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School of Engineering and Sciences



**Reception of Multiple Users in Reconfigurable Wireless Networks**

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**Juan Manuel Velázquez Gutiérrez**

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Major in Communication systems


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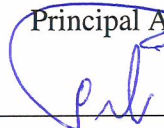
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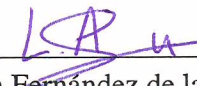
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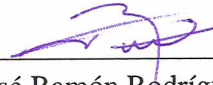
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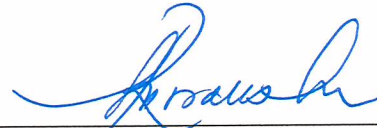
  
\_\_\_\_\_  
Dr. César Vargas Rosales  
ITESM  
Principal Advisor

  
\_\_\_\_\_  
Dr. Mahdi Zareei  
ITESM  
Co-Advisor

  
\_\_\_\_\_  
Dra. Leyre Azpilicueta Fernández de las Heras  
ITESM  
Committee Member

  
\_\_\_\_\_  
Dra. Rafaela Villalpando Hernández.  
ITESM  
Committee Member

  
\_\_\_\_\_  
Dr. José Ramón Rodríguez Cruz.  
UANL  
Committee Member

  
\_\_\_\_\_  
Dr. Rubén Morales Menéndez.  
Dean of Graduate Studies  
School of Engineering and Sciences

Monterrey, Nuevo León, December, 2018

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# Dedication

To my parents, Francisco Javier Velázquez Ibarrarán and María Concepción I. Gutiérrez Herrera, because they are my guides in my life. To my brother, sisters, sister in-law and brother in-law -Francisco Javier, Carolina, María Concepción, María Cristina and Angel Omar- who remind me of the importance of family values. To my lovely niece, Cristina Dominika.



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# Reception of Multiple Users in Reconfigurable Wireless Networks

by

Juan Manuel Velázquez Gutiérrez

## Abstract

The next generation of wireless networks (WNs) will confront important challenges presented by the high density and convergence of wireless elements (WEs), their influence on our lifestyle, and the proliferation of new paradigms of wireless communication systems. Coexistence, mobility, and multiple access are some issues widely studied in order to ensure their operation. In this work, it is introduced a framework of WNs considering association schemes, networking, and coexistence aspects which are useful concepts to describe their features and the relationship among elements. Code division multiple access (CDMA) systems are considered as attractive models to future wireless communication systems (WCS) in order to affront the new and changing challenges.

Traditional direct sequences - code division multiple access (DS-CDMA) and double codification division multiple access (C2DMA) models provide engaging features given that offer continuous access to the wireless channel to every user, i.e. they permit coexistence of multiple users at the same time. These models are interference limited systems what make them vulnerable to usual issues in dynamic networks such as the Near-Far problem, Hidden and exposed terminals interference, etc. However, C2DMA model has been shown additional properties that makes it attractive to reconfigurable wireless networks (RWNs) due to its robustness to interference. It considers a double correlation technique which provides simultaneous reception and motivates the study of sequences used as codes in order to increase their use and visualise some applications. This work considers current and new sets of codes through an exhaustive search of useful sequences for both models. First, desired properties are identified for each model, next known sequence sets are tested with these properties. After that, an exhaustive search algorithm is used in order to select and group sequences to form all the possible code sets with desired properties. Finally, these models are tested in challenging scenarios in order to examine their performance and persuade about their potential in future WNs.

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# Chapter 1

## Introduction

Emerging wireless communication technologies and the diversity of wireless elements (WEs) have changed our lifestyle and the perspective of wireless communication networks (WCNs) study which have been taking importance in communication systems due to their flexibility, mobility, adaptability, expandability, versatility, and cost-effectiveness. Their use and implementation in different areas such as health (medical), Robotics, Manufacturing & automation, telecommunications, etc. which have motivated the development of new communication systems in order to explore new paradigms and redefine the scopes of existing technologies.

New trends, which incorporate communications systems, allow to interconnect diverse WEs with different attributes and qualities to construct wireless networks (WN) through a wide variety of technologies and mechanisms that ensure their operation. Thus, the association schemes of WEs operate collaborative or cooperative strategies that present certain drawbacks that limit their scopes and feasibility. They confront important impediments that are widely studied in order to improve their operation such as multiple user and coexistence problems, multiple access channel, energy efficiency, spectrum utilization, security, etc.

Reconfigurability and mobility are important challenges to WNs due to their dynamic topology and connectivity, flexible configuration, and constant redistribution of resources that require additional mechanisms in order to provide reliable conditions to wireless communication. Wireless communication systems (WCS) affront these issues through dedicated protocols and/or algorithms for signal processing that increase their complexity. Code division multiple access (CDMA) systems offer attractive features for WNs, e.g. channel sharing, bandwidth efficiency, and simultaneous transmissions, which establish an intrinsic coexistence scheme. Nevertheless, CDMA models proposed by reconfigurable wireless networks (RWN) also require complementary strategies that allow their implementation. In this work, C2DMA model, proposed by C. Barrera in [4], is considered as a promising model for these networks. It considers a double codification through spreading codes with particular properties that allows simultaneous reception and mitigates interference from the properties of codes; however, the available set of codes come from Walsh sequences and as consequence, the code sets are small. The study of known sequences used as codes and the exploration of new sets are important contributions in his work, as well as, the inspection of potential applications.

## 1.1 Motivation

High density and convergence of WEs have represented important challenges for next generation WCS and motivated the development of new strategies, schemes, mechanisms and protocols that ensure their operation. Thus, issues as mobility, coexistence problem, demand of spectrum and allocation of new wireless technologies are widely studied in order to search and explore new alternatives and increase their scopes. Systems based on CDMA technique provide important advantages to WN. However, the dynamic and intermittent behavior of future WNs requires the implementation of additional protocols, algorithms, and schemes to adapt them to changing conditions and as consequence, their complexity and feasibility increases.

C2DMA model promises powerful features to WN such as robustness to interferences and simultaneous reception of signals where the spreading codes play a crucial role. These features can be reached through a simple modification of traditional DS-CDMA model easy to implement. C. Barrera described that this system requires an additional correlation property known as *Double correlation* (DC), which consists in maintaining the correlation of all products of codes, and provided small sets of codes derived from Walsh-Hadamard sequences. On the other hand, I. Ayala explore complementary codes in [15] with favorable results which increases the interest to search and examine new DC code sets and permit explore new applications. C2DMA model can provide *cross-layer* features that helps to reduce the complexity of WEs through properties of codes., i.e. the powerful features of signals can avoid the implementation of some mechanisms and, as consequence, decrease the computational complexity and simplify the communication systems, network efficiency, and resource utilization. Finally, this model is consistent with new trends as *Green technologies* and *internet of everything* (IoE).

## 1.2 Problem Statement and Context

Future WNs must response to diverse challenges which can limit their scopes and implementations. The tendency of interconnect WEs in dynamic and spontaneous networks will require to provide compatible platforms with flexible and versatile schemes that allow their interaction without altering the performance of others active WEs, so connectivity, access channel, and coexistence issues have been taking importance due to their changing nature and limited resources. The attractive features of CDMA models depend on properties of codes, which establish the representation signals and robustness to interference. So, the scarcity of codes with useful properties is their main restraint and, as consequence, their implementation. Researches on sequences have shown that the number of families of sequences is limited because each additional property reduces their set size decreasing the number of users or WEs in a network. The search of new sequence sets with desired properties is essential to integration of these models for future WNs.

### 1.3 Research Questions

- Why the CDMA systems are attractive to future WNs?
- What is the role of spreading codes in CDMA systems? and How can they be classified?
- What are the desired properties of sequences to CDMA systems?
- Why the C2DMA model is attractive to RWN?

### 1.4 Solution Overview

This work considers that systems based on CDMA model have attractive features for future WNs, however the available codes with desired properties are reduced. Thus, the research on new useful code sets could increase the scopes and applications of these systems satisfying the requirements of new wireless technologies.

### 1.5 Objectives

In this project, the exploration and searching of sequences used as code is proposed in order to provide a new set of codes that allow to exploit the communication schemes based on CDMA systems for future WN. They will be considered useful sequence for DS-CDMA and C2DMA models.

- **Particular objectives**
  - Identify the useful properties and features of known families of sequences used as codes in CDMA systems.
  - Search new code sets useful to CDMA models that allow to explore their scopes and application for future WNs.
  - Test the feasibility of CDMA models in high density scenarios that represent important challenges for RWNs.

### 1.6 Contributions

In this thesis, a new wireless networks framework is proposed. It considers the association, networking, and coexistence concepts as important factors to describe wireless networks. An extensive research of binary sequences was made in order to explore their properties and functions in WCS. The survey *Sequence sets in Wireless Communication Systems* [49], published in *IEEE Communication Surveys & Tutorials* (Q1, JCR ratings), classifies a collection of codes in four families considering their correlation properties, so they are defined the traditional, complementary, interference-free window, and complex families.

Additionally, a multiple reception scheme based on C2DMA system was presented in *Reception of Multiple Users in Reconfigurable Networks* [50] and published in *Ad-Hoc and Sensors Wireless Networks* magazine, it compares the traditional DS-CDMA, using different families of codes, and C2DMA models in multi-user scenarios where hidden and exposed terminal interference are presented. The results showed that the traditional DS-CDMA model is vulnerable to hidden terminal interference and robustness to exposed terminal interference. In this model, the spreading codes cannot help to mitigate hidden terminal interference because it is a vulnerability model. On the other hand, C2DMA model showed robustness to hidden and exposed terminal interference, hence the properties of spreading codes play a crucial role. This feature permits the multiple reception of signal simultaneously what is attractive to RWN.

Finally, an exhaustive search of codes is presented from sequence with length  $N = 8$ . It considers useful properties of sequences for both systems, traditional DS-CDMA and C2DMA, such as No negative, balance, correlation, and shifting and some highlights from sequences with length  $N = 16$ . This way, new code sets are provided for both CDMA models in order to incorporate them to future applications.

## 1.7 Thesis Organization

This thesis is organized in 7 chapters. Chapter 2 considers the association, networking, and coexistence as study objects. Chapter 3 provides an overview of CDMA systems, their advantages and disadvantages, and a discussion about their use in current WN. An analysis of known sequence sets is developed in Chapter 4, while Chapter 5 describes a selection method to search new code sets with length  $N = 8$ . In Chapter 6, the new set of codes are proved and DS-CDMA and C2DMA are compared in order to visualize their performance in multiple user scenarios considering typical issues in RWN. Finally, the conclusions and possible future work are provided in Chapter 7.

# Chapter 2

## Wireless networks: Association, Networking, and Coexistence

In this chapter, wireless elements (WEs) are described and classified considering their nature. They are related through association schemes which define the priorities, partnership, and decision strategies considering their physical and technological limitations. Networking process is described in five phases and coexistence concept is introduced. A brief description of coexistence schemes based on multiple access techniques is given. Finally, important aspects of WNs are described and a collection of WN classifications are provided considering different features or attributes.

### 2.1 Introduction

High density of wireless elements (WEs) and their capability to perform multiple tasks represent important challenges in communications systems. Wireless networks (WNs) have been taken importance and influence nowadays because they can be exploited in significant areas for our lifestyle which motivates their study and development in order to explore their scopes and define new paradigms. Low cost of installation, easy implementation, flexibility of connection, mobility, high penetration, and increasing coverage are attractive advantages to wireless communication networks (WCNs) as well as infrastructure and/or access point. Besides, they confront certain drawbacks related to their limitations, scopes, and feasibility; issues as security, propagation loss, and throughput are some disadvantages for WNs.

The features of WNs are given by the WEs because they establish the relationship with other elements considering their physical and technological limitations. *Wireless elements* can be defined as electronic tools that have the capability to sense, act, process signals and communicate in order to perform specific functions [27]. Examples of WEs are shown in Figure 2.1. These elements are also known as *gadgets* and can be classified into four categories according to their nature:

1. *Mechanical*. These elements are composed by an electro-mechanical part, which can be an actuator or mechanical device, and a wireless system that sends and/or receives data signals; for example, these kind of WEs are widely used in manufacturing industry

and automation systems. Therefore, technologies as radio-frequency and Bluetooth are widely used for these WEs.

2. *Electronic*. They are devices that respond to environment and/or electrical signals in order to process them and perform according to specific task. They can share information through wireless signals. They can be exploited by technologies such as Wifi or Bluetooth.
3. *Programmable*. They can be mechanical or electronic elements with an additional feature, they have the capability to make local decisions through algorithms in order to modify their configuration and performance.
4. *Application*. They are software with the capability to operate and/or control the hardware resources, interfaces, and technologies to complete tasks. Their algorithms allow to process information and interact with other elements and/or software (applications).

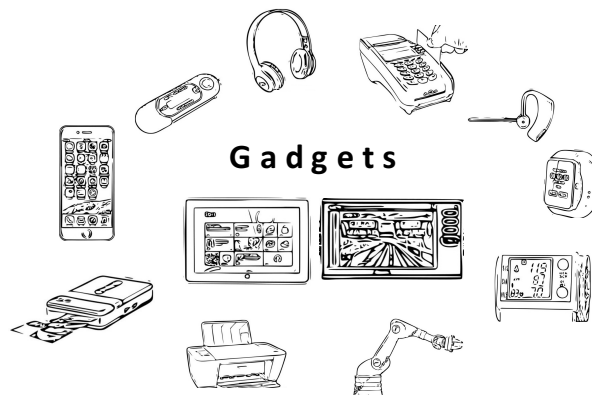


Figure 2.1: Gadgets: Devices, Machines, vehicles, sensors, etc

The incorporation of communication ability to interconnect WEs allows to associate them for constructing WNs. Different platforms can be exploited due to the convergence and integration of technologies increasing the options to construct networks. The association schemes describe the relationship among WEs, mechanisms, strategies, models, and agreements among WEs. On the other hand, the process to construct the network is known as *Networking* where the important handshakes are defined through protocols that define the rules to exchange information in order to ensure the communication among all the elements in the network. Furthermore, the concurrence of WEs involves the study of the coexistence concept due to the influence in the wireless channel and, as a consequence, in the communication. These important concepts will be extended in the following sections.

