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Towards a Full Integration of Physics and Math Concepts: The Path Full of Traps

Prof. Angeles Dominguez, Tecnologico de Monterrey (ITESM) and Universidad Andres Bello (UNAB)

Angeles Dominguez is a Professor of the Department of Mathematics within the School of Engineering, a researcher at the School of Education, and Associate Dean of Faculty Development at the School of Medicine and Health Sciences at the Tecnologico de Monterrey, Mexico. Also, she is currently collaborating with the School of Engineering at the University Andres Bello at Santiago, Chile. Angeles holds a bachelor degree in Physics Engineering from Tecnologico de Monterrey and a doctoral degree in Mathematics Education from Syracuse University, NY. Dr. Dominguez is a member of the Researchers' National System in Mexico (SNI-1) and has been a visiting researcher at Syracuse University, at UT-Austin and at Universidad Andres Bello. She teaches undergraduate courses in Mathematics, graduate courses in Education, and is a thesis advisor on the master and doctoral programs on education at the Tecnologico de Monterrey. Her main research areas are: models and modeling, use of technology to improve teaching and learning, gender issues in STEM education.

Itzel Hernandez-Armenta, Tecnologico de Monterrey

Itzel Hernandez-Armenta received a bachelor's degree in physics from the Universidad de las Américas Puebla and is currently a second-semester PhD student at the Educational Innovation program in Tecnologico de Monterrey. She has experience as a public communicator of science and astronomy for children, youth and teachers at the National Institute of Astrophysics, Optics and Electronics (INAOE) and since 2017 she is the coordinator of the Astronomical Society of Tecnológico de Monterrey, Monterrey campus. She is a collaborator of the Group of Research and Innovation of Physics Education and part of the Research Group in Educational Innovation. Her research interest topics involve university education in STEAM areas, multidisciplinary integrations of knowledge and gender studies in science education.

Dr. Jorge Eugenio de la Garza Becerra, Tecnologico de Monterrey (ITESM)

Jorge de la Garza is a Lecturer of the Physics Department within the School of Engineering at the Tecnologico de Monterrey, Monterrey, Mexico. He holds a bachelor degree in Physics Engineering and a doctoral degree in Education both from Tecnologico de Monterrey. Dr. de la Garza has been recognized by the government as a member of the Researchers' National System in Mexico (SNI). He also actively participates in the different initiatives of the University mainly those related to interdisciplinary approach of teaching and learning, looking to close the gap between how knowledge is created and how students learn. His main research areas are a) models and modeling, b) learning environments and c) problem solving.

Towards a full integration of physics and math concepts: A path full of traps

Abstract

Both mathematics and physics concepts have been closely interrelated since their formal beginnings in ancient times. Moreover, from a historical perspective, it is possible to identify how physics advanced as more complex mathematical ideas were available. In fact, it is hard to separate them either in or outside a classroom. However, in the classroom there are many instances that the teaching of one subject obstructs or creates barriers for the other. After five years of teaching a physics and math integrated course for freshman undergraduate students, a series of inconsistencies have been identified between both subjects. These inconsistencies can be perceived as traps that create conflicts between the concepts, interfering with students' learning. The instructors teaching the integrated course are aware of those problems and they are authentically concerned about what to do to create awareness for these conflicts that make learning and understanding harder for students. Moreover, they have some suggestions as to what to do or how to address those inconsistencies, so the teaching and learning of both disciplines (physics and mathematics) is improved.
In this study, the authors present some of these inconsistencies that arose while working in an integrated physics and mathematics course for first year undergraduate students (mostly kinematics and differential calculus). Some of the inconsistencies come from language, other from the framework of reference, and some others from the applications. As concluding remarks, the authors aim to provide some ways to alleviate that tension. The main objective is to have series of works focusing in the inconsistencies while searching for suggestions to mediate them and improve conceptual relationships that promote a better understanding for students.

