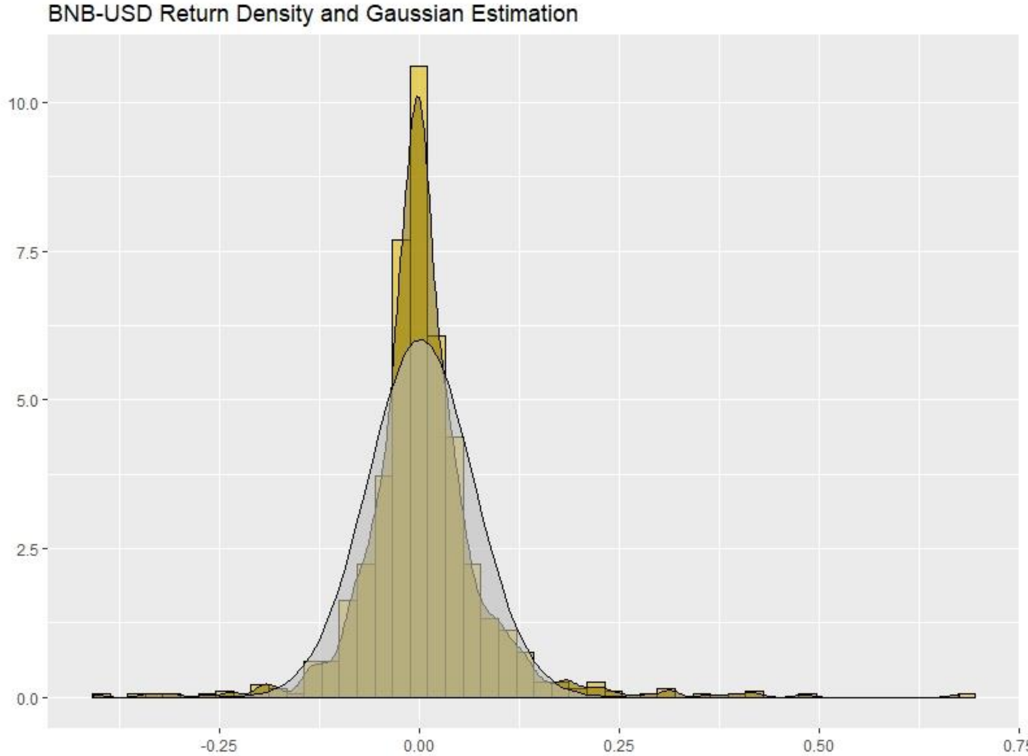
BSV-USD Return Density and Gaussian Estimation



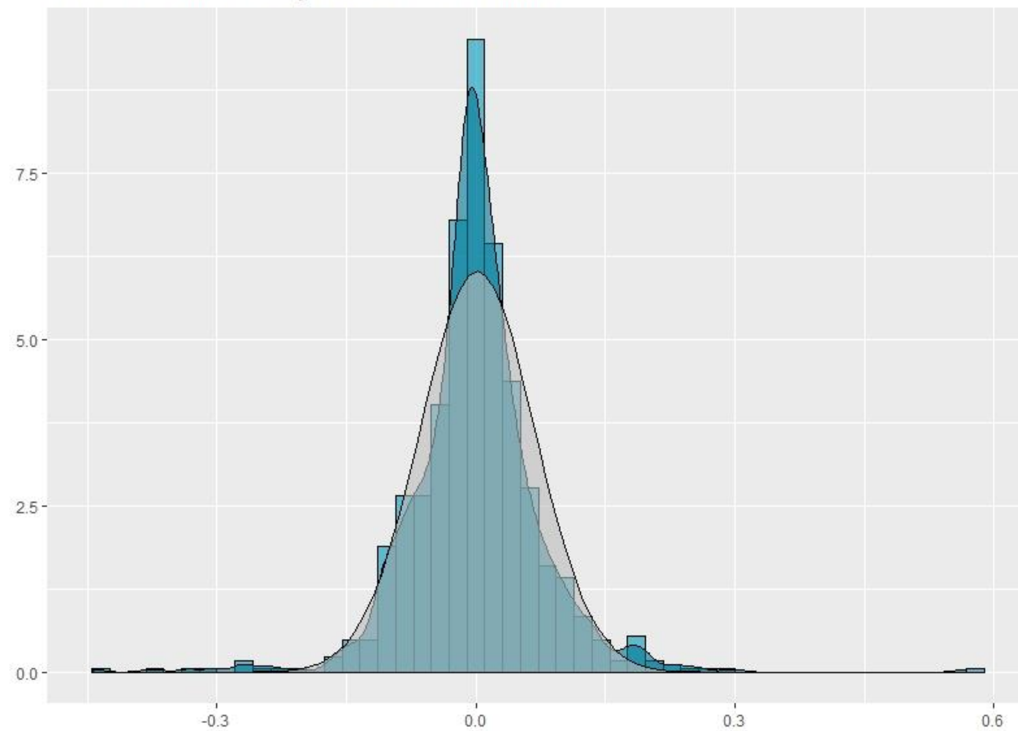
EOS-USD Return Density and Gaussian Estimation



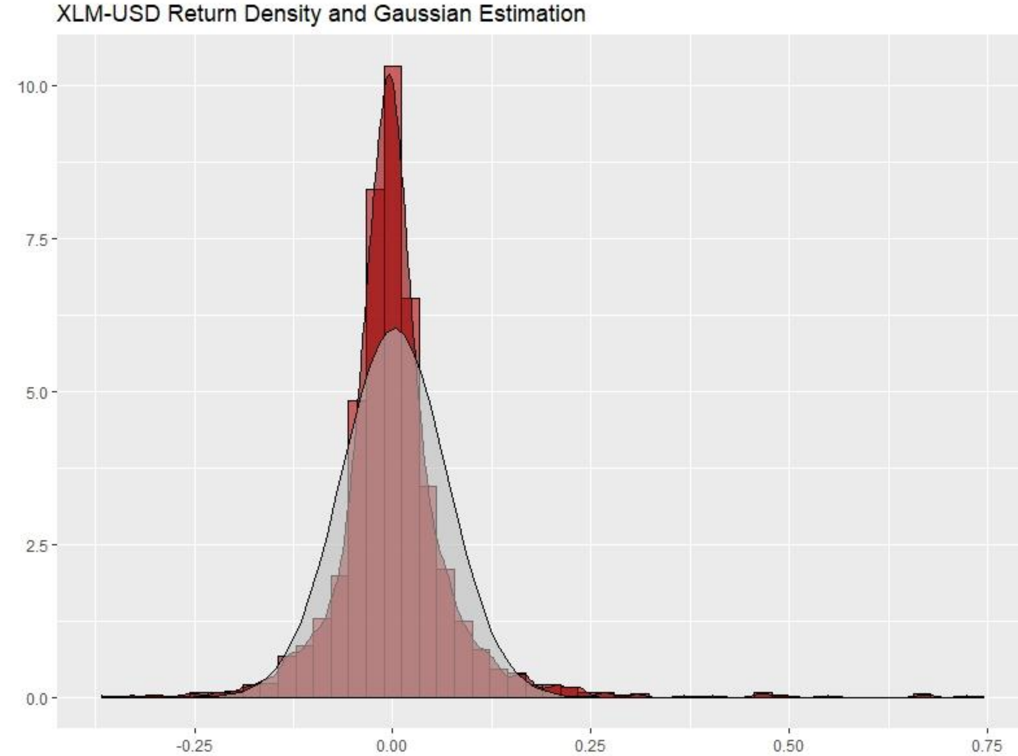
BNB-USD Return Density and Gaussian Estimation