## 2.2 Association schemes

Association schemes characterize the relationship among WEs in order to share information and resources according to the availability, capacity, and priority of each element subject to its disposition or configuration through reliable mechanisms and technologies to determine the

architecture schemes, i.e. they define the network structure considering the nature status (Active / No Active), the capacities, and the WEs limitations as Figure 2.2 shows. The association of elements allows to increase their reaches beyond their capabilities, e.g. increases coverage area and save resources; this is the main motivation to encourage the association strategies of elements.

This thesis, a description of *collaborative* and *cooperative* strategies as association strategies is provided in order to distinguish their features and differences that allow a better description of WN and their analysis.

In WNs, collaborative and cooperative terms are usually used indistinctly to describe the joint work of WEs because they describe schemes with similar functions. However, in this work interesting differences are considered, so collaborative strategies will be used to describe the association from the point of view of the WE and its priorities, while cooperative strategies will primarily consider network priorities to make decisions. The network structure is constructed through the relationship among WEs, communication models, and guidelines which are defined by an association scheme to assure the operation of WNs.

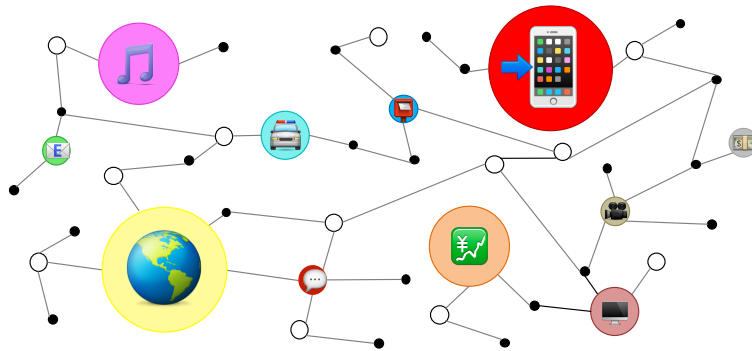


Figure 2.2: Association of wireless elements (WEs).

In contrast, elements require a common platform that allow them to interpret signals contained in the wireless channel and their formats to send information. This is the role of technology implemented in each element because it establishes technical strategies, schemes, models, algorithms, and methodologies in order to provide the reliability, connectivity, accessibility, scalability, flexibility, and compatibility among all the members and the communication model. All these aspects allow the association of WEs.

### 2.2.1 Collaborative scheme

These schemes construct networks considering individual communication links among WEs, i.e. each node pair (receiver and transmitter) defines resources and transmission agreements according to their capacity and disposition, it implies weak compromises. This is because each element is focused on its primary tasks and only shares resources when its operation is not at risk, i.e. WE priorities are over WN priorities. In this way, the local decisions only involve the node pairs. In collaborative strategies, information network can restrict information about

other elements in the network and functions, e.g. hierarchical or cellular architectures, so each node reserves the connectivity according to individual evaluation of its environment and tasks.

These schemes are mainly used by temporal networks where not all the elements have benefits. i.e. elements that help others to perform a specific task. For example, in spontaneous networks a WE can request access to a network and use its resources to share information.

### 2.2.2 Cooperative scheme

This scheme contemplates strategies where each request is considered as a primary task which requires strict rules and processes to assign resources. Thus, the compromises among elements are strong because all the members have influence in the network and the contributions are in benefit of the WN by providing the access to the structure and by sharing resources, i.e. WN priorities are over WE priorities. An important feature of cooperative schemes is that each WE decision can have impact in the operation of the other members in the network. So, the local decisions are made for the convenience of the network. Architectures based on cooperative strategies can be used as infrastructure in telecommunication services or control systems because it ensures the operation of all the elements in the network.

## 2.3 Networking

*Networking* is the process that describes the mechanism to construct wireless networks, the assignation of resources, and the connectivity strategies in the network. It consists of a collection of steps or phases that allows each WE (also known as *node* or *user*) to discover its environment and establish a reliable communication following the protocols or standards adopted to form the network. In the following subsection, networking process is introduced and important aspects are considered.

### 2.3.1 Networking process

The main goal of networking is to incorporate WEs (nodes) as users of the communication networks and construct the WN in order to provide the benefits of the association to all the members. It requires to define mechanisms and schemes to assign the resources according to a certain strategy. Thereby, a basic description of Networking process is given considering five phases that are followed by the nodes in order to explore the wireless channel and to obtain relevant information about their neighbors in order to establish the network structure, and the transmission rules. See Figure 2.3. Thus, the nodes get the instructions and parameters to join to the network.

#### Observation

The node explores the wireless channel with the purpose of perceiving the existence of signals and/or estimating its condition and availability. Spectrum sensing techniques provide important information to WEs in order to discover their environment, avoid interference,



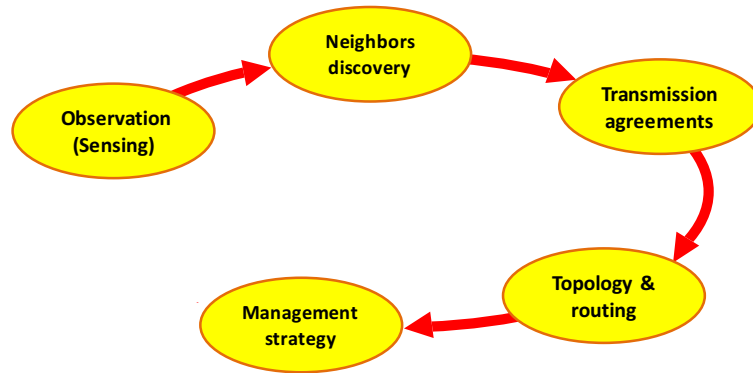


Figure 2.3: Networking process.

make decisions to transmit signals. They play an important role in networking. There are direct methods related with frequency domain approaches, indirect methods related with time domain or energy measuring; so common spectrum sensing techniques are Energy detector based, waveform based, cyclostationary based, Radio identification based, Match-Filtering based, etc., [45], [53].

### Neighbors discovery

Each node has the capability to recognize the nodes in its influence area with compatible technology and distinguish information signals from wireless channel. So, the *neighbors* are nodes which can establish direct radio links and can be identified as potential members in the network. They can share information about their neighbors and communication agreements.

### Transmission agreements

This phase consists on the conciliation among nodes to define the protocols, strategies, and signal formats in order to allow the interconnection, interactivity, and provide the transmission parameters. It is characterized by wireless communication technologies which establish the use of resources and the rules of transmission; so important issues related with the transmissions are established such as the frequency of operation, the signal representation, the modulation format, the codification, the power transmission, etc.

The communication mechanisms or protocols define the models or techniques used by the WEs according to their capability and application. They include important issues such as access channel, reception strategies, data transfer, privacy and security, etc. New trends seek to exploit models based on association schemes (diversity strategies) in order to improve or optimize a certain task, so collaborative communication strategies and cooperative detection techniques among others are widely studied.

### Topology and routing

The available communication links describe a mapping that defines the interconnection among nodes and establishes the basis of network topology. Hence the nature of WEs, association

schemes and transmission modes (simplex, half and full duplex) are relevant to determine the structure and network architecture. The arrangement of a network can have *static* or *dynamic* configuration. Thus, in a static arrangement the links are unchangeable and used widely as fixed structure in service networks. While a dynamic arrangement provides temporal configuration among nodes, i.e. the communication links are activated or reconfigured in order to change the topology. This kind of topology is exploited by structureless networks. Usually, the topologies are described according the figure formed by nodes connectivity, so they can be characterized as bus, star, ring, mesh, tree, etc.

Routing strategies define the paths that follow the information from the source up to reach its destination. The optimum route between origin and destination nodes is calculated through algorithms considering the network topology and weight parameters as criteria to evaluate the efficiency of each path and all the links among intermediate nodes (multi-hop strategy). The weight parameters are associated with communication or physical parameters such as link utilization, time delay, distance, hops, congestion, interference, energy consumption, etc.

### **Management strategy**

It is the mechanism that ensures the WNs operation and provides confidentiality, integrity, and functionality through a collection of protocols, strategies, and/or algorithms which control, and manage the resources of the nodes and the network. They try to compensate the network vulnerabilities, transmission parameters, and nodes connectivity. So, there are strategies based on security, routing, efficiency, power control, attack avoidance, etc. There are usually three kind of strategies widely used in WNs which provide interesting features according to network definition.

1. Centralized. This strategy is based on an unique entity that defines and makes important decisions. This central element is recognized by the WEs and provides the authentication requested by each element to access to the network as members. Besides, this strategy also provides easy implementation and hard network control because the decisions are made with the knowledge of the entire network. Continuous feedback is required to update the stage network. The communication rules and node functions are established previously and regulated by the central element that provide stability and consistent resource allocations. However, the centralized strategies have important disadvantages such as scalability, high processing capacity, traffic increase in the network and more latency.
2. Hierarchical. It consists of grouping elements in subsets and assign them a head-element to manager locally the subset. These head-elements are also grouped to form an upper layer which is managed by a new head-element, this iterative process constructs a hierarchical structure. In this way, the management is fragmented and the complexity to make decisions decreases. Each layer defines the head-elements according to their capabilities and/or strategic location.
3. Distributed. In this strategy, the decisions are made by each element according to the

environment and neighbors. Strategies based on association schemes can provide conciliation among neighbors without a head-entity through reference information and by adding flexibility to balance the needs of each WE. It implies independent decisions can cause conflicts with other elements and, as consequence, decrease its performance and stability. These strategies are usually based on collaborative schemes due to their weak compromises and the restricted access to resources.

Additionally, hybrid schemes combine the benefits of central, hierarchical, and/or distributed schemes and are exploited in large networks in order to reduce the complexity and latency but establish global rules regulating some issues that ensure the operation of all the members in the network. Figure 2.4 shows a representation of networking phases in a WNs operating simultaneously.

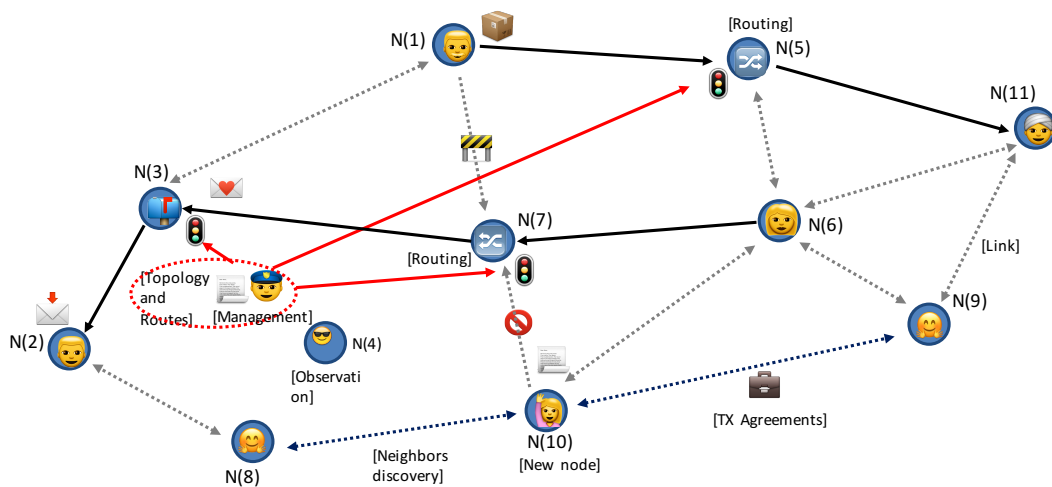


Figure 2.4: Networking.

## 2.4 Coexistence in wireless networks

Coexistence consists in the presence of active WEs or multiple signals in a given area that alters the conditions of wireless channel hindering or blocking the wireless communication. It implies important issues in WNs because these signals introduce interference, which can be generated by natural or electromagnetic radiation factors and decreases the wireless system performance. Thus, the coexistence phenomena are studied when two or more WEs combine or overlap time, frequency, and/or space domains, e.g. elements that use the nearby or same channels and share space of influence and simultaneous transmissions take place in the same channel, etc. For this reason, coexistence is widely being studied for next generation WNs in order to affront important challenges such as spectrum scarcity and high density of WEs.

In WCS, this phenomenon is known as *Coexistence problem* and can be defined as the difficulty or impossibility of establishing a reliable communication among WEs within a given area caused by interfering signals present in the wireless channel. The increasing number

of WEs, their closeness, simultaneous transmissions and their influence in wireless channel imply important challenges. There are attractive strategies for WNs that allow to share the channel with different users. These schemes are known as Coexistence schemes and have important impact on robustness and mitigation of interference levels, they consist on the use or reservation of resources to users during a period of time.

### 2.4.1 Coexistence schemes

Coexistence schemes try to establish the best conditions to communication systems and ensure operation of multiple WEs. Strategies based on multiple access techniques are useful because they distribute the resources following orthogonal or semi-orthogonal models [21]. Basically, these strategies can be described as temporal, spectral, spatial, coded or combination of models [24], [39], [51].

#### Temporal schemes

These schemes allow a single transmission in a certain wireless channel for periods of time, i.e. only one WE is transmitting information (active mode) while the rest are in standby, passive or no active mode. This contention avoids interference among WEs and allows the exploitation of the entire channel exclusively by one element. The transmission periods can be orderly or opportunistically meaning that there are schemes which assign time slots according to a plan where the synchronization plays an important role or a competition model. Examples of temporal schemes are

- Time division multiplexing (TDM) techniques

These techniques are widely used in digital technologies. They establish a transmission period subdivided in  $N$  intervals or time slots assigned to users to access the wireless channel. In time division multiple access (TDMA) the intervals have the same duration and are occupied by demand or cyclically repeating time slot. In centralized systems, the time slots are assigned by the manager element while no centralized systems, usually, employ competitive strategies.