Introduction

Science education calls for mediators to not only understand but to merge and readapt knowledge before being introduced to students, taking into account that relation making is crucial. Considering historical and philosophical contexts which led to scientific progress may be valuable for science and math courses. This kind of knowledge networks, among other educational profits, allow students to face their own alternative notions and compare them with scientific ideas through time [1, p.161].

Several efforts have been made to prompt physics teachers to incorporate a historical view about the scientific development of models and theories. Some of those efforts include professional development courses in which "epistemic activities were designed that focused on the development of theories, with their epistemological and ontological dimensions, at historical stages of many fields of physics" [2, p.404]). However the need to incorporate an interdisciplinary approach with other fields, such as mathematics, to this insight is identified.

Educative integration of physics with math promotes the constitution of shared conceptual structures, however, “the very mention to Physics *and* Mathematics suggests that these subjects would be distinct and, by extension, that they could be disentangled by means of a competent philosophical discourse (...) despite being distinct subjects, there is a continuity between them” [3, p.646]. These associations are not inconsequential nor trivial and need to be addressed by educational research.

Conceptual gaps may have compelling explanations linked to history and philosophy of science, which should be carefully researched and deemed. Yet a more empirical viewpoint, studied directly on physics and mathematics courses, is suitable to piece together the complexity of the matter. During the last five years the authors have focused on creating and giving a course called *Fis-Mat* (which stands for Physics and Mathematics in Spanish). In this course, the total time destined for a regular *Physics I* course (mechanics) and a *Mathematics I* course (differential calculus) is merged to use the whole time to offer an integrated version in which the content of both subjects is covered [4], [5], [6].

Since the beginning, it was self-evident that to accomplish and implement such course we could not depend solely on the syllabus of both courses and decided to use models and modeling theory to base the course. Over the years a series of *incidents* have happened in the classroom where some fundamental differences on how physics and math are taught have shown us that in some cases, the content in one course is not what the other one needs. Moreover, there are some extreme cases in which the way concepts are covered in one of the courses predispose students to make mistakes in the other course.

Over time we have reported some research on students’ use of models, and how we work in the classroom [6]. It has become apparent that these discrepancies were a whole new area of research and in this work we start looking for options on how to mediate the differences encountered, pointing out that most of the issues encountered become apparent only when both courses are taught together. It is our belief that university education should mediate these differences to promote the integration of knowledge and build stronger connections among disciplines.

Our learning from the integrated course

Physical models can be introduced using Modeling Instruction [7] and [8] as a grounding strategy. This is made by launching inquiry or investigation activities around a physical situation in which students can go through research, deduction, hypothesis testing, refining of thoughts and conclusion communication [9]. If new representations are needed, the professor introduces them prior the new model construction. Then, students actively solve the presented situation by discussing it in small groups of peers, with sporadic whole group interventions to ensure all students are moving at the same pace.

By the end of the modeling instruction session, new representations made by each group are shown to the class by a reasoning explanation and model presentation. Thus, an incremental development is achieved and a robust model is build. Modeling instruction used in the classroom is represented in Figure 1 [4].