XTZ-USD Return Density and Gaussian Estimation



XLM-USD Return Density and Gaussian Estimation



Annex 2

Table 10. Gaussian parameters for period 1

Series	Mean	Standard Deviation
USD	0.0005933	0.03449061
CNY	0.00071511	0.03464577
GBP	0.00089863	0.03524327
JPY	0.00051414	0.03476215
EUR	0.00081746	0.03504697
RUB	0.00136578	0.03839077
KRW	0.00075834	0.03489146
CAD	0.00084306	0.03491956

Sample mean and standard deviation for period 1

Elaborated by author

Table 11. Gaussian parameters for period 2

Series	Mean	Standard Deviation
USD	0.00554768	0.04177746
CNY	0.00550901	0.04179899
GBP	0.00552473	0.04205814
JPY	0.00570988	0.04225495
EUR	0.00540553	0.04203149
RUB	0.0053665	0.04260797
KRW	0.00540639	0.04190852
CAD	0.0054936	0.04224592

Sample mean and standard deviation for period 2

Elaborated by author

Table 12. Gaussian parameters for period 3

Series	Mean	Standard Deviation
USD	-0.0008871	0.03983615
CNY	-0.0007696	0.03997954
GBP	-0.0007558	0.03963739
JPY	-0.0009572	0.03992305
EUR	-0.0007618	0.03980056
RUB	-0.0006794	0.03986886
KRW	-0.0006859	0.03965073
CAD	-0.0007635	0.03977409

Sample mean and standard deviation for period 3

Elaborated by author

Table 13. Normality tests´ p-values for period 1

Series	AD	SF	LL	CVM	JB
USD	3.70×10^{-24}	4.57×10^{-19}	3.18×10^{-26}	7.37×10^{-10}	0
CNY	3.70×10^{-24}	5.14×10^{-19}	5.13×10^{-26}	7.37×10^{-10}	0
GBP	3.70×10^{-24}	8.48×10^{-19}	5.25×10^{-26}	7.37×10^{-10}	0
JPY	3.70×10^{-24}	9.80×10^{-19}	8.94×10^{-26}	7.37×10^{-10}	0
EUR	3.70×10^{-24}	1.16×10^{-19}	2.60×10^{-26}	7.37×10^{-10}	0
RUB	3.70×10^{-24}	3.96×10^{-19}	3.45×10^{-26}	7.37×10^{-10}	0
KRW	3.70×10^{-24}	2.96×10^{-19}	2.29×10^{-26}	7.37×10^{-10}	0
CAD	3.70×10^{-24}	1.42×10^{-19}	2.07×10^{-26}	7.37×10^{-10}	0

p-values for the normality test under the null hypothesis that samples come from Gaussian distribution. Results for period 1

Elaborated by author

Table 14. Normality tests´ p-values for period 2

Series	AD	SF	LL	CVM	JB
USD	3.70×10^{-24}	7.00×10^{-16}	3.50×10^{-27}	7.37×10^{-10}	0
CNY	3.70×10^{-24}	6.12×10^{-16}	6.63×10^{-27}	7.37×10^{-10}	0
GBP	3.70×10^{-24}	1.09×10^{-16}	2.47×10^{-27}	7.37×10^{-10}	0
JPY	3.70×10^{-24}	1.01×10^{-16}	2.24×10^{-27}	7.37×10^{-10}	0
EUR	3.70×10^{-24}	1.22×10^{-16}	3.79×10^{-27}	7.37×10^{-10}	0
RUB	3.70×10^{-24}	2.10×10^{-16}	1.10×10^{-27}	7.37×10^{-10}	0
KRW	3.70×10^{-24}	1.31×10^{-16}	7.26×10^{-27}	7.37×10^{-10}	0
CAD	3.70×10^{-24}	6.45×10^{-16}	5.91×10^{-27}	7.37×10^{-10}	0

p-values for the normality test under the null hypothesis that samples come from Gaussian distribution. Results for period 2

Elaborated by author

Table 15. Normality tests´ p-values for period 3

Series	AD	SF	LL	CVM	JB
USD	3.70×10^{-24}	8.37×10^{-12}	1.89×10^{-17}	7.37×10^{-10}	0
CNY	3.70×10^{-24}	7.16×10^{-12}	1.16×10^{-17}	7.37×10^{-10}	0
GBP	3.70×10^{-24}	1.89×10^{-12}	1.21×10^{-17}	7.37×10^{-10}	0
JPY	3.70×10^{-24}	6.82×10^{-12}	6.45×10^{-17}	7.37×10^{-10}	0
EUR	3.70×10^{-24}	7.86×10^{-12}	7.40×10^{-17}	7.37×10^{-10}	0
RUB	3.70×10^{-24}	1.64×10^{-12}	5.29×10^{-17}	7.37×10^{-10}	0
KRW	3.70×10^{-24}	1.11×10^{-12}	3.04×10^{-17}	7.37×10^{-10}	0
CAD	3.70×10^{-24}	1.93×10^{-12}	1.22×10^{-17}	7.37×10^{-10}	0

p-values for the normality test under the null hypothesis that samples come from Gaussian distribution. Results for period 3

Elaborated by author

Table 16. NIG parameters for period 1

Series	Mu	Delta	Alpha	Beta
USD	0.00154973	0.01643816	13.3845177	-0.7808852
CNY	0.00160107	0.01647261	13.2511966	-0.7180989
GBP	0.00140049	0.01728209	13.5202783	-0.3959853
JPY	0.00173888	0.01775741	14.5421282	-1.000637
EUR	0.00187378	0.01787566	14.3916269	-0.8493242
RUB	0.00175328	0.02235079	14.8034164	-0.2558583
KRW	0.00183364	0.01807578	14.605989	-0.8682435
CAD	0.0018773	0.01733094	13.8383085	-0.8257623

NIG parameters for period 1

Elaborated by author

Table 17. NIG parameters for period 2

Series	Mu	Delta	Alpha	Beta
USD	0.00476355	0.01963383	10.0119824	0.40060928
CNY	0.00490142	0.01974805	10.0903326	0.3095503
GBP	0.00534242	0.02130837	11.0653821	0.09497425
JPY	0.00557589	0.02171893	11.2951092	0.0608597
EUR	0.00503606	0.0205177	10.4715609	0.18560599
RUB	0.00503418	0.02172827	10.9515931	0.17131322
KRW	0.00466985	0.02091286	10.83432	0.38471586
CAD	0.00495131	0.0204373	10.3436331	0.2759504