- Opportunistic techniques

They consist of exploitation on portions or the entire channel when it is not being used. This implies that the users require to sense the channel before using it in order to determine the availability and avoid interference. The time slots have variable duration and can be regulated or not. This technique is widely used in free bands where there are no restrictions, e.g. Industrial, Scientific and Medical (ISM) bands. Opportunistic technique is also attractive to cognitive radio (CR) technologies because it allows to use a licensed band when the primary user does not transmit, i.e. the primary user is in passive mode. In this way, secondary users can access to entire band or a portion of it as sub-band with the condition of stopping transmissions when the primary user transmits, so the availability depends on primary user activity. Note that this technique does not require manager elements because the decision to transmit is made by each user.

- Schedule techniques

In these techniques, the access to channel is regulated by manager elements. It is established a plan to assign time slots orderly. Each user requests the access to the manager element, which is accepted or denied, and allocated to a specific slot to transmit., e.g. Centralized networks.

### Spectral schemes

Radio spectrum is the portion of electromagnetic spectrum where the radio waves are propagated in the space traveling at the speed of light. Its access, uses, and exploitation are important issues in wireless communications because it is a limited resource; so it is regulated by governments and/or international institutions to provide telecommunication services.

The spectral schemes distribute portions of radio spectrum in order to separate the communication links and avoid interference in the frequency domain, i.e. the signals propagate in a communication link are over a frequency range  $|f_2 - f_1|$  which limit the bandwidth [47]. Thus, spectral schemes divide a wireless channel in sub-bands in order to increase the communication links, e.g. frequency division multiplexing (FDM) techniques. Basically, these schemes can be classified as follows,

- Single band (SB) transmissions. The communication link has a unique frequency band to transmit information. It requires that both WEs (transmitter and receiver) operate in the same band in order to establish the communication link, i.e. single input - single output (SISO) system. The signals can be modulated encoding the information in a manner suitable for transmission transporting the message signals through a radio band or wireless channel. Traditional wireless technologies are based on SB transmission, such as broadcast systems.
- Multiple band (MB) transmissions. In this case, the WEs have the capability to establish multiple communication links at the same time. It is necessary to allocate multiple bands in order to avoid interference. These technologies are known as multiple-carrier (MC) systems which integrate the signals to decode and recover the messages. The techniques based on MC systems have been studied in order to efficiently use the radio spectrum, so orthogonal FDM technologies are attractive for WNs.

### Spatial separation schemes

They propose the geographical separation of active WEs or their grouping into sets in order to protect the communication links by mitigating or avoiding interference from simultaneous transmissions in a given area. These strategies allow to reuse bands of radio spectrum. They are widely used in satellite communications and wireless mobile telephony systems and are studied in ad-hoc networks. Clusterization and Guard zones schemes are interesting models that use spatial separation.

- Clusterization. The elements of a network are spatially divided in clusters (subregions) forming subnets with their own structures. In the cluster, the WEs can exploit all the resources, however the power transmission is limited in order to avoid interference with

neighboring cells. This scheme allows reuse of the radio bands when the clusters are far enough from other cluster with the same bands.

- *Guard zones.* They define protecting zones for receiver elements establishing a minimum physical separation or inhibiting transmitters around a specific receiver. These schemes consider sensing techniques in order to establish the handshakes during the initial phase of communication; it is useful for operating environments with low mobility and high density of nodes, [48].

### Coded schemes

These schemes essentially change the signal representation through patterns that code the information and add new useful characteristics to wireless systems. The patterns have important properties that allow to recover the original messages through signal processing techniques. So, in WCS the patterns are known as *code signals* that provide important features such as correlation, spread spectrum, and security for continuous access to the entire channel for all WEs permitting simultaneous transmissions in a single channel [47].

## 2.5 Aspects of Wireless network.

The diversity of applications and the emerging communication paradigms have motivated by the study and analysis of WNs. Thus, in this work it is considered that association, networking, and coexistence are useful concepts to explain them because they can describe their features beyond their technologies and sizes as Figure 2.5 shows.

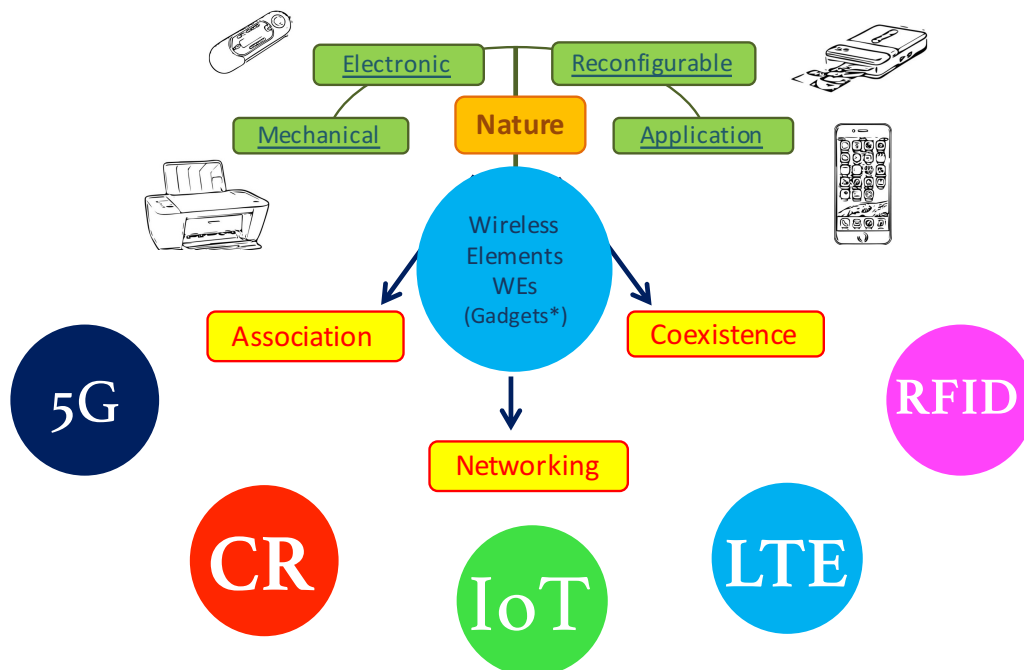


Figure 2.5: Wireless network concept.

Network architectures are related to the association schemes, the connectivity of WEs through networking process, and WEs capabilities. Thereby, the description of network structure, their features, scopes, and limitations are given by association schemes. Hence, the nature, availability, and capacities of WEs are important because their tasks and functions are defined. While networking establishes the connections among WEs in order to define the topology and technical guidelines, it also considers the incorporation of nodes in order to increase the flexibility and scalability of the network. Besides, the communication standards define the protocols and transmission agreements that ensure the network operability through management strategies. Finally, the coexistence schemes provide the necessary strategies to distribute the resources and allow the simultaneous transmissions ensuring the data transmission and reduce the effects of interference.

New trends in WCS promise high data rate, low latency, energy efficiency, and high traffic density that represent important challenges for WNs. For this reason, new issues are considered in wireless network design and motivate the development of new strategies to deal with them; thus, issues as interference, energy efficiency, spectrum utilization, and security are widely studied, which are introduced in following subsections.

### **Interference**

In WNs, interference can be defined as the distortion of signals due to the presence of agents that can degrade the performance of receivers or disrupt the communication. These distortions can be divided into two categories: interference caused by natural phenomena, which cannot be eliminated, and interference caused by transmissions factors, which can be reduced using different techniques and signal properties. Interference can be classified according to the phenomena that caused it, common examples are, [44], [39], [21],

- Multiple access interference (MAI). It is caused when multiple communications links access to channel.
- Multipath interference (MPI). It is produced by different propagation paths to reach a specific receiver.
- Multiuser interference (MUI). It is caused when simultaneous communication links exist in the same channel from different WEs.
- Inter-Symbol Interference (ISI). It is caused by the delay spread characteristics of the channel where two different symbols from the same transmission session are received overlapping one another.

The techniques used in coexistence schemes reduce the interference increasing the WCS performance, so they can be classified as (a) mitigation techniques, which reduce the effects of interference through filtering signals in the same channel, correlation properties, orthogonality of signals, etc, (b) cancellation techniques, which eliminate signal interference, (c) avoidance techniques, which separate the signals in a certain domain as time, frequency, space, and (d) interference management, which controls the signal transmission. For example, Multiple access techniques combine these techniques and exploit signal properties to construct coexistence strategies.

### Spectrum utilization and efficiency

Spectrum utilization is regulated by governments and international institutions in order to allocate frequency spectrum portions to technologies and services satisfying important requirements as propagation and coverage area, maximizing its use and distribution in benefit of nations and communication systems through spectrum management plans allowing the development of new technologies and services establishing a frequency allocation chart and assigning concession and operation licenses, [5]; so primary and secondary users are licensed users to use and exploit the spectrum in a certain band with a specific service. However, there are free bands that do not require permission to be accessed such as Industrial, Scientific and Medical (ISM) bands, these bands are used by technologies with low power transmission and small scale, e.g. ZigBee, Wifi, and Bluetooth that require coexistence schemes to avoid harmful interference.

On the other hand, spectrum efficiency refers to the amount of data bandwidth that a specific technology can extract from a certain frequency spectrum with the fewest transmission errors, [40], i.e. the maximum information rate transmitted over a given bandwidth in a specific WCS and reliable performance; hence spectrum efficiency may also be called *bandwidth efficiency* which is usually expressed as bits per second per hertz ( $bits/s/Hz$ ), [18]. Thus, wireless technologies seek to use the spectrum more efficiently through associative strategies (collaborative and cooperative strategies), new communication formats (modulation and multiplexing techniques), dynamic radio environments, etc.

### Energy efficiency

It refers to the efficient use and consumption of energy by WEs, which has been taking a tremendous interest in WNs because energy supplies are limited. It can be studied by WE consumption or by WN consumption. The first one considers the individual consumption of each element. The strategies seek to prolong the lifespan of the battery, by reducing the operations per WE and the algorithm complexity, by cross layering techniques, by embedded systems, etc. While the second one considers strategies based on WN consumption. They seek low power transmission by using associative models to save energy, power and routing control to reduce the volume of information transferred, etc.

It is important to consider the new trends in WNs that explore new *green sources* for WEs as alternative sources for extracting energy from the surrounding environment and converting it into consumable electrical energy. These processes are known as *energy harvesting* or *power scavenging* with interesting features such as low maintenance and cost. Therefore, different forms of ambient energies are widely studied to recharge and prolong the lifespan of battery as sunlight, mechanical energy, thermal energy, and RF energy, [6], [33]. Additionally, there are strategies that propose the wireless transfer power through inductive coupling, magnetic resonance coupling, and electromagnetic radiation. They also consider both wireless transfer networks, which are assistant networks with unique goal to provide energy to WE, and simultaneous wireless information and power transfer networks (SWIPT), e.g. broadcast systems.



## Security

The protection of information and network connectivity ensuring are important aspects in WNs that are related to data confidentiality, integrity, and non-repudiation of information [42]. However, wireless nature makes WNs vulnerable to malicious and/or unauthorized users and puts integrity and privacy of the information at risk; thereby, security strategies provide authentication and protection methodologies in order to blind the information and ensure the performance of WCS, which represent important challenges. In this way, each technology seeks to develop robust protocols and algorithms according to its features, they have to be resistant to malicious WEs as eavesdropping nodes, which are unauthorized receivers that try to extract information from wireless channel, and jamming nodes, which try to degrade the signals and, as consequence, the WN performance.

Security methods can be instrumented in different phases or layers in WCS, i.e. security methods can be based on physical layer, MAC, network management, or access to session techniques, etc. Then, techniques as cryptography, multiple access, channel coding, key words, etc are widely studied. New trends propose cross layer functions, coding signals, and proactive or reactive systems based on associative mechanisms, which involve actions from authorized users in networking process, in order to increase the reliability and reduce risks; this way techniques as relay networking, physical layer network coding, secrecy capacity, channel coding, and power approaches have been proposed, [42], [26], [52].

## 2.6 WNs classification

The description of WNs is an important issue in their study in order to determine their capabilities and reaches. Aforementioned, WEs play an important role because define the association schemes, features and network structures considering their nature, and the physical and technological limitations.

The WEs diversity allows to construct WNs with different features, however it is possible to identify their particularities, aspects, and/or applications in order to characterize them and define a classification based on any selection criteria. The following WNs classifications are useful examples that allow to describe them from different points of view.

### 2.6.1 Classifications based on WEs

They characterize the WNs from any attribute, feature, or nature of the WEs. It is possible to identify, at least, three different classifications based on WE which are

- Considering the type of WE.
  - *Kind of WE.* When all the elements are the same or have similar features, the network is defined as *homogeneous* WN; otherwise when WN has different kind of WEs or have different features, the network is defined as *heterogeneous*.

- *Particular element.* They consider a particular element in WN as criteria, e.g. vehicular ad-hoc networks (VANETs), which are mobile networks that are related with the communications among vehicles.
- *Nature of WE.* The nature of WEs is a useful criteria to describe a WN, for example, re-programmable WEs can construct reconfigurable WN or virtual networks can be constructed from any application installed in smart devices with wireless access.
- Considering WEs size. These classifications are widely used to describe networks from size of elements. For example, micro-WNs measure the size of each element in micrometers. These nomenclatures appear as a consequence of the minutirization of the WEs.
- Considering any WE capability. They refer to a particular capability of WEs as mobility, sensibility to environment, reconfiguration of WEs, or perform a task or action. For example, mobile WNs define networks that allow the element mobility and adjust the parameters in order to ensure the communication.

## 2.6.2 Classifications based on WN aspects

In these cases, WNs are described by features

- Considering coverage area. They contemplate the area that cover all the elements in the networks, i.e. the entire coverage. This way, these WNs can be defined as
  - *Wireless personal area network (WPAN).* They are used to describe small WNs that cover an area around a person, approximately 10 meters, which allow the connectivity *point-to-point* (P2P) with technologies with low power transmission; they are widely used in industrial control networks composed of wireless sensors and actuators, and private personal networks composed of wireless handset tools, for example wearable networks that connect smart elements as phones, watches, headphones, etc.
  - *Wireless local area network (WLAN).* They have coverage area, approximately, less than 100 meters and can be used to interconnect WPAN or cover small private areas, for example, WLAN are used as access point to internet service in *Home-Office* WNs.
  - *Wireless metropolitan area network (WMAN).* These networks allow to establish the wireless communication among WEs within a metropolitan area, approximately less than 5 Kms. They require technologies with large range and/or broadband access. They are also used to interconnect smaller WNs or to provide a service in specific zones, for example private networks between multiple office buildings in a city or mobile networks as cellular services.
  - *Wireless wide area network (WWAN).* They can be considered as an extension to WMAN that defines coverage areas less 15 Kms. Besides, WWAN can also be used as backbone network for wireless services within a wide area.