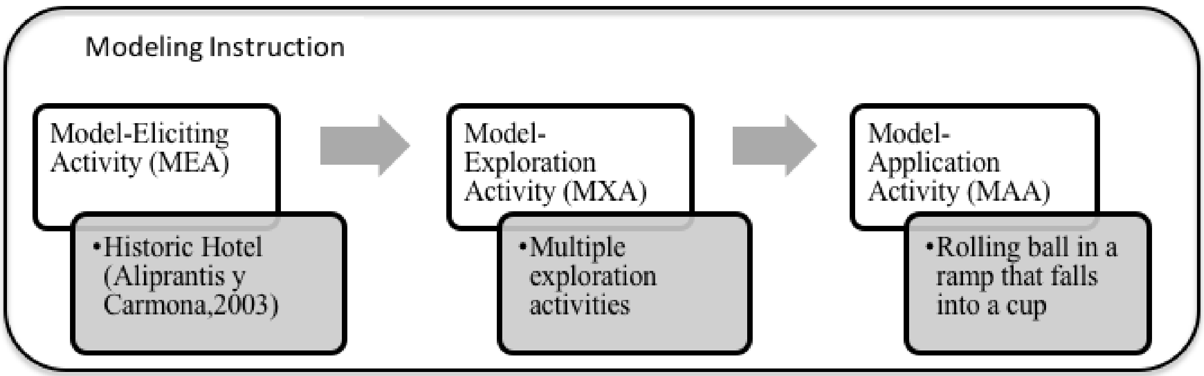

Fig. 1. Sequence for model development embedded in modeling instruction [6].

Representations are the media used to reproduce and represent physical phenomena, expressing relationship between variables, communicating and synthesizing ideas about an event and their characteristics, considering that “different media emphasize (and de-emphasize) different aspects of the systems they were intended to describe” [10, p. 12]. During the process of modeling construction [6], the more representations used, the more robust the model would be, by using different representations in a coherent and articulated way [11].

Table 1 shows the content of *Physics I* and *Mathematics I* courses at the university where the integrated Fis-Mat course is taught. We would like to point out that in the case of the Mathematics course, calculus content varies from traditional courses not only on what is taught but when is taught to favor that concepts are discussed when needed in the physics content [4]. This is represented in Fig. 2 by showing how Physics and Mathematics content is connected to promote robust connections among concepts.

TABLE I
COURSE CONTENT FOR PHYSICS 1 AND MATHEMATICS I

Physics content	**Calculus content**
Vectors	• Linear model
• Motion at constant speed	• Quadratic model
• Motion with constant acceleration	• Derivatives
• Constant Acceleration quantitative	• Euler's method
• Motion in Two Dimensions	• Non-continuous functions
• Energy	• Integral
• Work	• Line integral
• Forces	• Applications of derivatives and integrals
• Forces of friction	• Applications of mathematical models
• Momentum	
• Forces of spring and circular motion	
• Rotational and harmonic motion	

After three years of covering the content and focusing on the relationships between all the concepts, a new sight of the matter has been achieved. In Figure 2, we show physics concepts and mathematical relationships among them in such a way that they will keep being true regardless of which specific scenario is being covered. This type of arrangement of the content also permits to initiate the course with different topics and eventually building all the relationships along the program, instead of forcing a specific order.

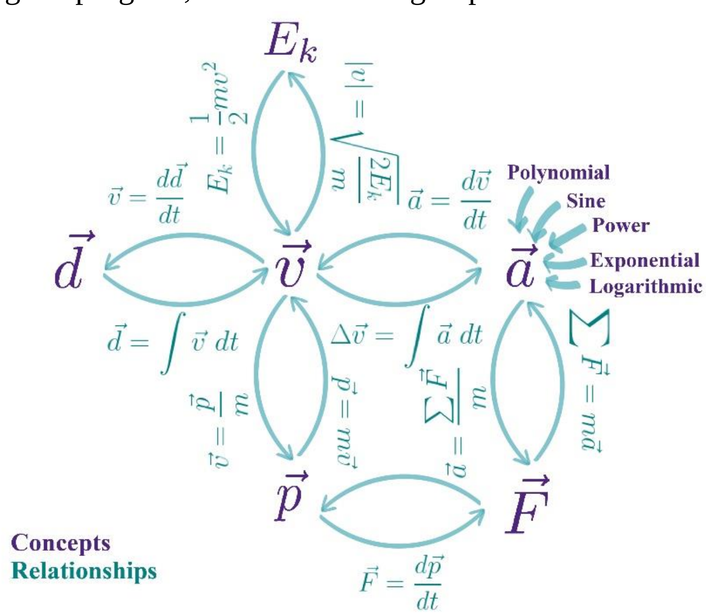

Fig. 2. An integrated view of physics and math concepts in our syllabus.