NIG parameters for period 2

Elaborated by author

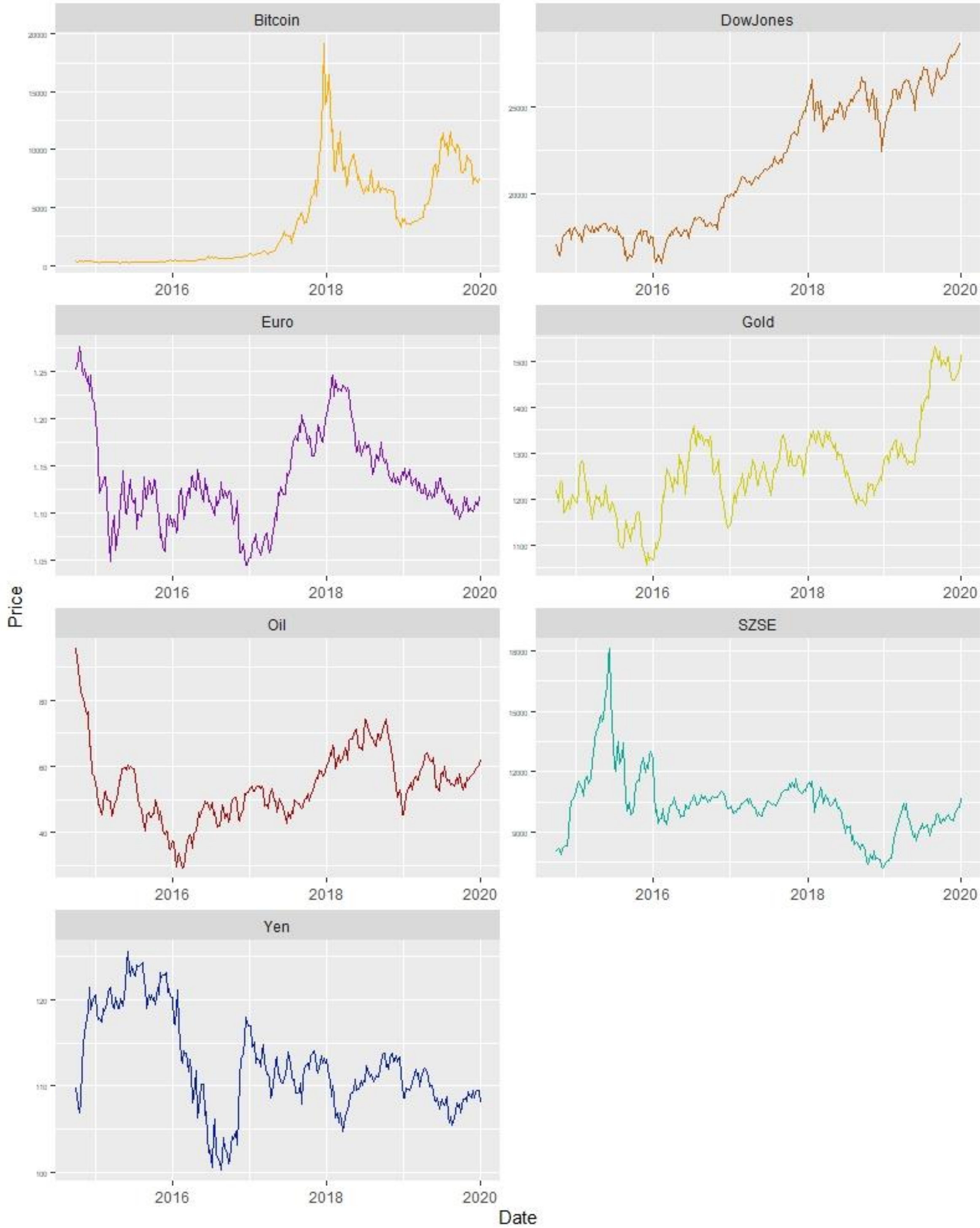
Table 18. NIG parameters for period 3

Series	Mu	Delta	Alpha	Beta
USD	0.00280472	0.02077849	11.8696497	-2.0759134
CNY	0.00262456	0.02085664	11.7902838	-1.8960877
GBP	0.00289485	0.02112235	12.2047418	-2.075692
JPY	0.00326557	0.02103738	12.156312	-2.3908889
EUR	0.00302283	0.02061438	11.8002435	-2.1321365
RUB	0.00254179	0.02116651	11.9985926	-1.8064159
KRW	0.00306941	0.02124813	12.3888875	-2.159749
CAD	0.0028861	0.02183632	12.6831333	-2.0918932