- *Wireless regional area network (WRAN)*. They have coverage areas less 40 Kms that allow interconnect different cities and/or remote rural populations. Furthermore, they can be used as infrastructure for broadband technologies or broadcast services as TV, AM and FM radio.
  - *Wireless global area network (WGAN)*. These describe WNs that cover portions of planet; these networks are dedicated to specific services and regulated by international institutions. Satellite technologies are attractive to these networks because they support several technologies and cover large areas.
- Considering Association scheme. It refers to associative strategies among WEs to perform to a specific task. Recall, the association strategies are described as collaborative or cooperative schemes.
  - Considering coexistence scheme. It considers the distribution strategy of resources and access channel technique. Aforementioned, these strategies are classified as temporal, spectral, spatial, and coded.
  - Considering WN Architecture. The architecture describes the network structure, their organization and configuration. Network architecture describes the full layout of WEs connections and their communication links with detail. WN architectures can be described using to the physical or virtual figure that form their elements (topology) or using a hierarchical strategy.
  - Considering WN routing. They consider the nature of routing strategies to define the path that follow the information signals from source element up to reach its destination element. For example, when the path between two elements is established previously, the strategy is known as static routing, otherwise when the paths can change the strategy is known as dynamic routing. Other option is to consider an specific criteria known as *weight parameter* such as distance among nodes, traffic in the communication link, the number of relay elements (hops) that intervene during all the path between source and destination nodes, etc. For example, the direct communication among nodes is known as single hop communication while when more relay elements (or hops) intervene is called multi-hop communication.
  - Considering WN management scheme. It considers the management and control mechanisms that ensure the WN operation. Aforementioned, the management schemes are defined as centralized, hierarchical, or distributed. This criteria also considers the capability of management scheme to adapt its performance to new conditions, i.e. the ability to change network feature to resolve or optimize any unexpected challenge. This ability is introduced to management schemes through *smart* algorithms which can resolve any issues using alternative solutions or learning algorithms and optimize the solution. Additionally, they are also considered the capability to implement through manual or autonomous configuration, e.g. self-organization WNs.

### 2.6.3 Classifications based on others aspects

- Considering communication paradigms. They contemplate the paradigms that describe the proposals that resolve important challenges through describing their concepts, guidelines, and technologies. Internet of things (IoT), Cognitive Radio (CR), Green wireless systems, and 5G of WCS are some attractive paradigms to next generation of WNs.
- Considering the band of operation. They consider the frequency range that operate the WEs, for example the dedicated bands assigned by governments to a specific technology or service (satellite, aeronautic, TV -Radio waves, etc.) or any characteristic of the frequency used (infrared, acoustic, etc.). Usually, it is also used the wavelength ( $\lambda$ ) as indirect parameters using the relation  $\lambda = C/f$  where  $C$  is speed of light and  $f$  is the frequency of operation, e.g. millimeters networks.
- Considering the technology. Sometimes the technology implemented in WEs is enough to define a network because it implies the use of specific protocols, mechanisms, and/or models, e.g. Bluetooth, Zigbee, LTE, WiMAX, etc.
- Considering the utilization. This classification considers the application areas, for example it is possible to describe WNs as industrial, wearable, social, medical, etc. On the other hand, it is common to describe WN according to their proprietor, i.e. private or public WNs.

Figure 2.6 shows different classifications base on element features, WN aspects, and other useful criteria.

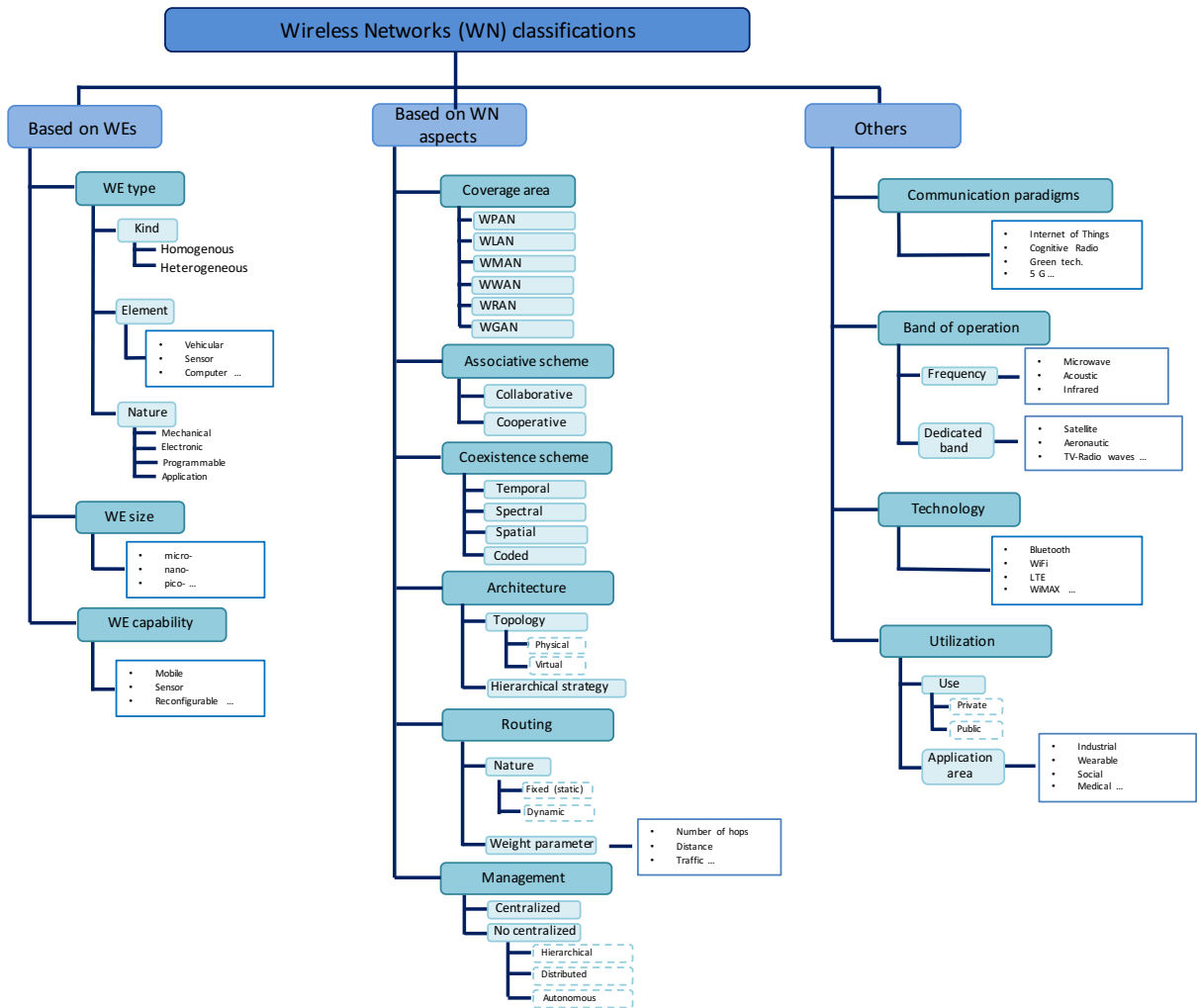


Figure 2.6: Different classifications of WNs.

# Chapter 3

## Reconfigurable Wireless Networks and CDMA systems

In this chapter, reconfigurable wireless networks (RWN) are introduced considering their attractive features for next generation of WNs such as adaptability, flexibility and their needs. The description of two code division multiple access (CDMA) models are given considering their advantages and disadvantages. Besides, relevant impediments for RWN based on CDMA models are presented and some complementary schemes used to decrease their effects.

### 3.1 Introduction

Increasing number of WEs and their applications have been changing our lifestyle and, as consequence, the perspective on WCN. The emerging paradigms in wireless communication systems (WCS), which promise high data rate, low latency, energy efficiency, and high traffic density, have motivated the develop of new technologies, models, and strategies in order to offer more services based on WEs connectivity. New trends incorporate communication ability to interconnect WEs that allow their association and construct WN through different platforms. The convergence and integration of wireless technologies in WEs represent a collection of challenges for networking. Additionally, the high density of elements also represents an important challenge due to closeness of WEs and the phenomenas that it involves, e.g. coexistence and multiuser access problems, multiuser and multiple access interferences (MUI and MAI), etc.

Figure 3.1 shows the growth and connectivity of WNs since 2014 and the perspective for 2021 and the assortment of WE from Ericsson mobility report (Q4-2015), [35]; the diversity of networks are based on different schemes according to the WEs capabilities and the available resources. Therefore, the next generation of WNs must operate under several challenging conditions, [14].

Network reconfiguration is an attractive feature because allows to adjust the WEs operation in order to ensure the communication among WN members. This way, reconfigurable wireless networks (RWN) can be defined as dynamic networks with the capability to modify

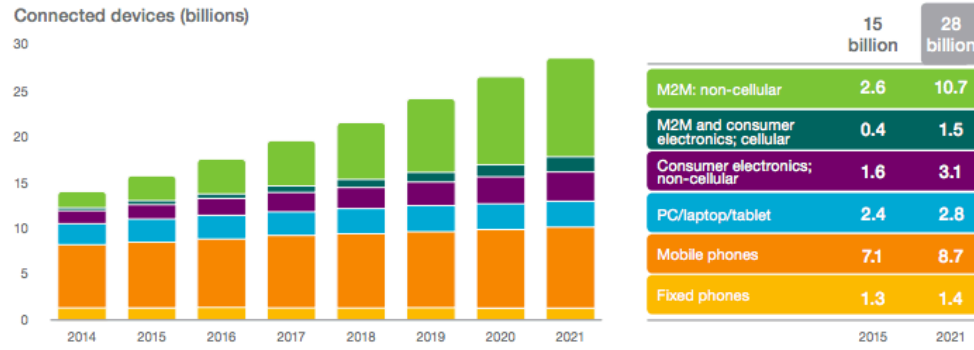


Figure 3.1: Present and perspective of connectivity of devices.

its configuration and communication parameters through management schemes that define the priorities to take local decisions and the use of their resources, i.e. WEs can change their association scheme, topology, routing plan or power transmission according to new requirements. For example, Self-organization networks, which are RWN that respond autonomously to frequent changes in their environment, erratic WE status and needs of their members through *smart* process, are attractive to spontaneous networks without infrastructure such as ad-hoc. In Figure 3.2 is represented a RWN with a temporal configuration, Active and No active elements, and communication links that follow only the available nodes.

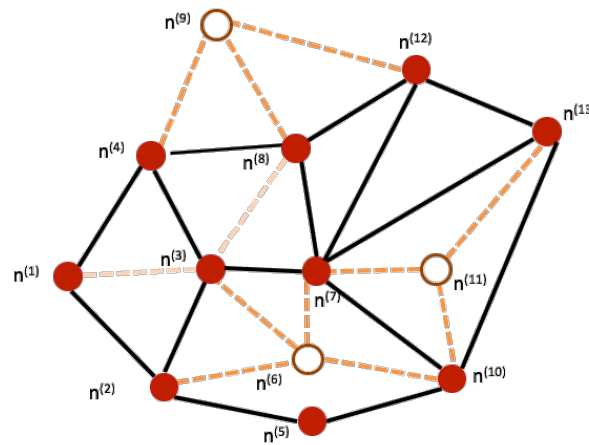


Figure 3.2: Reconfigurable Wireless Networks.

RWN confront important impediments due to simultaneous communication and closeness of their WEs. Thus, coexistence schemes are keys because they provide the distribution of resources and access channel in order to allocate users with reliable conditions to communicate. Techniques based on multiple access schemes have been proposed to provide , uniformly or flexibility, the resources considering the temporal, frequency, and spatial parameters, or combinations to separate users, [21].

In this work, the systems based on code division multiple access (CDMA) technique are

considered attractive to RWN because they provide engaging features given that offers continuous access to entire channel for all users, i.e. CDMA technique permits to coexist multiple users and simultaneous transmissions using a single channel, [47], it represents a set of challenges in multiuser WCS since it implies the presence of more interference signals in the wireless channel that can degrade or interrupt the communication. Although CDMA systems are interference limited system, there are useful strategies that help to reduce the interferences effects and improve the receiver performance, so mitigation, cancellation, suppression and alignment techniques are widely used.

On the other hand, the dynamic nature of RWN also presents relevant phenomena that decrease the CDMA system performance such as interferences caused by hidden and exposed terminals and *Near-Far* problem, which are shown in Figure 3.3, and they are described as follows

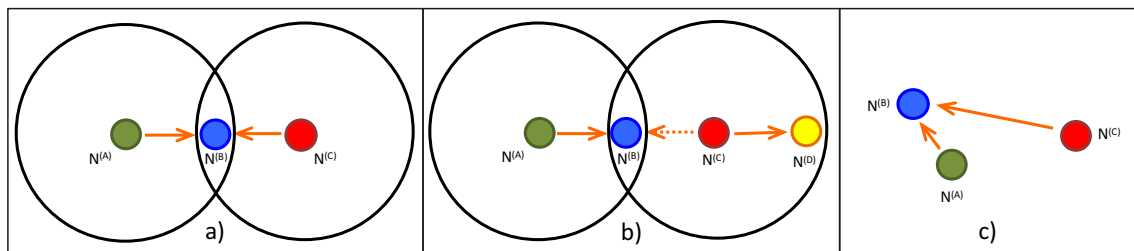


Figure 3.3: a) Hidden terminal interference, b) Exposed terminal interference, and c) Near-Far problem.