All the mathematical relationships are written in the most general way possible. Normally in a course the simplest version of the relationship between concepts is covered, leaving the job of finding the full relationship for later courses, normally focusing only in specific equations. Looking at Fig. 2, we can see mainly two kinds of relationships between physical concepts, algebraic, which in Physics 1 consist mostly of multiplying or dividing by mass, and calculus, by derivatives and integrals. An extra layer of future complexity is mentioned in the idea that the acceleration could be represented by any kind of function (normally different functions are covered as part of the Calculus 1 course). Not always a lot of class time is dedicated for this, but the seeds that relationships should not change are planted here. This creates a big dependence on concepts normally covered in Calculus 1 course without context and makes all the content in both courses codependent.

Likewise, it is possible to clearly see how other later courses simply connect to the proposed relationships. In a course like electricity and magnetism, electrical and magnetic forces are covered, which connects directly to forces in Figure 2, simply explaining the proposed model. Also, the relationships between gravitational force, gravitational field and gravitational energy, are the base relationships that will always be true between those concepts.

A series of inconsistencies that have been found in our attempt to integrate physics and mathematics in the course are now presented. A later section in this article will focus on covering one of them, while others will be covered in future publications.

- **Scalar vs vector mathematics.** *Mathematics I* is a scalar course while *Physics I* is a vector course. There is a difference on how some concepts are covered, how graphs are done, and how to do algebra. Most problems in physics use vectors while most *Mathematics I* courses only cover scalar concepts.
- **Graphs.** How graphs are used and explained in both courses not only is different, it is in some cases contradictory. This is related with the use of scalar vs vector mathematics; still, it should be possible to find more general ways to construct graphs so that they mean and are built the same way for both courses. Not only that, but eventually whatever is considered a graph in any course should have similar meanings no matter where it is covered.
- **Terms, pseudo terms and synonyms.** When teaching any physics course, or when a different course like mechanics is taught, variables and concepts are named differently from course to course. The use of words link greatly with how the brain access information and an artificial fragmentation of knowledge can be created when we are not careful.
- **Fragmentation of ideas**. Even inside the same course context, the way we normally introduce concepts creates gaps between them. Phrases like ‘only chapter 4 and 5 come in the exam’ might sound familiar. It is possible to rebuild the content in such way that

everything built before is still relevant and true even when working with different concepts. An example of this can be found in Fig. 2.

- **Specific vs general cases.** In physics, students normally tend to learn the *specific* equations of the *specific* case and whenever they have the *specific* conditions use such equation. It is possible to build a more general model which works in every scenario and the student would need to understand and read their model to limit it to the situation they are solving.
- **Opportunities created using technology.** With all the benefits of using technology in the classroom, often we do not take time to find which new aspects of our course can appear when using new devices or software. For example, sampling error, or void data, appear regularly while using motion sensors in the classroom. In another example, students may find that although they tried to walk with constant speed for five seconds, graphs only show it happened for one second.
  The need arises to recognize such opportunities and exploit them, thus, students learn to read and understand and better build their understanding.
- **What a textbook actually is.** Textbooks are often used as an outline for course planning. Most textbooks are built as repositories of information on a concept or knowledge area and tend to be ordered in a certain way, sometimes by time of discovery, and other times in order of 'difficulty'. This order may not have authentic or significant for developing cognitive interconnections. Also, books often promote remembering how to do specific problems which we have identified as a problem itself.

Inconsistencies regarding velocity

In this article, misconceptions and incongruencies around the concept of *velocity* are analyzed, opening the floor for further work. In the following section, three velocity definitions are compared, two of them stated in physics textbooks and one taken from a calculus book.

Regarding the presence of physical concepts in mathematics textbooks, we present an example of [12, Ch. 2, pp. 88], in which *average velocity* is defined as the change in distance divided by the change in time. Later, the idea of velocity as a series of small intervals that eventually would become the tangent of a position graph is presented, without making an explicit reference to the concept of derivative [12, Ch. 2, pp. 148].