NIG parameters for period 3

Elaborated by author

Annex 3


Bitcoin
DowJones
Euro
Gold
Oil
SZSE
Yen
Price
Date
2016
2018
2020

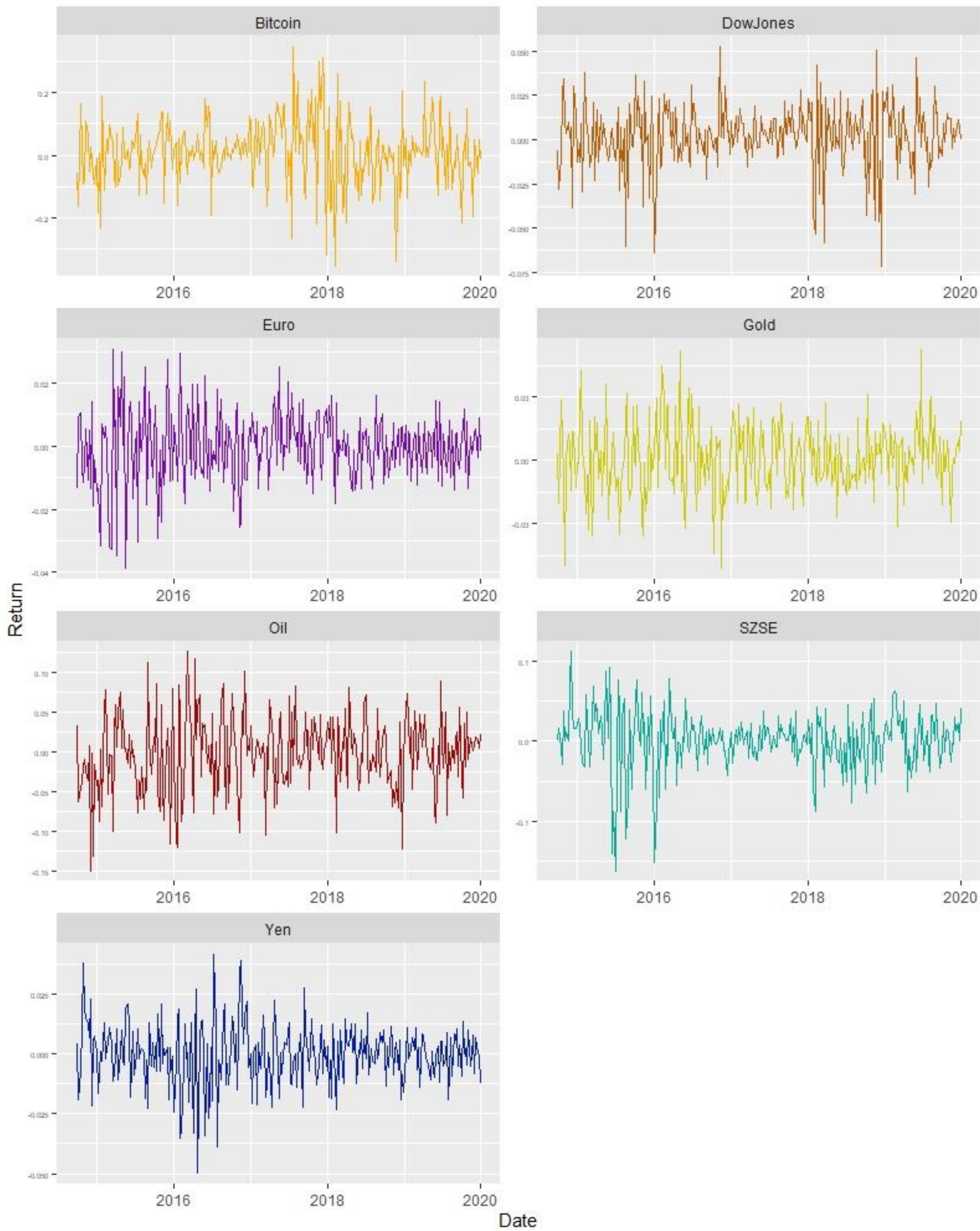

Bitcoin
DowJones
Euro
Gold
Oil
SZSE
Yen
Return
Date
2016
2018
2020

Table 29. Gaussian Parameters for Currency data

	E[x]
Bitcoin	0.01063192
Euro	-0.0004658
Yen	-3.98E-05

Sample expected value vector for currencies portfolio

Elaborated by author

Table 29. Gaussian Parameters for Currency data

V[x]	**Bitcoin**	**Euro**	**Yen**
Bitcoin	0.01128532	4.35E-05	3.10E-05
Euro	4.35E-05	0.00013065	-6.52E-05
Yen	3.10E-05	-6.52E-05	0.00016086

Sample covariance matric for currencies portfolio

Elaborated by author

Table 30. Gaussian Parameters for Commodity data

	E[x]
BTC	0.01063192
Gold	0.00079702
Oil	-0.0014662

Sample expected value vector for commodity portfolio

Elaborated by author

Table 30. Gaussian Parameters for Commodity data

V[x]	**Bitcoin**	**Gold**	**Oil**
Bitcoin	0.01128532	2.79E-05	4.47E-05
Gold	2.79E-05	0.00028495	6.05E-05
Oil	4.47E-05	6.05E-05	0.00218681

Sample covariance matric for commodity portfolio

Elaborated by author

Table 31. Gaussian Parameters for Index data

	E[x]
BTC	0.01063192
Dow Jones	0.00187192
SZSE	0.00100964

Sample expected value vector for Index portfolio

Elaborated by author

Table 31. Gaussian Parameters for Index data

V[x]	**Bitcoin**	**Dow Jones**	**SZSE**
Bitcoin	0.01128532	0.00019463	0.00024558
Dow Jones	0.00019463	0.00033758	0.00021696
SZSE	0.00024558	0.00021696	0.00136894

Sample covariance matric for Index portfolio

Elaborated by author

Table 32. NIG Parameters for Currency data

	E[x]
Bitcoin	0.01063065
Euro	-0.0004657
Yen	-3.99E-05

Expected value vector for Currency portfolio

Elaborated by author

Table 32. NIG Parameters for Currency data

V[x]	**Bitcoin**	**Euro**	**Yen**
Bitcoin	0.01141237	3.24×10^{-5}	3.11×10^{-5}
Euro	3.24×10^{-5}	0.0001302	-6.36×10^{-5}
Yen	3.11×10^{-5}	-6.36×10^{-5}	0.00015868

Covariance matric for Currency portfolio

Elaborated by author

Table 33. NIG Parameters for Commodity data

	E[x]
Bitcoin	0.01063131
Gold	0.00079725
Oil	-0.0014667

Expected value vector for Commodity portfolio

Elaborated by author

Table 33. NIG Parameters for Commodity data

V[x]	**Bitcoin**	**Gold**	**Oil**
Bitcoin	0.01101986	2.50×10^{-5}	2.97×10^{-5}
Gold	2.50×10^{-5}	0.00028838	5.59×10^{-5}
Oil	2.97×10^{-5}	5.59×10^{-5}	0.00221962

Covariance matric for Commodity portfolio

Elaborated by author

Table 34. NIG Parameters for Index data

	E[x]
BTC	0.01062988
Dow Jones	0.00187115
SZSE	0.00100803

Expected value vector for Index portfolio

Elaborated by author

Table 34. NIG Parameters for Index data

V[x]	**Bitcoin**	**Dow Jones**	**SZSE**
BTC	0.01223759	0.00017689	0.00020145
Dow Jones	0.00017689	0.00033415	0.00019478
SZSE	0.00020145	0.00019478	0.0012939

Covariance matric for Commodity portfolio

Elaborated by author

References

1. Al Mamun M., Uddin G. S., Suleman M. T. & Kang S. H. (2019) Geopolitical risk, uncertainty and Bitcoin investment. *Physica A, 540* (2020) 123107. Retrieved from https://doi.org/10.1016/j.physa.2019.123107
2. Almudhaf Fahad (2018). Pricing efficiency of Bitcoin Trust. *Applied Economics Letters, 25*(7), 504-508, Retrieved from https://doi.org/10.1080/13504851.2017.1340564
3. Alvarez-Ramirez J., Rodrigues E. and Ibarra-Valdez C. (2017). Long-Range Correlations and Asymmetry in the Bitcoin Market. *Physica A, 492*, 948-955. Retrieved from 10.1016/j.physa.2017.11.025
4. Anderson Niall H, Hall Peter and Tritterington (1994). Two-Sample Test Statistics for Measuring Discrepancies between Two Multivariate Probability Density Functions Using Kernel-Based Density Estimates. *Journal of Multivariate Analysis, 50*, 41-54
5. Anderson T. W. and Darling D. A. (1952). Asymptotic Theory of Certain “Goodness of Fit” Criteria Based on Stochastic Processes. *Annals of Mathematical Statistics, 23*(2), 193-212. Retrieved from doi:10.1214/aoms/1177729437
6. Angel James J. and McCabe Douglas (2015). The Ethics of Payments: Paper, Plastic, or Bitcoin? *Journal of Business Ethics, 132*, 603-611. Retrieved form 10.1007/s10551-014-2354-x
7. Bachélier L. (1900). The Theory of Speculation. *Annales scientifiques de l´École Normale Supérieure, 3*, 21-86. Retrieved from https://doi.org/10.24033/asens.476
8. Baek C. and Elbeck M. (2015). Bitcoins as an investment or speculative vehicle? A first look. *Applied Economics Letters, 22*(1), 30-34. Retrieved from http://dx.doi.org/10.1080/13504851.2014.916379
9. Bagnold R. A. (1956). The flow of cohesionless grains in fluids. Phil. Trans. *R. Soc. Lond. A, 249*, 235-297
10. Bagnold R. Añ (1954). *The physics of blown sand and desert dunes*. London:Methuen