- **Hidden terminal.** It refers to the mutual interference caused by transmitters that send information to the same receiver using the same channel; it occurs when each transmitter is not visible or hidden to another one and degrade the receiver performance. In Figure 3.3a, the transmitters  $N^{(A)}$  and  $N^{(C)}$  send information simultaneously to  $N^{(B)}$ ; note that node  $N^{(C)}$  is not visible to  $N^{(A)}$  and vice versa, thus  $N^{(C)}$  is a hidden terminal to  $N^{(A)}$  and, similarly,  $N^{(A)}$  is a hidden terminal to  $N^{(C)}$ .
- **Exposed terminal.** It is a transmitter that send information within coverage area of a certain receiver which is communicating with another node. It is illustrated in Figure 3.3b, in this case  $N^{(A)}$  transmits to  $N^{(B)}$  and, simultaneously,  $N^{(C)}$  transmits to  $N^{(D)}$ ; however the signal transmitter by  $N^{(C)}$  is also reached by  $N^{(B)}$  and, as consequence, interferes to transmitter signal from  $N^{(A)}$ . Given that  $N^C$  is visible or exposed to  $N^B$ ,  $N^C$  is known as Exposed terminal.
- **Near-Far problem.** It refers to the difference of received power from two transmitters due to their distance to the receiver. So, the near transmitter interferes with transmitted signal from far transmitter. In Figure 3.3c, the nodes  $N^{(A)}$  and  $N^{(C)}$  transmit to  $N^{(B)}$  with the same transmitted power; nevertheless, the receive signal from  $N^{(A)}$  is stronger to  $N^{(C)}$  and alters the transmitter signal from  $N^{(C)}$ . Note that *Near-Far* problem is related with Hidden and Exposed terminal interferences.



## 3.2 Code division multiple access (CDMA)

Code Division Multiple Access (CDMA) is a multiple access technique that combines code division multiplexing (CDM) strategy and Spread Spectrum (SS) technique. CDM assigns codes to separate users in a single channel and SS technique spread the information signal over a wide frequency band through spread sequences independent to the data, see Figure 3.4, [16]. Therefore, CDMA is accomplished multiplying the data signal with a spread code increasing the transmission band, i.e. the information signal with narrowband  $B_i$  is expanded to spreading signal bandwidth  $B_t$ . Therefore, *Processing Gain* defines the relationship between information signal and spreading signal as  $G_P = B_t/B_i$ , where  $B_t \gg B_i$ .

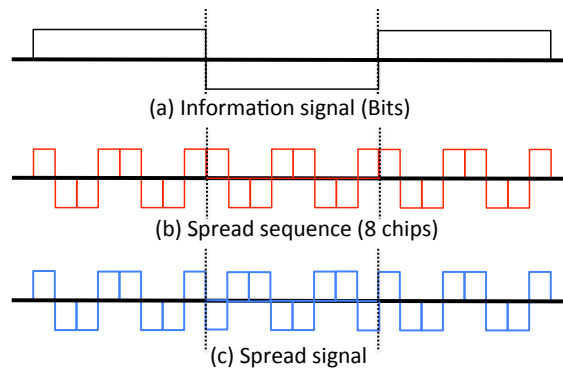


Figure 3.4: CDMA signals.

SS systems have interesting characteristics such as a) *low power spectrum density* that drops the SS signals with a factor  $1/N_S$  (where  $N_S$  is the spreading factor) and introducing low interference into the wireless channel and make them difficult to detect, b) *immunity against jamming attacks* that reduces and cancels undesired signals caused by harmful sources, c) *high time resolution* due to fast resolution of correlation functions that measure the relationship among signals and follow the order of the chip duration, and d) *multiple access* that allows to share the wireless channel to multiple users through properties of spreading sequences used as codes, [16], [2], [36], [37].

## 3.3 Types of schemes based on CDMA system

There are two primary CDMA schemes used in WNs: Direct Sequences (DS) and Frequency Hopping (FH); the spreading codes play an important role to establish the transmission bandwidth, signal representation, and decision factors. Other hybrid schemes combine the CDMA systems with different techniques as time and frequency division in order to add particular features, [39].

In DS-CDMA scheme, the signal is spread to the entire communication channel and coded with the transmitter or receiver codes. On the other hand, FH-CDMA scheme divides the communication channel in sub carriers. The signal is transmitted hopping to different sub

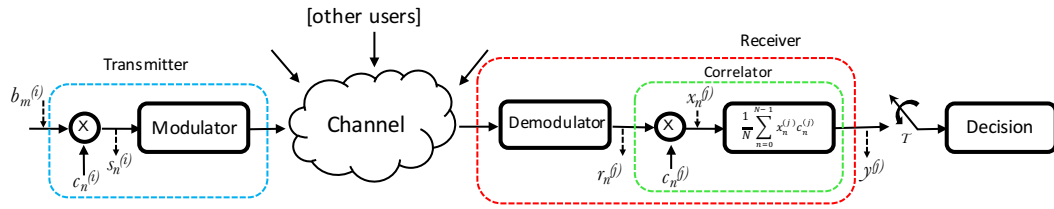


Figure 3.5: Traditional DS-CDMA system diagram block

carriers periodically describing a *hopping pattern* which is defined by the spreading code, [37]. In this thesis, it is of interest the analysis and study of DS-CDMA systems for single and multiple users scenarios considering BPSK modulation applied in RWN while FH-CDMA systems are not considered. Besides, it is introduced an attractive scheme based on DS-CDMA system proposed by C. Barrera in [4] that encodes the information signal using two codes which provides powerful features to RWN.

### 3.3.1 Direct - Sequences CDMA (DS-CDMA) system

The principle of DS-CDMA system consists in assigning a sequence as code that encodes and spreads the information signal into entire channel. Figure 3.5 shows the diagram that describes the single user communication link of traditional DS-CDMA system and it is described as follows.

Considering a transmitter  $i$  that generates a stream of information with  $m$  bits  $b_m^{(i)}$  with energy  $\pm\mathcal{E}_b$  which are multiplied by a code  $c_n^{(i)} \in \{\pm 1\}$ ,  $n = 0, \dots, N - 1$ ; where  $N$  is the number of chips. Then, the spread signal  $s_n^{(i)}$  is modulated and transmitted to the wireless channel. On the other hand, the received signal is the sum of all signals present in the channel such as information, interference and noise due to DS-CDMA system allows transmit to multiple users sharing the same channel at the same time. The received signal  $r_n^{(j)}$  is demodulated, decoded with the receiver  $j$  ( $c_n^{(j)}$ ) and the despreading operation is performed to obtain the decision variable  $y^{(i)}$  which is achieved correlating the signal with the receiver code to recover the information. Flikkema in [17] introduces a useful mathematical model in discrete time. Considering synchronous transmissions and additive white gaussian noise ( $W_n$ , AWGN) during the reception of the  $n - th$  chip with mean value of zero and standard deviation  $\sqrt{N_0/2}$ , and the propagation effects are ignored.

Multiuser receiver consists of a bank of correlators with multiple branches, [34], so the number of branches is defined by the number of elements in the network less one. Hence it is assumed that RWN topology has already been established and every nodes know their neighbors and their sequences. This way, the configuration of bank of correlator is shown in Figure 3.6

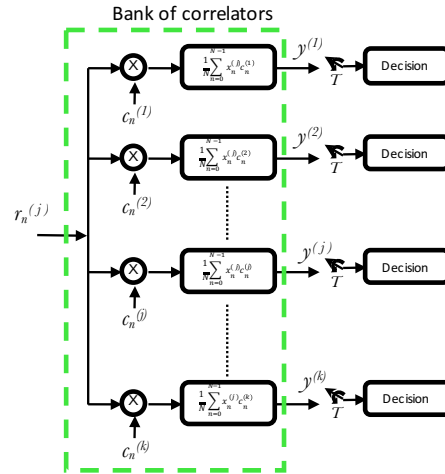


Figure 3.6: Receiver for multiuser scenario

The decision variable in branch  $i$  is given by

$$y^{(i)} = \frac{1}{N} \sum_{n=0}^{N-1} b_m^{(i)} c_n^{(i)} c_n^{(i)} + \frac{1}{N} \sum_{n=0}^{N-1} \sum_{\substack{j=1 \\ j \neq i}}^{\Psi} b_m^{(j)} c_n^{(j)} c_n^{(i)} + \frac{1}{N} \sum_{n=0}^{N-1} W_n^{(i)}$$

This way, it is easy to note that the first term in  $y^{(i)}$  contains the information sent using the code  $i$  which is also called *information term*. The second term defines the interference caused by other transmissions in the channel and, finally, the third term denotes the AWGN spread by the code  $c_n^{(i)}$ . Again, the properties of codes play an important role.

CDMA systems exhibit additional features to SS system which are described as follows, [37], [39].

- ADVANTAGES

- *Selective Addressing Capacity*. Each transmitted signal is identified by the transmitter or receiver code.
- *Multipath interference*. Considering synchronous reception, the interference caused by copies of transmitted signal due to different propagation phenomena are mitigated using the properties of codes.
- *Multiple access interference*. It is mitigated using a set of codes with *good* correlation property.
- *Low probability of interception*. The information signal is recovered only with the code assigned.
- *Resistant to Exposed terminal interference*.

- DISADVANTAGES

- *Asynchronous reception*. The correlation properties of the sequences can be broken out of the synchrony which decrease the receiver performance.

- *Power control.* The receiver is vulnerable to *Near-Far problem*, so power control strategies are used to manage the transmission power of each user and avoid or mitigate thus problem.
- *Vulnerable to Hidden terminal interference.*

CDMA system have been adopted by cellular mobile communication technologies with centralized configuration, add protocols and implement complementary strategies to manage the system. Examples of this technologies are the IS-95 standard, UMTS and Satellite systems, [16].

### 3.3.2 CDMA with double codification (C2DMA) system

It is a new model proposed by C. Barrera in [4] that is based on traditional DS-SS system and provides attractive features to RWN. Figure 3.7 shows the diagram block of C2DMA system for single user case. Note that it considers a double codification strategy which requires additional properties to spreading codes. The mathematical models for single user and multiple users are provided by C. Barrera considering synchrony, BPSK modulation and AWGN channel, while propagation effects are ignored.

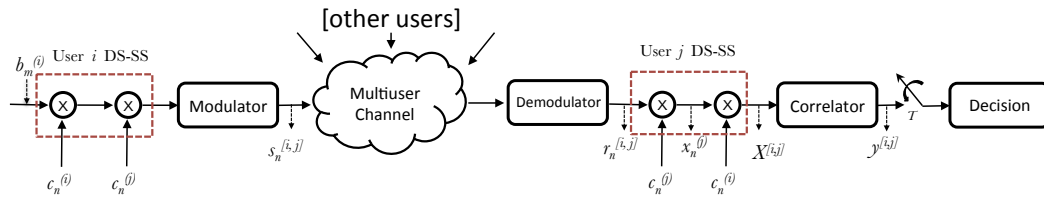


Figure 3.7: DS-C2DMA system diagram block for single user scenario.

When multiple transmissions take place simultaneously using the same communication channel, the received signal is decoded first by the received code  $c_n^{(k)}$  while a bank of correlators decode the possible transmitters code  $c_n^{(v)}$  in each  $v$ -th branch corresponding.

Then, when the receiver is the destination of the information, i.e.  $k = j$  and  $v = i$ , the decision variable in  $i$ -th branch is given by

$$\begin{aligned}
 y_n^{[i,j]} &= b_m^{(i)} + \frac{1}{N} \sum_{n=0}^{N-1} \sum_{\substack{v \in \Psi \\ v \neq i}} b_m^{(v)} c_n^{(v)} c_n^{(i)} \\
 &+ \frac{1}{N} \sum_{n=0}^{N-1} \sum_{\substack{k \in \Omega \\ k \neq j}} \sum_{\substack{v \in \Psi \\ v \neq i}} b_m^{(v)} c_n^{(v)} c_n^{(k)} c_n^{(j)} c_n^{(i)} \\
 &+ \frac{1}{N} \sum_{n=0}^{N-1} W_n^{[i,j]}
 \end{aligned} \tag{3.1}$$

Hence, it is possible to distinguish four terms. The first one is the information term received in the  $i$ -th branch from node  $i$  to node  $j$ ; the second term contains the sum of signals

send to node  $j$  from nodes different to  $i$ , i.e. the hidden terminals to  $i$ -th branch; on the other hand, the third term contains the sum of signals that are not related with nodes  $i$  and  $j$  from other members in the RWN, i.e. they are exposed terminals to node  $j$ . Finally, the fourth term is the AWGN spread by the code in the  $i$ -th branch. The properties of spreading sequences are of top importance because each property adds features increasing the robustness of the receiver. For example, the balance property that reduces the DC voltage component. The correlation properties, which measure the similarity of sequences, play an important role, so autocorrelation (AC) property, which compares with time-shifted versions of itself and helps to recover the information and mitigates multipath interference; crosscorrelation (CC) property, which compares two different sequences, and double correlation property (DC), which consists of the orthogonality of all products of sequences that mitigate the MAI and MUI. Now, recall that two sequences are said to be orthogonal if the sum of the element-wise product with each other is zero. These properties are exploited to distinguish the origin-destination pairs.

This model provides powerful features to RWN, so its advantages and disadvantages are

- ADVANTAGES

- *Simultaneous reception.* The use of double codification allows to receive multiple signals, simultaneously, from different transmitters.
- *Resistance to Hidden interference.* This interference can be mitigate because double codification robustness the correlation properties of codes.
- *Cross-layer system.* C2DMA system simplifies some process to communication system because the signal properties can reduce the complexity of protocols used in others layers.
- *Codification.* The signal is coded using two different codes, so to retrieve the information it is necessary to known both transmitter and received codes; on the contrary the signal cannot be decoded. This feature provides any kind of information security.

- DISADVANTAGES

- *Spreading codes.* C2DMA system needs an additional property to reach its best performance, double correlation. This condition reduce the set size of codes that can be used in C2DMA system.
- *Receiver complexity.* The receiver requires more hardware to decode the information.
- *Asynchronous reception.* Similar to traditional DS-CDMA system, C2DMA system requires synchrony reception to maintain the correlation properties among the sequences.