In chapter 2, *instantaneous velocity* is defined as the evaluation of a distance equation, divided by the time interval when this is very small. Perhaps this is to connect the idea of a tangent line in the distance equation [12, Ch. 2, pp. 152]. Still in chapter 2, average velocity is defined once again as a displacement over time [12, Ch. 2, pp. 150], now considering the *limit* when the change becomes infinitely small. This is an introduction for the use of the limit rule to get a derivative of the distance function.

Concernedly, physics relationships between the mathematical concepts around velocity are introduced in a slight manner. The first solution of *speed* is made within the context of a problem, where it is simply stated that the speed of a particle is "the absolute value of the velocity". In chapter 3, under a subsection called *Physics*, they define average velocity as $\frac{\Delta\, distance}{\Delta\, time}$, and instantaneous velocity as $\frac{dt}{ds}$, now deriving from a distance function to get a velocity function [12, Ch. 3, pp. 197]. Later, in chapter 13, while working with vector functions, the velocity vector is understood as $r'(t)$ and the speed as the magnitude at that same specific time [12, Ch. 13, pp. 857-858].

As for the physics books references, on [13] Physics for scientists and engineers with modern physics, average velocity in the *x* direction is defined as $\frac{\Delta x}{\Delta t}$, stating that "the average velocity of a particle v_x, is defined as the displacement of the particle, Δx, divided by the interval of time, Δt, in which the displacement occurs" [13, Ch. 2, pp. 21]. Subsequently, average speed notion is determined by specifying that velocity and speed are different concepts for physics, although they "are the same in day to day life" [13, Ch. 2, pp. 22]. When instantaneous velocity in the *x* direction is brought, it is presented as the limit when a change in time approaches to zero of the $\frac{\Delta x}{\Delta t}$, rewriting the expression as $v_x = \frac{ds}{dt}$ [13, Ch. 2, pp. 24]. In following paragraphs in the book, *instantaneous speed* is explained as the magnitude of the instantaneous velocity.

Still in chapter 2, change in position is viewed as the integral of velocity in terms of time [13, Ch. 2, pp. 40]. In chapter 4, authors redefine the average velocity specifying that it is a vector and as such should follow vector rules [13, Ch. 2, pp. 72]. From then on, the definition is the change in the displacement vector over the change in time, instead of the previous version where displacement was defined as only in the *x* direction.

Also, on [14] chapter 2, *speed* is first introduced as follows: "An object's average speed is the distance traveled divided by the time interval required to travel that distance" [14, Ch. 2, pp.35]. It is later specified the relation with a traveled distance, and not with a component in any direction. Later, the *x* component of velocity is explained as the *x* component of its displacement by a length of time. It is particularly interesting to note that still in chapter 2, in every occasion the expression $v_x = \frac{\Delta x}{\Delta t}$ arises, it is pointed that it is only true *if* the velocity is constant. On following sections, the indication is no longer made when now the expression is defined as $\bar{v} = \frac{d\bar{r}}{dt}$ [14, Ch. 2, pp.49]. Also, definition of average velocity is compared to instantaneous velocity, stating that speed is *always* an instantaneous speed. As it should have been stated in the previous examples [13] and [14].

From these three conceptual previews, it is possible to point out small but important differences, mainly in the nature of the approached concepts presented both in physics and calculus books.

For the physics examples, building since the beginning the idea of a vector component relative to velocity is of an extreme importance, while for the calculus book it is not mentioned until later. Further observations on how each of the concepts around the idea of velocity are used during a whole class structure should be made. For example, with graphic representations made under both mathematical and physical background. While both approaches may seem similar, serious matters are provoked.

An example, often found in the classroom, happens when you ask students which object has a higher velocity, one traveling at 5 m/s or one traveling at -5 m/s. From a pure mathematical point of view, students tend to say that obviously a velocity of 5 m/s is higher than one of -5 m/s. This question becomes more interesting when you look at it from a physics point of view, where both velocities have the same magnitude, and as such, have the same speed.