11. Bariviera A. F., Basgall M. J., Hasperué W. and Naiouf M. (2017). Some Stylized Facts of the Bitcoin Market. *Physica A, 484*, 82-90. Retrieved from 10.1016/j.physa.2017.04.159
12. Barndorff-Nielsen O. (1977) Exponentially decreasing distributions for the logarithm of particle size. *Proceedings of the Royal Society A, 353*(1674), 401-4019. Retrieved from https://doi.org/10.1098/rspa.1977.0041
13. Barndorff-Nielsen O.E. and Shepard N. (2001). Modelling by Lévy Processed for Financial Econometrics. In *Lévy Processes*, Birkhäuser, Boston. Retrieved from https://doi.org/10.1007/978-1-4612-0197-7_13
14. Barndorff-Nielsen Ole E. (1997) Normal Inverse Gaussian Distribution and Stochastic Volatility Modelling. *Scandinavian Journals of Statistics, 24*(1), 1-13, Retrieved from https://onlinelibrary.wiley.com/doi/abs/10.1111/1467-9469.00045
15. Basel Committee on Banking Supervision (2019). Minimum capital requirements for market risk. *Bank for International Settlements*. Retrieved from https://www.bis.org/basel_framework/
16. Baur D. G., Thomas D., Konstantin K. (2018) Bitcoin, gold and the US dollar – A replication and extension. *Finance Research Letters, 25* (2018), 103–110. Retrieved from http://dx.doi.org/10.1016/j.frl.2017.10.012
17. Bouchaud Jean-Phillipe and Potters Marc (2001) More stylized facts of Financial markets: leverage effect and downside correlations. *Physica A, 299*, 60-70. Retrieved from https://doi.org/10.1016/S0378-4371(01)00282-5
18. Bouoiyour J. & Refk S. (2016) Bitcoin: A beginning of a new phase? *Economics Bulletin, 36*(3), 1430–1440. Retrieved from https://www.researchgate.net/publication/305810738
19. Bouoiyour Jamal and Selmi Fefk (2015). Bitcoin Price: Is It Really that New Round of Volatility can be on way*? CAAT University of Pau*, France ESC, Tunis Business School of Tunis, Tunisia, paper No. 65680
20. Bouri E., Hussain S. S. J. & Roubaud D. (2019) Cryptocurrencies as hedges and safe-havens for US equity sectors. *The Quarterly Review of Economics*

and Finance (2019). Retrieved from https://doi.org/10.1016/j.qref.2019.05.001

21. Bouri Elie, Jalkh Naji, Molnár Peter and Roubaud David (2017). Bitcoin for energy commodities before and after the December 2013 crash: diversifier,hedge or safe heaven? *Applied Economics, 49*(50), 5063-5073. Retrieved from https://doi.org/10.1080/00036846.2017.1299102
22. Brooks Chris (2014). *Introductory Econometrics for Finance*. Cambridge, UK: Cambridge University Press.
23. Brunnermeier Markus K. and Oehmke Martin (2012). Bubbles, Financial Crises, and Systemic Risk, Working paper 18398, *National Bureau of Economic Research* 1050 Massachusetts Avenue Cambridge, MA 02138
24. Buchholz Martis, Delaney Jess, Parker Joseph and Warren Jeff (2012). Bits and Bets Information, Price Volatility, and Demand for Bitcoin. *Economics, 312*, 2-48. Retrieved from https://www.reed.edu/economics/parker/s12/312/finalproj/Bitcoin.pdf
25. Bueno Pedro Bonillo, Fortes Emilio Aragon and Valchoski Konstantinos (2017). Speculative investment, heavy-tailed distribution, and risk management of Bitcoin exchange rate returns. *Journal of Progressive Research in Social Sciences, 5*(1). Retrieved from http://scitecresearch.com/journals/index.php/jprss/article/view/1225
26. Caginalp Gundez, Porter David and Smith Vernon (2001) Financial Bubbles: Excess Cash, Momentum, and Incomplete Information. *The Journal of Psychology and Financial Markets, 2*(2), 80-99. Retrieved from http://www.pitt.edu/~caginalp/JPFM2000.PDF
27. Calin Ovidiu (2015). *An Informal Introduction to Stochastic Calculus with Applications*. Michigan, USA: World Scientific Publishing Co.
28. Cambell John Y., Lo Andrew W. and MacKinay A. Craig (1996). *The Econometrics of Financial Markets*. New Jersey, USA: Princeton University Press
29. Cermak V. (2017) Can Bitcoin become a viable alternative to fiat currencies? An empirical analysis of Bitcoin's volatility based on a GARCH

model. *SSRN Electronic Journal*. Retrieved from: 10.2139/ssrn.2961405

30. Charfeddine L., Benlagha N. & Maouchi Y. (2020) Investigating the dynamic relationship between cryptocurrencies and conventional assets: Implications for financial investors. *Economic Modelling, 85* (2020), 198-217. Retrieved from https://doi.org/10.1016/j.econmod.2019.05.016
31. Cheah Eng-Tuck and Fry J. M. (2015). Speculative bubbles in Bitcoin markets? An empirical investigation into the fundamental value of Bitcoin. *Economic Letters, 130*, 32-36. Retrieved from 10.1016/j.econlet.2015.02.029
32. Cheung Adrian (Wai-Kong), Roca Eduardo and Su Jen-Je (2015). Crypto-currency bubbles: an application of the Phillips Shi-Yu (2013) methodology on Mt. Gox Bitcoin prices. *Applied Economics, 47*(23), 2348-2358. Retrieved from http://dx.doi.org/10.1080/00036846.2015.1005827
33. Chu J, Nadarajah S. and Chan S. (2015). Statistical Analysis of the Exchange Rate of Bitcoin. *PLoS ONE, 10*(7). Retrieved from 10.1371/journal. pone.0133678
34. Cian P., Raicaniova M. and Kancs D. A. (2016). The economics of BitCoin price formation. *Applied Economics, 48*(19), 1799-1815. Retrieved from 10.1016/j.procs.2017.11.013
35. Coase Ronald H. (1992). The Institutional Structure of Production. *Occasional Papers, 28*, 1-16. Retrieved from http://chicagounbound.uchicago.edu/occasional_papers
36. Cont Rama (2001) Empirical properties of asset returns:stylized facts and statistical issues. *Quantitative Finance, 1*, 223-236. Retrieved from http://rama.cont.perso.math.cnrs.fr/pdf/empirical.pdf
37. Corbet S., Meegan A., Larkin C., Lucey B. & Yarovaya L. (2018) Exploring the dynamic relationships between cryptocurrencies and other financial assets. *Economics Letters, 165* (2018), 28–34. Retrieved from https://doi.org/10.1016/j.econlet.2018.01.004
38. Corelli A. (2018) Cryptocurrencies and Exchange Rates: A Relationship and Causality Analysis. *Risk, 6*(4), 111. Retrieved from 10.3390/risks6040111