## 3.4 Related works

This section reviews some schemes used in traditional DS-CDMA system for RWN. Besides, two of schemes, which also consider two codes, are presented in order to contrast their features with C2DMA system.

### 3.4.1 DS-CDMA systems and RWN

The attractive features of DS-CDMA system for RWN has motivated the development of useful strategies that allow their implementation. Therefore, strategies as power control, clusterization, scheduling, random access, and guard zone seek mitigate interference problems and save resources.

- Power control strategies. They are strategies that manage the power transmission of WEs in order to avoid interference caused by other WEs around a specific receiver, long term fading effects, and regulate their energy consumption, i.e. power control strategies ensure the sufficient transmit power to establish the communication link minimizing the interference caused to vicinity nodes, [1].
- Clusterization strategies. These strategies are based on dividing the networks in sub-networks reducing the number of nodes and defining similar structure to centralized systems where a head node that acts as manager node that implement simple power control strategies as open loop or closed loop.
- Scheduling or contention less strategies. These strategies establish an orderly transmissions manner, the resources are assigned through fixed or demand scheduling and can be complemented with other schemes such as TDMA and FDMA, [37].
- Random access or contention strategies. These strategies describe a process that define when is possible send information without interfering with the transmission of others transmitters. A decision factor is established in order to reduce the probability of collision. Due to the transmitters do not have the knowledge that the channel have been used, it is possible that the transmissions can fail. Hence the importance of the random decision factor. Random access strategies can be divided in two groups, the repeat random access and random access with reservation, [37].
- Guard zone strategies. They are based on protecting receiving nodes through a minimum physical separation and extended inhibiting transmitters around a receiver. It considers carrier-sense multiple access (CSMA) protocol to establish the handshakes during the initial phase of communication; it is useful for operating environments with low mobility and high density of nodes, [48].

Nevertheless, the implementation of these strategies increase the complexity system and latency. The coordination and handshakes process also decrease the transmission capacity. Thus, new researches seek reduce the complexity and operation.

### 3.4.2 CDMA systems using two codes

The concept to use two codes in a CDMA system is not new, so in this subsection they are presented two approaches that consider two codes to transmit information. These approaches are also alternative to next generation of WNs, however they have important disadvantages to consider.

#### **System 1: Modulation technique for Spread Spectrum multipath channels.**

H. Zare proposed a modulation and demodulation technique of information for a CDMA system in [54], it consists in assigning two orthogonal sequences as codes for transmitting the bit stream with different codes, i.e. the binary bit ones are spread by one of the assigned codes and zeros by the other code. In this system, receivers contain two branches for demodulation of information using their code assigned. This proposal has as main advantage that it considers the MPI and MAI effects and the performance is close to that of single user scenario. However, the number of active users is limited by the number of codes available in a set  $M$ , i.e. the number of active users are given by  $M/2$ ; so, the number of active users and the complexity of the receivers represent their main disadvantages.

#### **System 2: Coding scheme for Packets in CDMA Ad-Hoc networks.**

A coding scheme is presented by D. Liu in [32] defined as Transceiver Packet Coding Scheme (TPCS). It assigns each nodes of the network orthogonal sequences as codes to transmit packets. The scheme considers that each node knows its neighbors and their corresponding code assigned. Thus, if node  $i$  needs to send information to node  $j$ , the packet is constructed with a header and information data where the header contains the receiver code. For transmission, the whole packet is modulated using the spreading code of transmitter node  $i$ . The receiver despreads the packet with each of neighboring codes so that the origin of the information is known. Afterwards, the receiver code (recovered in the header) is used in order to extract the information. If node  $j$  is not the intended receiver, then the result of the multiplication will be zero due to the orthogonality. The system is extended to the case of multiuser detection (MUD). As advantages of this system is that the number of users is equal to number of available codes  $M$ . Also, this system resolves the hidden and exposed terminal problem, and allows multiple reception using MUD in perfect asynchronous system. Nevertheless, the use of a header depend on processing gain is an important disadvantages due to it consumes bandwidth for control information. Besides, the complexity of the receiver increases when MUD is used due to the need to get the correlation of signals received and some matrix manipulation.

## 3.5 RWN based on CDMA systems: A discussion

In this chapter, four CDMA models were presented which have been explored in RWN. They are traditional DS-CDMA, C2DMA, *System 1* (Modulation technique), and *System 2* (Transceiver Packet Coding Scheme, TPCS) that provide attractive features for RWN. Besides, theses systems also provide interesting properties to new trends in WCS, for example,

spectrum utilization and efficiency increasing the the availability of spectrum to others technologies. However, it is possible to distinguish differences among CDMA models that are discussed in this subsection.

Therefore, traditional DS-CDMA system requires the implementation of auxiliary strategies seeking to reduce interference phenomena and increase capacity, so *Near-Far* problem and hidden terminals are important issues to this model. This way, the strategies mentioned in subsection 3.4.1 are incorporated to WCS in order to satisfy the RWN requirements. It implies a strong association among the members of the network and robust management mechanisms, i.e. this model is based on cooperative schemes. All these additional requirements increase the complexity of the system, which is an important disadvantage. Furthermore, the correlation properties of sequences used as spreading codes play a crucial role because they provide the multiple access capability and mitigate interference. On the other hand, C2DMA system provides powerful features to RWN because it allows simultaneous reception and resistance to hidden terminal interference through cross-layer solution, i.e. the double codification strategy, which can be implemented since physical layer, helps to resolve important problems as *Near-Far* without auxiliary strategies implemented in WCS. This is an interesting feature because C2DMA model only requires incorporate the double codification strategy to confront important issues. However, these features depend on spreading code and their correlation and double-correlation properties which play a key role in this system. C. Barrera describes the double-correlation property and provides small sets of sequence that satisfy all the properties. His results promise attractive features. Nevertheless, he restricted his study to Walsh-Hadamard sequences. I. Ayala in [15] explored a different family of spreading codes: complementary codes, and his results suggest that new codes can improve the performance of C2DMA system.

Aforementioned, *System 1* requires two codes to form a single communication link. It reduces the number of available codes and, as consequence, the number of users in the network. The combination of bit streams requires synchrony in order to restore the information stream. Its main advantage is to consider MPI and MAI effects. In contrast, *System 2* provides robustness to hidden and exposed terminals and multiple user reception but its header strategy consumes bandwidth for control information which is relevant disadvantage because it reduces the bandwidth efficiency. These systems confront satisfactorily interference issues using interesting strategies, however they consume resources (codes or bandwidth) in order to carry out their implementation. Besides, they require additional process to recover the information that increase their complexity.

Note that all CDMA models introduced seek additional tools in order to confront the RWN challenges and, as consequence, the complexity of WCS increases. On the other hand, C2DMA model promises robustness to interference and simultaneous reception of signals which are attractive features to future WNs; its implementation only requires a single modification of traditional model. The properties of codes are the key of this model. For this reason, this work considers that the study of spreading sequence sets are an essential issue in order to explore new scopes and applications of the system. Finally, C2DMA system was presented in



[50], as part of this work, where their attractive features are exposed as simultaneous reception and robustness to hidden and exposed terminal interference that allow the association and coexistence of nodes.

# Chapter 4

## An analysis of known sequence sets

In this chapter, known spreading code sets are studied based on their attractive properties for WCS. Conventional sets of codes used in CDMA system are analyzed in order to inspect their correlation behavior considering synchronous and asynchronous reception. An extensive survey of sequences used as codes in communication systems is presented in [49] which was elaborated as part of this work.

### 4.1 The CDMA system and spreading codes: properties

The properties of codes are crucial, so they are introduced in wireless communication terms, particularly focused on the CDMA system, in order to define their impact and usefulness. Recall that each property adds useful features which improve its performance.

- *Balance.* This property helps to reduce the DC voltage component in the spread signal. Thus, it is tested following the equation 4.1 for each sequence; so considering binary codes  $c_{n,s}^{(i)} \in \{\pm 1\}$  are called *balanced* only if the sum of every elements is equal to zero for codes with *even* length and  $\pm 1$  for *odd* length. It suggests that balanced codes of even length do not introduce levels of DC voltage which make them attractive to CDMA system.

$$Bal = \sum_{s=0}^{N-1} C_{n,s}^{(i)} = 0 \quad (4.1)$$

- *Correlation properties.* They measure the similarity among codes which provide the capability to distinguish and/or attenuate signals from wireless channel impairments due to their values define the code set usefulness; hence the importance of their study. Thus, autocorrelation plays an important role for despreading signals and recovering the information, i.e. it allows to detect the presence of a code in the channel. On the other hand, Cross-correlation property attenuates the encoded signals with different codes, i.e. the signals are not despreading, so it reduces harmful signals from different transmitter-receiver pairs and provide the multiple access capability, e.g. it mitigates the MAI and MUI in the receiver.

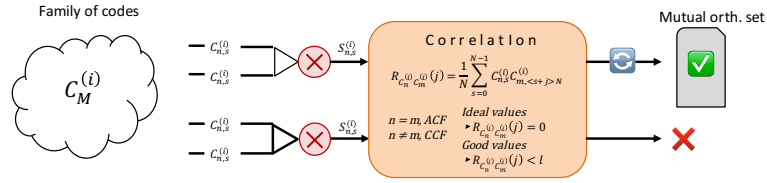


Figure 4.1: Double-Correlation property.

Desired values of correlation are given in equations 4.2 and 4.3 for ACF and CCF respectively, which are also known as *ideal values*. Besides, they are also considered *good* values, low values and do not represent significant interference to receiver. The main lobe, in desired phase, is much bigger than side lobes for ACF and close to zero for CCF, which are described through correlation matrices and Side-lobes graphs, see Section 4.2.

$$R_{C_n^{(i)}}(j) = \frac{1}{N} \sum_{k=0}^{N-1} C_{n,k}^{(i)} C_{n,\langle k+j \rangle_N}^{(i)} = \begin{cases} 1, & j = 0, \\ 0, & j \neq 0, \end{cases} \quad (4.2)$$

where  $\langle k + j \rangle_N = (k + j) \text{-mod } N$ ,  $j = 0, 1, \dots, N - 1$ .

$$R_{C_n^{(i)} C_m^{(k)}}(j) = \frac{1}{N} \sum_{s=0}^{N-1} C_{n,s}^{(i)} C_{m,\langle s+j \rangle_N}^{(k)} = 0, \quad (4.3)$$

Note that Equation (4.3) is valid for sequences in different families, i.e., for  $i \neq k$  as well as for  $i = k$  as long as  $n \neq m$ .

Finally, the C2DMA system requires a particular case of correlation property due to double codification scheme which is called *Double-correlation* that consists in maintaining the correlation of all products of codes as shows Figure 4.1.

- Shifting. This property satisfies the correlation properties considering the reception of shifted version of codes which provides asynchronous reception capability. This is an attractive feature to CDMA system because it allows robustness to MPI.

## 4.2 Correlation matrix and Side-lobes graph

In the study of properties of sequences, they are two helpful tools that allow to analyze their behavior and usefulness to CDMA: Correlation matrix and Side-lobes correlation graph.

### 4.2.1 Correlation matrix.

The correlation values for each pair of codes ( $C_n^{(i)}$ ,  $C_m^{(i)}$ ) in a specific phase  $\phi$  are sorted in a matrix  $M_\phi(n, m)$  described in equation 4.4

$$M_\phi(n, m) = R_{C_n^{(i)} C_m^{(i)}}(\phi) \quad (4.4)$$

Note that the main diagonal of the matrix corresponds to the ACF values and the others to the CCF. For example, considering the WH set of four sequences as codes, i.e.  $c_1^{(1)} = (1, 1, 1, 1)$ ;

$c_2^{(1)} = (1, -1, 1, -1)$ ;  $c_3^{(1)} = (1, 1, -1, -1)$  and  $c_4^{(1)} = (1, -1, -1, 1)$ , the correlation matrices are defined as follow considering all the shifts ( $\phi = 0, 1, 2, 3$ )

$M_{\phi=0}$	$c_1^{(1)}$	$c_2^{(1)}$	$c_3^{(1)}$	$c_4^{(1)}$	$M_{\phi=1}$	$c_1^{(1)}$	$c_2^{(1)}$	$c_3^{(1)}$	$c_4^{(1)}$
$c_1^{(1)}$	<b>1</b>	0	0	0	$c_1^{(1)}$	<b>1</b>	0	0	0
$c_2^{(1)}$	0	<b>1</b>	0	0	$c_2^{(1)}$	0	<b>-1</b>	0	0
$c_3^{(1)}$	0	0	<b>1</b>	0	$c_3^{(1)}$	0	0	0	<b>1</b>
$c_4^{(1)}$	0	0	0	<b>1</b>	$c_4^{(1)}$	0	0	<b>-1</b>	0

$M_{\phi=2}$	$c_1^{(1)}$	$c_2^{(1)}$	$c_3^{(1)}$	$c_4^{(1)}$	$M_{\phi=3}$	$c_1^{(1)}$	$c_2^{(1)}$	$c_3^{(1)}$	$c_4^{(1)}$
$c_1^{(1)}$	<b>1</b>	0	0	0	$c_1^{(1)}$	<b>1</b>	0	0	0
$c_2^{(1)}$	0	<b>1</b>	0	0	$c_2^{(1)}$	0	<b>-1</b>	0	0
$c_3^{(1)}$	0	0	<b>-1</b>	0	$c_3^{(1)}$	0	0	0	<b>-1</b>
$c_4^{(1)}$	0	0	0	<b>-1</b>	$c_4^{(1)}$	0	0	<b>1</b>	0

### 4.2.2 Side-lobes correlation graph

It can be a two-dimensional (2-D) coordinated plane where the vertical line (y-axis) defines the correlation values and the horizontal (x-axis) refers to phase between received and reference codes, i.e. the number of shifts out of phase in asynchrony reception case or the in phase reception for synchrony case. For example, consider the codes  $c_2^{(1)}$  and  $c_3^{(1)}$  from the previous example, Figure 4.2a shows the autocorrelation values for all shifts of  $c_2^{(1)}$  where the received and the reference codes are the same. On the other hand, Figure 4.2b shows the cross-correlation values for all shifts between  $c_2^{(1)}$  and  $c_3^{(1)}$ . Note that each peak is called as *lobe*.