Conclusion

Quite enough inconsistencies can become apparent when teaching physics and mathematics integrated courses. Some of them are minor opportunities to make concepts easier to understand, while others simply make both approaches inconsistent. The hope for a deeper and grounded analysis of these issues could be reason to engage in future research, in the search for a fix from root of those issues.

One of the objectives of higher education institutions, is to take knowledge and make it available and understandable for students, taking the time to make sure of its coherence and the integration between different areas of knowledge. This final step is discouraged by the way different courses are normally separated and do not always become apparent.

We hope to enter in greater detail in later studies while looking for more ideas, finding profoundness on the source for the issues to find solutions to reduce the differences between disciplinary teaching differences in mathematics and physics courses.

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References

[1] F. Seroglou, and P. Koumaras, “The contribution of the history of physics in physics education: A review,” *Science, education and culture*, pp. 327-346, Dordrecht, Springer, 2001.

[2] P. Garik, L. Garbayo, Y. Benétreau-Dupin, C. Winrich, A. Duffy, N. Gross, and M. Jariwala, "Teaching the conceptual history of physics to physics teachers." *Science & Education*, vol. 24, no. 4, pp. 387-408, 2015.

[3] F. B. Kneubil, and M. R. Robilotta, "Physics teaching: mathematics as an epistemological tool," *Science & Education*, vol. 24, no. 5-6, pp. 645-660, 2015.

[4] A. Dominguez, G. Zavala, & J. A. Alanís, "Integrated Physics and Math course for Engineering Students: A First Experience", in *Proc. of the 120th ASEE Annu. Conf. and Expo.*, Atlanta, GA, 2013, pp. 23.766.1 - 23.766.9.

[5] A. Dominguez & J.E. de la Garza, "Closing the gap between physics and calculus: Teaching innovations in an integrated course", in *Proc. of the 122nd ASEE Annu. Conf. and Expo.*, Seattle, 2015, pp. 26.353.1 - 26.353.14.

[6] A. Dominguez, J. De la Garza, and G. Zavala, "Models and Modelling in an Integrated Physics and Mathematics Course". In G.A. Stillman, W. Blum, & M. Salett Biembengut (Eds.), *Mathematical Modelling in Education Research and Practice: Cultural, Social and Cognitive Influences, International Perspectives on the Teaching and Learning of Mathematical Modelling*, New York, pp. 513–522, Springer, 2015.

[7] I. A. Halloun, and D. Hestenes, "Modeling instruction in mechanics", *Am J of Phys*, vol. 53, no. 11, pp. 1043-1055, 1987.

[8] D. Hestenes, "Modeling theory for math and sciences education". In R. Lesh, P. L. Galbraith, C. R., Haines, and A. Hurford (Eds.), *Modeling students' mathematical modeling competencies*, pp. 13-41, New York, Springer, 2010.

[9] E. Brewe, "Modeling theory applied: Modeling instruction in introductory physics", *Am J of Phys*, 76(12), 1155-1160, 2008.

[10] R. A. Lesh, and H. M. Doerr, "Foundations of a Models and Modeling Perspective on Mathematics Teaching, Learning, and Problem Solving". In R. A. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning and teaching*, pp. 3-33, Mahwah, Erlbaum, 2003.

[11] A. Dominguez & J.E. de la Garza, "Representations in an integrated Physics and Mathematics course based on models", in *Proc. of the 123rd ASEE Annu. Conf. and Expo.*, New Orleans, 2016.

[12] J. Stewart, *Essential calculus: Early transcendentals*, Thompson Learning. 2002.

[13] R. A. Serway, and J. W. Jewett, *Physics for scientists and engineers with modern physics*, Cengage Learning, 2008.

[14] E. Mazur, C. H. Crouch, D. Pedigo, P. A. Dourmashkin, and R. J. Bieniek, *Principles & Practice of Physics*, Pearson, 2015.