39. Davidson Laura and Block Walter E. (2015). Bitcoin, the Regression Theorem, and the emergence of a new medium of exchange. *The Quarterly Journal of Austrian Economics, 18*(3), 311-338. Retrieved from https://mises.org/library/bitcoin-regression-theorem-and-emergence-new-medium-exchange
40. Demsetz Harold (1990). *Ownership, Control and the Firm.* Oxford, England: Blackwell Publushing
41. Diba Behzad T. and Grossman Herschel I. (1988). Explosive Rational Bubbles in Stock Prices? *The American Economic Review, 78*(3), 520-530. Retrieved from https://www.jstor.org/stable/1809149?seq=1
42. Dicker David A. and Fuller Wayne A. (1979). Distribution of the Estimators for Autoregressive Time Series with a Unit Root. *Journal of the American Statistical Association, 74*(36), 427-431. Retrieved from 10.2307/2286348
43. Dimitrova V., Fernández-Martinez M., Sánchez-Granero J. E. and Segovia Trinidad (2019). Some comments on Bitcoin market (in)efficiency. *PLoS ONE, 14*(7). Retrieved from https://doi.org/10.1371/journal.pone.0219243
44. Dobrow Robert P. (2014). *Probability with Applications and R*. Northfield, USA: Wiley & Sons, Inc.
45. Duffie D. and Pan J. (1997). An Overview of Value at Risk. *Journal of Derivatives, 4*, 7-49. Retrieved from http://dx.doi.org/10.3905/jod.1997.407971
46. Eberlein E. and Keller U. (1995). Hyperbolic distributions in finance. *Bernoulli, 1*(3), 281-299. Retrieved from 10.3150/bj/1193667819
47. Eberlein E. and Prause K. (2000). The Generalized Hyperbolic Model: Financial Derivatives and Risk Measures. In Mathematical Finance, *Bachelier Congress 2000*, Berlin Springer Finance, 245-267. Retrieved from 10.1007/978-3-662-12429-1_12
48. European Central Bank (2015). Virtual currency schemes – a further analysis. *Eurosystem*. Retrieved from 10.2866/662172
49. Frisby D. (2004) *Bitcoin: The Future of Money?* UK: Unbound

50. Fry John and Cheah Eng-Tuck (2016). Negative bubbles ans shocks in crypto-currency markets. *International Review of Financial Analysis, 47*, 343-352, Retrieved from https://doi.org/10.1016/j.irfa.2016.02.008
51. Gil-Alana L. A., Aikins A. E. J. & Romero R. M. F. (2020) Cryptocurrencies and stock market indices. Are they related? *Research in International Business and Finance, 51*(2020). Retrieved from 101063. https://doi.org/10.1016/j.ribaf.2019.101063
52. Godsiff P. (2015). Bitcoin: Bubble or Blockchain. In Agent and Multi-Agent Systems: Technologies and Applications, *Springer*, 191-203. Retrieved from 10.1007/978-3-319-19728-9_16
53. Greenspan A. (2002). *Economic Volatility*. At a symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, August 30, 2002
54. Grinberg Reuben (2001). Bitcoin: An Innovative Alternative Digital Currency. *Hasting Science & Technology Lay Journal, 4*, 159-208. Retrieved from https://ssrn.com/abstract=1817857
55. Harvey D. L., Leybourne S. J., Sollis R. and Taylor A. M. R. (2016). Tests for explosive financial bubbles in the presence of non-stationary volatility. *Journal of Empirical Finance, 38*, 548-574. Retrieved from https://doi.org/10.1016/j.jempfin.2015.09.002
56. Haubo A. D. (2015). Bitcoin, gold and the dollar – A GARCH volatility analysis. *Finance Research Letters, 16*, 85-92. Retrieved from https://doi.org/10.1016/j.frl.2015.10.008
57. Hull John C. (1997). Options, *Futures, and Other Derivatives.* USA: Pearson Pretice Hall
58. Hussain S. S. J., Bouri E., Roubaud D. & Kristoufek L. (2019) Safe haven, hedge and diversification for G7 stock markets: Gold versus bitcoin. *Economic Modelling* (2019). Retrieved from https://doi.org/10.1016/j.econmod.2019.07.023
59. J.P. Morgan/Reuters (1996). *RiskMetrics – Technical Document*. New York, USA Fourth Edition

60. Jarque Carlos M. and Bera Anil K. (1980). Efficient tests for normality, homoscedasticity, and serial independence for regression residuals. *Economic Letters, 6*(3), 255-259. Retrieved from https://EconPapers.repec.org/RePEc:eee:ecolet:v:6:y:1980:i:3:p:255-259
61. Ji Q., Bouri E., Keung M. L. and Rouband D. (2019). Dynmic connectedness and integration in cryptocurrency markets. *International Review of Financial Analysis, 63*, 257-272. Retrieved from https://doi.org/10.1016/j.irfa.2018.12.002
62. Joerg Osterrieder (2017). The Statistics of Bitcoin and Cryptocurrencies. *Advances in Economics, Business and Management Research, 26*. Retrieved from 10.2991/icefs-17.2017.33
63. Jovan Matej and Ahčan Aleš (2017) Default prediction with the Merton-type structural model based on the NIG Lévy process. *Journal of Computational and Applied Mathematics, 311*, 414-422. Retrieved from https://doi.org/10.1016/j.cam.2016.08.007
64. Juhász Páter, Stéger József, Kondor Dániel and Vattay Gábor (2018). A Bayesian approach to identify Bitcoin users. *PLos ONE, 13*(12), 1-21. Retrieved from https://doi.org/10.1371/journal.pone.0207000
65. Katsiampa P. (2017) Volatility estimation for Bitcoin: A comparison of GARCH models. *Economics Letters, 158* (2017), 3–6. Retrieved from https://doi.org/10.1016/j.econlet.2017.06.023
66. Kloter P., Kartajaya H. and Setiawa I. (2010) *Marketing 3.0*. Barcelona, Spain: LID Editorial
67. Kurek Robert (2015). Bitcoin vs. Legal and Tax Regulation in Poland and Wolrdwide. *Research Papers of Wroclaw University of Economics, 39*(7), 154-161. Retrieved from 10.15611/pn.2015.397.11
68. Kwiatkowski Denis, Phillips Peter, Schmidt Peter and Shin Yongcheol (1992) Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics, 54*(3), 159-178. Retrieved from http://www.sciencedirect.com/science/article/pii/0304-4076(92)90104-Y