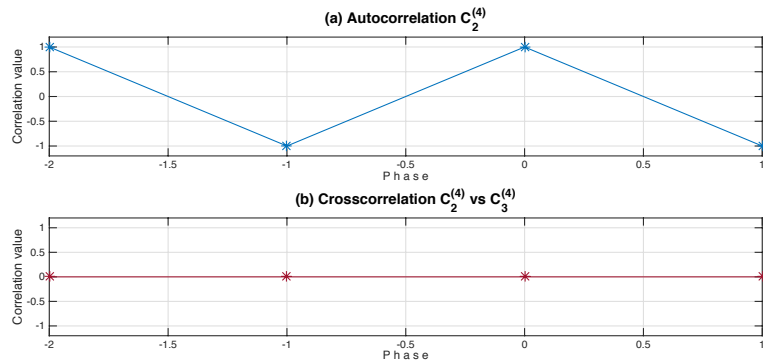


Figure 4.2: Side-lobes graph. (a) Autocorrelation and (b) Cross-correlation.

A three-dimensional (3-D) space is defined to describe a Side-lobes graph that represents the correlation values, there are two cases considered: (a) Side-lobe graph based on a *correlation matrix* that describes all the correlation values (z-axis) from the number of code  $C_{n1}^{(i)}$  (x-axis) versus the number of code  $C_{n2}^{(j)}$  (y-axis) where  $\{n1, n2\} \in n$ , and (b) Side-lobe graph based on *Shifts* that describes the correlation value (z-axis) from a set of codes  $C_n^{(i)}$  (y-axis) for every shift (x-axis).

### 4.3 Analysis of code sets

In this section, a collection of known sets of codes are analyzed through their properties. Figure 4.3 shows a classification of the sequences sets in four families according to their correlation properties: Traditional, complementary, interference-free window, and complex which are widely studied in [49]. Some advantages, disadvantages, and challenges for these families are shown in Table 4.1.

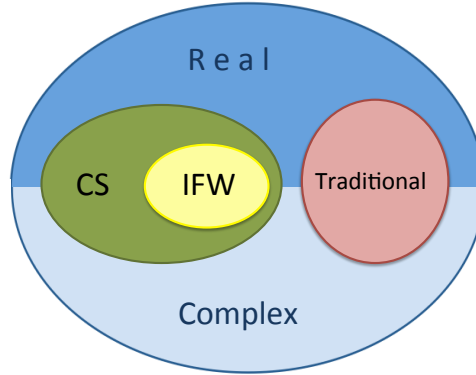


Figure 4.3: Sequences used as spreading codes in WCN: families of codes

This work tests the properties of binary code sets with elements  $\{\pm 1\}$  and length  $N = 8$  from Traditional, complementary and interference-free window families. It will test every desired property for traditional DS-CDMA and C2DMA models in order to inspect their behavior. Figure 4.4 shows the methodology used to study the properties of sequences and tools as Correlation matrix and Side-lobe graphs are introduced in order to examine their correlation values and compare them, so their properties can be studied.

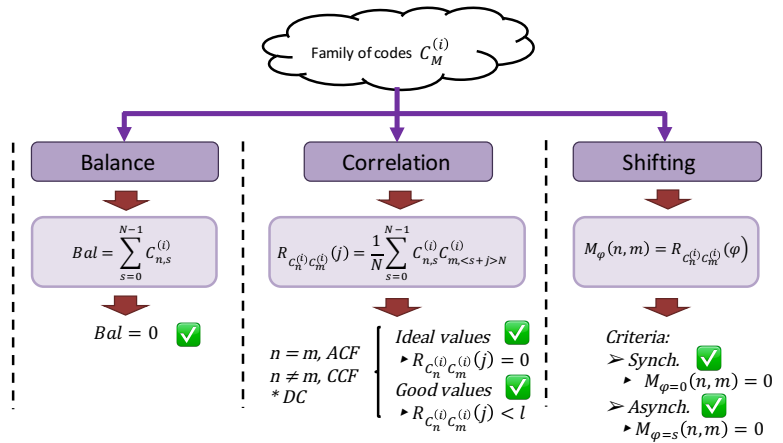


Figure 4.4: Verifying properties of code sets.

Table 4.1: Advantages - Disadvantages and Challenges of families of code sets

Family	Advantages of family	Disadvantages of family	Challenges
<b>Traditional</b> [19], [22], [20],[29], [3], [28], [41], [12], [23],[13].	<ul style="list-style-type: none"> <li>• They have been widely studied.</li> <li>• Large lengths of codes.</li> <li>• LFSR: Easy construction methods.</li> <li>• LFSR: Low complexity.</li> <li>• Random properties.</li> <li>• LFSR: Easy to implement.</li> <li>• NLFSR: Unpredictability. (Security)</li> </ul>	<ul style="list-style-type: none"> <li>• Unbalanced relationship of ACF and CCF.</li> <li>• Invariable data rate.</li> <li>• Irregular pattern of CCF.</li> <li>• Poor GCF out of phase.</li> </ul>	<ul style="list-style-type: none"> <li>• Balance: complexity vs. security.</li> <li>• Sequences that satisfy both GCF.</li> </ul>
<b>Complementary</b> [7], [46], [55].	<ul style="list-style-type: none"> <li>• Set of codes with good values of GCF.</li> <li>• Support asynchronous scenarios for WCS.</li> <li>• Easy construction methods.</li> <li>• Support MC systems.</li> </ul>	<ul style="list-style-type: none"> <li>• Small number of sequences.</li> <li>• Short length of sequences.</li> <li>• Invariable data rate.</li> </ul>	<ul style="list-style-type: none"> <li>• Increase the set size with loss of GCF properties.</li> </ul>
<b>Interference Free Window</b> [8], [10], [30].	<ul style="list-style-type: none"> <li>• Perfect GCF within a Window.</li> <li>• Variable data rate.</li> </ul>	<ul style="list-style-type: none"> <li>• Large sequences vs small window.</li> <li>• Elaborate construction methods</li> <li>• Reduce the spectral efficiency.</li> </ul>	<ul style="list-style-type: none"> <li>• Increase the window size without increasing the code length.</li> <li>• Increase the flock size.</li> </ul>
<b>Complex</b> [25], [43], [11].	<ul style="list-style-type: none"> <li>• Large sequences using unitary roots.</li> <li>• Large set size of codes with good GCF.</li> </ul>	<ul style="list-style-type: none"> <li>• Difficult to implement.</li> <li>• Complex analysis.</li> </ul>	<ul style="list-style-type: none"> <li>• Apply them in current WCS.</li> </ul>

### 4.3.1 Hadamard sequences sets

Walsh, Paley, and Golay Hadamard sets with length  $N = 8$  are considered. Walsh and Paley sequences composed only with  $+1$  are not considered, e.g.  $W_1^{(8)} = (+1, +1, +1, +1, +1, +1, +1, +1)$  is not considered for Walsh sequences with length  $N = 8$  which reduces the set size  $M = 8 - 1 = 7$ . Each code set is studied as follows

#### Walsh-Hadamard set

Considering Sylvester's construction method for  $N = 8$ , so  $W_n^{(7)}$  set is defined as follows. Note that the sequences composed only with  $+1$  it is not considered which reduce the set size to  $M - 1$ .

$$W_1^{(7)} = (+1, -1, +1, -1, +1, -1, +1, -1);$$

$$W_2^{(7)} = (+1, +1, -1, -1, +1, +1, -1, -1);$$

$$W_3^{(7)} = (+1, -1, -1, +1, +1, -1, -1, +1);$$

$$W_4^{(7)} = (+1, +1, +1, +1, -1, -1, -1, -1);$$

$$W_5^{(7)} = (+1, -1, +1, -1, -1, +1, -1, +1);$$

$$W_6^{(7)} = (+1, +1, -1, -1, -1, -1, +1, +1);$$

$$W_7^{(7)} = (+1, -1, -1, +1, -1, +1, +1, -1);$$

★ **BALANCE PROPERTY**

Tables 4.2 shows the *Bal* values for  $W_n^{(7)}$  verifying the Balance property for these sequences

Table 4.2: Balance: Walsh-Hadamard set

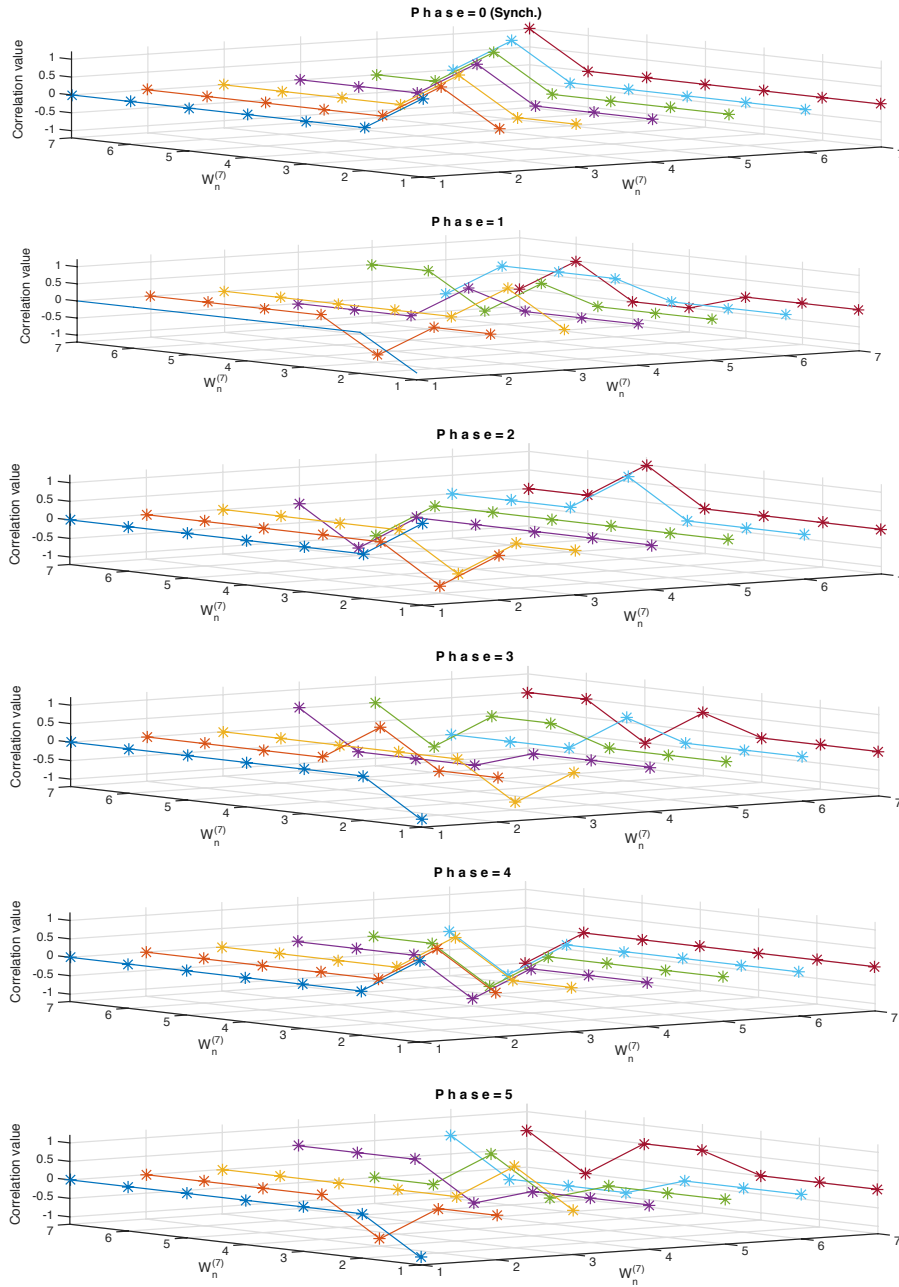
$W_n^{(7)}$	Bal	meet?	$W_n^{(7)}$	Bal	meet?	$W_n^{(7)}$	Bal	meet?	$W_n^{(7)}$	Bal	meet?
$W_1^{(7)}$	0	YES	$W_3^{(7)}$	0	YES	$W_5^{(7)}$	0	YES	$W_7^{(7)}$	0	YES
$W_2^{(7)}$	0	YES	$W_4^{(7)}$	0	YES	$W_6^{(7)}$	0	YES			