69. Lambert Elizabeth E. (2016) The Internal Revenue Service and Bitcoin: A Taxing Relationship. *Virginia Tax Renevue, 35*(1), 88-115. Retrieved from https://heinonline.org/HOL/LandingPage?handle=hein.journals/vrgtr35&div=6&id=&page=
70. Lilleställ Jostein (2002) Some crude approximation, calibration and estimation procedures for NIG-variates. *Discussion Papers, 85*(373). Retrieved from https://www.researchgate.net/publication/318523427_Some_crude_approximation_calibration_and_estimation_procedures_for_NIG-variates
71. Lilliefors Hubert W. (1967). On the Kolmogorov-Smirnov Test for Normality with Mean and Variance Unknown. *Journal of the American Statistical Association, 62*(318), 399-402. Retrieved from 10.2307/2283970
72. Lintilhac P. S. and Tourin A. (2017) Model-based pairs trading in the Bitcoin markets. *Quantitative Finance, 17*(5), 703-716. Retrieved from http://dx.doi.org/10.1080/14697688.2016.1231928
73. Mai Feng, Shan Zhe, Bai Qing, Wang (Shane) Xin and Chiang Roger H.L. (2018). How Does Social Media Impact Bitcoin Value? A Test of the Silent Majority Hypothesis. *Journal of Management Information Systems, 35*(1). 19-53. Retrieved from https://doi.org/10.1080/07421222.2018.1440774
74. Malik Muhammad Irfan and Rehman ur Atiq (2015) Choice of Spectral Density Estimator in Ng-Perron Test: A Comparative Analysis. International *Econometric Review, 7*(2), 51-63. Retrieved from https://doi.org/10.33818/ier.278040
75. Mantegna Rosario N., Palágyi Zoltán and Stanley Eugene (1999). Applications of Statistical Mechanics to Finance. *Physica A*, 274, 216-221. Retrieved from https://doi.org/10.1016/S0378-4371(99)00395-7
76. Markowitz Harry (1952). Portfolio Selection. *The Journal of Finance, 7*(1), 77-91, Retrieved from https://doi.org/10.1111/j.1540-6261.1952.tb01525.x
77. Menger Carl (1892). *Principles of Economics*. Auburn, EUA: Ludwig Von Mises Institute 2009

78. Min L., Kontosakos E. E., Pantelous A. A. and Zhou J. (2019). Cryptocurrencies: Dust in the wind? *Physica A, 525*, 1063-1079. Retrieved from https://doi.org/10.1016/j.physa.2019.03.123
79. Nakamoto Satoshi (2009). Bitcoin: A Peer-tp-Peer Electronic Cash System. Retrieved from https://bitcoin.org/bitcoin.pdf
80. Ng Serena and Perron Pierre (2003) LAG Length Selection and the Construction of Unit Root Tests with Good Size and Power. *Econometrica, 69*(6), 1519-1554. Retrieved from https://doi.org/10.1111/1468-0262.00256
81. Nguyen Tram, de Bodisco Christopher and Thaver Ranjini (2018) Factors Affecting Bitcoin Price in the Cryptocurrency Market: An Empirical Study. *International Journal of Business and Economics Perspectives, 13*(1), 106-125. Retrieved from https://www.researchgate.net/publication/327601839_FACTORS_AFFECTING_BITCOIN_PRICE_IN_THE_CRYPTOCURRENCY_MARKET_AN_EMPIRICAL_STUDY
82. Nuñez J. A., Contreras-Valdez M. I., Ramirez-García A. and Sánchez-Ruenes E. (2018). Underlying Assets Distribution in Derivatives: The BRIC Case. *Theoretical Economics Letters, 8*, 502-513. Retrieved from 10.4236/tel.2018.83035
83. O´Donnell Matt (2018). Beyond Bitcoin. *Policy, 34*(1), 30-33. Retrieved from https://search.informit.com.au/documentSummary;dn=518585614093685;res=IELAPA
84. O´Sullivan Andrea (2018). Ungoverned or Anti-Governance? Bitcoin Threatens the Future of Western Institutions. *Journal of International Affairs, 71*(2), 90-102. Retrieved from https://www.jstor.org/stable/e26552322
85. Pan Wenliang, Tian Yuan, Wang Xueqin and Zhang Heping (2018). Ball Divergence: Nonparametric two sample test. *Annals of Statistics, 46*(3), 1109-1137. Retrieved from https://projecteuclid.org/euclid.aos/1525313077
86. Pankaj Jain, Jain Thomas, McInish H., and Miller Jonathan L. (2019). Insights from bitcoin trading. *Financial Management, 48*, 1031-1048. Retrieved from 10.1111/fima.12299

87. Phillips P. C. and Yu J. (2011). Dating the timeline of financial bubbles during the subprime crises. *Quantitative Economics, 2*(3), 455-491. Retrieved from
88. Phillips P. C., Shi S. and Yu J. (2015). Testing for Multiple Bubbles: Historical Episodes of Exuberance and Collapse in the S&P500. *International Economic Review, 52*(1), 1043-1078. Retrieved from 10.2139/ssrn.2327609
89. Phillips Peter and Perron Pierre (1986). Testing for a Unit Root un Time Series Regression. *Biométrica, 75*(2), 335-346. Retrieved from 10.1093/biomet/75.2.335
90. Polasik Micha, Piotrowska Anna Iwona, Wisniewski Tomas Piotr, Kotkowski Radoslaw and Lightfoot Geoffrey (2016) Price Fluectuations and the Use of Bitcoin: An Empirical Inquiry. *International Journal of Electronic Commerce, 20*(1), 9-49. Retrieved from 10.1080/10864415.2016.1061413
91. Poyser Obryan (2019) Exploring the dynamics of Bitcoin´s price: a Bayesian structural time series approach. *Eurasian Economic Review, 9*, 29-60. Retrieved from https://doi.org/10.1007/s40822-018-0108-2
92. Qarni Owais Muhammad, Gulzar Saqib, Fatima Syeda Tamkeen, Khan Majid Jamal and Shafi Khurram (2019) Inter-Markets Volatility Spillover in U.S. Bitcoin and Financial Markets. *Journal of Business Economics and Management, 20*(4), 694-714. Retrieved from https://doi.org/10.3846/jbem.2019.8316
93. Raymaekers Wim (2015). Cryptocurrency Bitcoin: Disruption, challenges and opportunities. *Journal of Payments Strategy & Systems, 9*(1), 30-40. Retrieved from https://www.semanticscholar.org/paper/Cryptocurrency-Bitcoin%3A-Disruption%2C-challenges-and-Raymaekers/d0be65eb6b770dc400d58c7b53c65e47eb3e69c8
94. Rogojanu Angela and Badea Liana (2015). The issue of “true” money in front of the Bitcoin´s offensive. *Theoretical and Applied Economics, 2*(603), 77-90. Retrieved from https://www.researchgate.net/publication/326270607

95. Roman José Andre (2015). Bitcoin: Assessing the Tax Implications Associated with the IRS´s Notice Deeming Virtual Currencies Property. *Developments in Banking Law*, 451-457. Retrieved from https://www.bu.edu/rbfl/files/2015/07/Roman.pdf
96. Rus Cosmin, Rakos Ileana-Sorina and Negru Nicoleta (2018). Brief Analysis of the Bitcoin Phenomenon by Private Users. *Annals of the University of Petrosani, Electrical Engineering, 20*. Retrieved from https://www.upet.ro/annals/electrical/doc/2018/07%20Rus.pdf
97. Rydberg T. H. (1999). Generalized Hyperbolic Diffusion Processes with Applications in Finance. *Mathematical Finance, 9*(2), 183-200. Retrieved from 10.1111/1467-9965.00067
98. Ryznar Margaret (2019). The Future of Bitcoins Futures. *Houston Law Review, 56*(3), 539-563. Retrieved from https://houstonlawreview.org/article/7550-the-future-of-bitcoin-futures
99. Sahoo Pradipta Kumar (2017). Bitcoin as digital money: Its growth and future sustainability. *Theoretical and Applied Economics, 24*(4), 53-64. Retrieved form http://store.ectap.ro/articole/1306.pdf
100. Segal Zachary (2018). Taking Back Bitcoin. *Touro Law Review, 34*(4), 1375-1407. Retrieved from https://digitalcommons.tourolaw.edu/lawreview/vol34/iss4/21
101. Senarathne Chamil W. (2019) The Leverage Effect and Information Flow Interpretation for Speculative Bitcoin prices: Bitcoin Volume vs ARCH Effect. *European Journal of Economic Studies, 8*(1), 77-84. Retrieved from https://www.researchgate.net/publication/332368561_The_Leverage_Effect_and_Information_Flow_Interpretation_for_Speculative_Bitcoin_Prices_Bitcoin_Volume_vs_ARCH_Effect
102. Shapiro S. S. and Francia R. S. (1972). An Approximate of Variance Test for Normality. *Journal of the American Statistical Association, 67*(337), 215-216, Retrieved from 10.1080/01621459.1972.10481232
103. Shen Hao, Xuanjin Meng, Rongjie Guo, Yuyan Zhao, Siyi Ding and Xiaojin Meng (2017). Heavy-tailed distribution and risk management of gold returns.

International Journal of Academic Research in Economics and Management Sciences, 3, 15-24. Retrieved from 10.6007/IJAREMS/v6-i3/3147

104. Shi Fa-Bin, Sun Xiao-Qian, Gao Jin- Hua, Xu Li, Wei Shen and Cheng Xue-Qi (2019). Anomaly detection in Bitcoin market via price returns analysis. *PLoS ONE, 14*(6). Retrieved from https://doi.org/10.1371/journal.pone.0218341

105. Taleb Nasser (2019). Perospective Applications of Blockchain and Bitcoin Cryptocurrency Technology. *TEM, 8*(1), 48-55. Retrieved from DOI: 10.18421/TEM81-06

106. Thode Henry C. (2002). *Testing for Normality*. New York, USA: Marcel Dekker Inc.

107. Thum M. (2018). The economic cost of Bitcoin mining. *CESifo Forum, 19*(1), 43-45. Retrieved from https://www.cesifo.org/DocDL/CESifo-Forum-2018-1-thum-bitcoin-march.pdf

108. Trautman Lawrence (2014). Virtual Currencies Bitcoin & What Now After Liberty Reserve, Silk Road, and Mt. Gox? *Richmond Journal of Law and Technology, 20*(4), 1-108. Retrieved from https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2393537

109. Trejo B. R., Núñez J. A. and Lorenzo A. (2006). Distribución de los rendimientos del mercado mexicano accionario. *Estudios Económicos, 21*, 85-118. Retrieved from https://www.redalyc.org/articulo.oa?id=59721105

110. Van Hout Marie Claire and Bingham Tim (2013). Responsible vendors, intelligent consumers: Silk Road, the online revolution in drug trading. *International Journal of Drug Policy*. Retrieved from http://dx.doi.org/10.1016/j.drugpo.2013.10.009

111. Vo Nhi N. Y. and Xu Guandong (2017). The volatility of Bitcoin returns and its correlation to financial markets. *International Conference on Behavioral Economic, Socio-cultural Computing*, October 2017, 16-18. Retrieved from 10.1109/BESC.2017.8256465

112. Von Mises Ludwig (1912). *The Theory of Money and Credit*. Indianapolis, EUA: Liberty Fund

113. Westernhoff Frank and Franke Reiner (2009) Converse Trading Strategies, Intrinsic Noise and the Stylized Facts of Financial Markets. *Quantitative Finance,* 12(3), 425-436. Retrieved from 10.1080/14697688.2010.504224
114. Yermack David (2015). Is Bitcoin a Real Currency? An Economic Appraisal. Handbook of Digital Currency:Bitcoin, Innovation, *Financial Instruments, and Big Data*. Chapter 2
115. Zhang W., Wang P., Li X. and Shen D. (2018). Some stylized facts of the cryptocurrency market. *Applied Economics, 50*, 5950-5965, Retrieved from Doi:10.1080/00036846.2018.1488076
116. Zheng-Zheng Li, Ran Tao, Chi-Wei Su, Oana-Ramona Bobont (2018). Does Bitcoin bubble burst? *Quality and Quantity Springer, 53*(1),91-105. Retrieved from 10.1007/s11135-018-0728-3
117. Zimmer Z. (2017). Bitcoin and Potosí Silver: Historical Perspectives on Cryptocurrency. *Technology and culture, 58*(2), 307-334. Retrieved from 10.1353/tech.2017.0038

Published Articles

1. Nuñez J. A., Contreras-Valdez M. I., Ramirez-García A. and Sánchez-Ruenes E. (2018). Underlying Assets Distribution in Derivatives: The BRIC Case. *Theoretical Economics Letters, 8*, 502-513. Retrieved from 10.4236/tel.2018.83035
2. Cerecedo Hernández, Daniel, Franco-Ruiz, Carlos Armando, Contreras-Valdez, Mario Iván, & Franco-Ruiz, Jovan Axel. (2019). Explosion in Virtual Assets (Cryptocurrencies). *Revista mexicana de economía y finanzas*, *14*(4), 715-727. Retrieved from https://dx.doi.org/10.21919/remef.v14i4.374
3. Núñez J. A., Contreras-Valdez M. I., Franco-Ruíz C. A. (2019). Statistical analysis of bitcoin during explosive behavior periods. *PLoS ONE, 14*(3). Retrieved from https://doi.org/10.1371/journal. pone.0213919

Curriculum Vitae

Mario Iván Contreras Valdez

Información
Fecha de nacimiento: 30 de mayo, 1994
Correo electrónico: marioivan.contrerasv@gmail.com
Celular: 55 59 43 29 54

Educación

Instituto Tecnológico y de Estudios Superiores de Monterrey - EGADE Business School
Campus Ciudad de México — Graduación: julio 2020
Dr. Ciencias Financieras — Promedio: 97

Instituto Tecnológico y de Estudios Superiores de Monterrey
Campus Ciudad de México — Graduación: diciembre 2016
Lic. Economía y finanzas — Promedio: 93
Mención Honorífica

Cevro Institut School of Political Studies
Praga, República Checa
Central Europe – Political, Social and Economic Challenges — Verano 2015
Intercambio Internacional — Calificación: A

Actividades de liderazgo

- Participación Mentor del desafío Comprometidos-2019
- Participación concurso Contacto BANXICO, ganador.
- Miembro de la Asolación de Alumnos de Economía (SAECO), área de logística.

Actividades Extraacadémicas

- Estudios de matemáticas en la facultad de ciencias de la UNAM.
- Participación concurso “Banquero Central” y “GBM Homebrokers”.
- Guionista y locutor de programa de radio “La Minuta” Concepto Radial CCM.
- Participación Congreso Bancario Universitario 2013.

Otras habilidades

Idiomas
Ingles Avanzado

Paquetería

Office, R, Python, STATA, E-Views, Bloomberg, Visual Basic, Raptor, Economatica, SPSS

This document was typed in using Microsoft Word by Mario Iván Contreras Valdez