★ **CORRELATION PROPERTIES**

● **Autocorrelation and cross-correlation**

$\phi = 0$	$W_1^{(7)}$	$W_2^{(7)}$	$W_3^{(7)}$	$W_4^{(7)}$	$W_5^{(7)}$	$W_6^{(7)}$	$W_7^{(7)}$	$\phi = 1$	$W_1^{(7)}$	$W_2^{(7)}$	$W_3^{(7)}$	$W_4^{(7)}$	$W_5^{(7)}$	$W_6^{(7)}$	$W_7^{(7)}$
$W_1^{(7)}$	+1.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_1^{(7)}$	-1.0	0.0	0.0	0.0	0.0	0.0	0.0
$W_2^{(7)}$	0.0	+1.0	0.0	0.0	0.0	0.0	0.0	$W_2^{(7)}$	0.0	0.0	+1.0	0.0	0.0	0.0	0.0
$W_3^{(7)}$	0.0	0.0	+1.0	0.0	0.0	0.0	0.0	$W_3^{(7)}$	0.0	-1.0	0.0	0.0	0.0	0.0	0.0
$W_4^{(7)}$	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$W_4^{(7)}$	0.0	0.0	0.0	+0.5	+0.5	+0.5	-0.5
$W_5^{(7)}$	0.0	0.0	0.0	0.0	+1.0	0.0	0.0	$W_5^{(7)}$	0.0	0.0	0.0	-0.5	-0.5	+0.5	-0.5
$W_6^{(7)}$	0.0	0.0	0.0	0.0	0.0	+1.0	0.0	$W_6^{(7)}$	0.0	0.0	0.0	-0.5	+0.5	+0.5	+0.5
$W_7^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	+1.0	$W_7^{(7)}$	0.0	0.0	0.0	-0.5	+0.5	-0.5	-0.5
$\phi = 2$	$W_1^{(7)}$	$W_2^{(7)}$	$W_3^{(7)}$	$W_4^{(7)}$	$W_5^{(7)}$	$W_6^{(7)}$	$W_7^{(7)}$	$\phi = 3$	$W_1^{(7)}$	$W_2^{(7)}$	$W_3^{(7)}$	$W_4^{(7)}$	$W_5^{(7)}$	$W_6^{(7)}$	$W_7^{(7)}$
$W_1^{(7)}$	+1.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_1^{(7)}$	-1.0	0.0	0.0	0.0	0.0	0.0	0.0
$W_2^{(7)}$	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	$W_2^{(7)}$	0.0	0.0	-1.0	0.0	0.0	0.0	0.0
$W_3^{(7)}$	0.0	0.0	-1.0	0.0	0.0	0.0	0.0	$W_3^{(7)}$	0.0	+1.0	0.0	0.0	0.0	0.0	0.0
$W_4^{(7)}$	0.0	0.0	0.0	0.0	0.0	+1.0	0.0	$W_4^{(7)}$	0.0	0.0	0.0	-0.5	+0.5	+0.5	+0.5
$W_5^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	+1.0	$W_5^{(7)}$	0.0	0.0	0.0	-0.5	+0.5	-0.5	-0.5
$W_6^{(7)}$	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	$W_6^{(7)}$	0.0	0.0	0.0	-0.5	-0.5	-0.5	+0.5
$W_7^{(7)}$	0.0	0.0	0.0	0.0	-1.0	0.0	0.0	$W_7^{(7)}$	0.0	0.0	0.0	+0.5	+0.5	-0.5	+0.5
$\phi = 4$	$W_1^{(7)}$	$W_2^{(7)}$	$W_3^{(7)}$	$W_4^{(7)}$	$W_5^{(7)}$	$W_6^{(7)}$	$W_7^{(7)}$	$\phi = 5$	$W_1^{(7)}$	$W_2^{(7)}$	$W_3^{(7)}$	$W_4^{(7)}$	$W_5^{(7)}$	$W_6^{(7)}$	$W_7^{(7)}$
$W_1^{(7)}$	+1.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_1^{(7)}$	-1.0	0.0	0.0	0.0	0.0	0.0	0.0
$W_2^{(7)}$	0.0	+1.0	0.0	0.0	0.0	0.0	0.0	$W_2^{(7)}$	0.0	0.0	-1.0	0.0	0.0	0.0	0.0
$W_3^{(7)}$	0.0	0.0	+1.0	0.0	0.0	0.0	0.0	$W_3^{(7)}$	0.0	-1.0	0.0	0.0	0.0	0.0	0.0
$W_4^{(7)}$	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	$W_4^{(7)}$	0.0	0.0	0.0	-0.5	-0.5	-0.5	+0.5
$W_5^{(7)}$	0.0	0.0	0.0	0.0	-1.0	0.0	0.0	$W_5^{(7)}$	0.0	0.0	0.0	+0.5	+0.5	-0.5	+0.5
$W_6^{(7)}$	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	$W_6^{(7)}$	0.0	0.0	0.0	+0.5	-0.5	-0.5	-0.5
$W_7^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	$W_7^{(7)}$	0.0	0.0	0.0	+0.5	-0.5	+0.5	+0.5
$\phi = 6$	$W_1^{(7)}$	$W_2^{(7)}$	$W_3^{(7)}$	$W_4^{(7)}$	$W_5^{(7)}$	$W_6^{(7)}$	$W_7^{(7)}$	$\phi = 7$	$W_1^{(7)}$	$W_2^{(7)}$	$W_3^{(7)}$	$W_4^{(7)}$	$W_5^{(7)}$	$W_6^{(7)}$	$W_7^{(7)}$
$W_1^{(7)}$	+1.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_1^{(7)}$	-1.0	0.0	0.0	0.0	0.0	0.0	0.0
$W_2^{(7)}$	0.0	-1.0	0.0	0.0	0.0	0.0	0.0	$W_2^{(7)}$	0.0	0.0	-1.0	0.0	0.0	0.0	0.0
$W_3^{(7)}$	0.0	0.0	-1.0	0.0	0.0	0.0	0.0	$W_3^{(7)}$	0.0	+1.0	0.0	0.0	0.0	0.0	0.0
$W_4^{(7)}$	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	$W_4^{(7)}$	0.0	0.0	0.0	+0.5	-0.5	-0.5	-0.5
$W_5^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	$W_5^{(7)}$	0.0	0.0	0.0	+0.5	-0.5	+0.5	+0.5
$W_6^{(7)}$	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$W_6^{(7)}$	0.0	0.0	0.0	+0.5	+0.5	+0.5	-0.5
$W_7^{(7)}$	0.0	0.0	0.0	0.0	+1.0	0.0	0.0	$W_7^{(7)}$	0.0	0.0	0.0	-0.5	-0.5	+0.5	-0.5

Figure 4.5 shows the 3-D Side-lobe graphs for all phases which represent the correlation matrices.





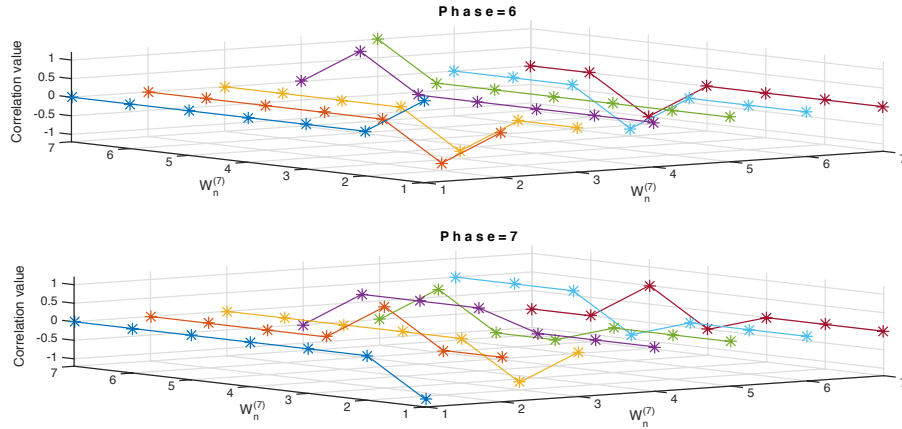


Figure 4.5: Correlation matrices for  $W_n^{(7)}$ : Phases.

• **Double-correlation**

C. Barrera provides a subset  $W_1^{DC}$  with four sequences that accomplish the Double-Correlation property from  $W_n^7$ , and the rest are allocated in other subset  $W_2^{DC}$ , i.e.

$W_1^{DC} = \{W_1^{(7)}, W_2^{(7)}, W_3^{(7)}, W_4^{(7)}\}$  and  $W_2^{DC} = \{W_5^{(7)}, W_6^{(7)}, W_7^{(7)}\}$  respectively, see [4].

However, this work explores all the possible subsets that accomplish this property from  $W_n^{(7)}$  sets. Thus, DC sets are given as  $W_{\#}^{DC} = \{W_i^{(7)}, W_j^{(7)}, W_k^{(7)}, W_l^{(7)}\}$ , where  $i, j, k,$  and  $l$  are the sequence indexes from  $W_n^{(7)}$ . Note that all the subsets have the same size, i.e.  $M_{DC} = 4$ .

#	$i$	$j$	$k$	$l$	#	$i$	$j$	$k$	$l$	#	$i$	$j$	$k$	$l$	#	$i$	$j$	$k$	$l$
1	1	2	3	4	8	1	2	6	7	15	1	4	6	7	22	2	4	5	7
2	1	2	3	5	9	1	3	4	5	16	1	5	6	7	23	2	4	6	7
3	1	2	3	6	10	1	3	4	7	17	2	3	4	6	24	2	5	6	7
4	1	2	3	7	11	1	3	5	6	18	2	3	4	7	25	3	4	5	6
5	1	2	4	5	12	1	3	6	7	19	2	3	5	6	26	3	4	5	7
6	1	2	4	6	13	1	4	5	6	20	2	3	5	7	27	3	4	6	7
7	1	2	5	7	14	1	4	5	7	21	2	4	5	6	28	3	5	6	7

★ **SHIFTING PROPERTY**

It considers the correlation values for all shifts of the code pairs that represent the reception phase of signals. In synchrony reception systems it is desirable to reduce the out of phase signals that represent interfering copies while asynchrony reception systems only reduce some strategic phases. This property is analyzed through correlation matrices for all phases ( $\phi = 0 \dots N - 1$ ). This way, the analysis of shifting property is given as follows. Note that  $M_{\phi}(i, j)$  provides ACF values when  $i = j$ , otherwise it provides CCF.

$i$	$j$	$\phi$								$i$	$j$	$\phi$							
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$W_1^{(7)}$	$W_1^{(7)}$	+1.0	-1.0	+1.0	-1.0	+1.0	-1.0	+1.0	-1.0	$W_3^{(7)}$	$W_4^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$W_1^{(7)}$	$W_2^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_3^{(7)}$	$W_5^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$W_1^{(7)}$	$W_3^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_3^{(7)}$	$W_6^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$W_1^{(7)}$	$W_4^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_3^{(7)}$	$W_7^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$W_1^{(7)}$	$W_5^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_4^{(7)}$	$W_4^{(7)}$	+1.0	+0.5	0.0	-0.5	-1.0	-0.5	0.0	+0.5
$W_1^{(7)}$	$W_6^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_4^{(7)}$	$W_5^{(7)}$	0.0	+0.5	0.0	+0.5	0.0	-0.5	0.0	-0.5
$W_1^{(7)}$	$W_7^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_4^{(7)}$	$W_6^{(7)}$	0.0	+0.5	+1.0	+0.5	0.0	-0.5	-1.0	-0.5
$W_2^{(7)}$	$W_2^{(7)}$	+1.0	0.0	-1.0	0.0	+1.0	0.0	-1.0	0.0	$W_4^{(7)}$	$W_7^{(7)}$	0.0	-0.5	0.0	+0.5	0.0	+0.5	0.0	-0.5
$W_2^{(7)}$	$W_3^{(7)}$	0.0	+1.0	0.0	-1.0	0.0	+1.0	0.0	-1.0	$W_5^{(7)}$	$W_5^{(7)}$	+1.0	-0.5	0.0	+0.5	-1.0	+0.5	0.0	-0.5
$W_2^{(7)}$	$W_4^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_5^{(7)}$	$W_6^{(7)}$	0.0	+0.5	0.0	-0.5	0.0	-0.5	0.0	+0.5
$W_2^{(7)}$	$W_5^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_5^{(7)}$	$W_7^{(7)}$	0.0	-0.5	+1.0	-0.5	0.0	+0.5	-1.0	+0.5
$W_2^{(7)}$	$W_6^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_6^{(7)}$	$W_7^{(7)}$	+1.0	+0.5	0.0	-0.5	-1.0	-0.5	0.0	+0.5
$W_2^{(7)}$	$W_7^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$W_6^{(7)}$	$W_6^{(7)}$	0.0	+0.5	0.0	+0.5	0.0	-0.5	0.0	-0.5
$W_3^{(7)}$	$W_3^{(7)}$	+1.0	0.0	-1.0	0.0	+1.0	0.0	-1.0	0.0	$W_7^{(7)}$	$W_7^{(7)}$	+1.0	-0.5	0.0	+0.5	-1.0	+0.5	0.0	-0.5

Thus, ACF for all the codes are plotted using 2-D Side-lobe graph.

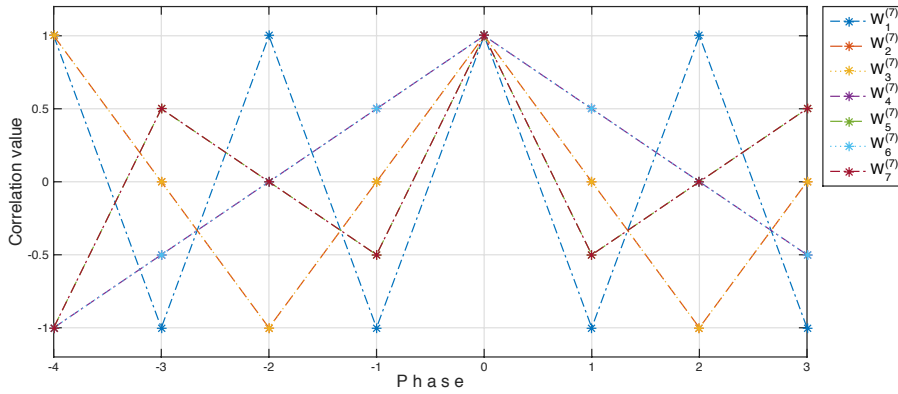


Figure 4.6: 2-D Side-lobe graph: ACF for  $W_n^{(7)}$  set.

### Paley-Hadamard set

It is obtained through software from The MathWorks Inc. for  $N = 8$ . Similarly to Walsh set, Paley set discards the sequences composed only with +1 element reducing, again, the set sizes to  $M - 1$ , i.e.  $P_n^{(7)}$  is defined as  $M = 7$  and  $N = 8$ .

$$\begin{aligned}
 P_1^{(7)} &= (+1, -1, +1, +1, -1, +1, -1, -1); \\
 P_2^{(7)} &= (+1, -1, -1, +1, +1, -1, +1, -1); \\
 P_3^{(7)} &= (+1, -1, -1, -1, +1, +1, -1, +1); \\
 P_4^{(7)} &= (+1, +1, -1, -1, -1, +1, +1, -1); \\
 P_5^{(7)} &= (+1, -1, +1, -1, -1, -1, +1, +1); \\
 P_6^{(7)} &= (+1, +1, -1, +1, -1, -1, -1, +1); \\
 P_7^{(7)} &= (+1, +1, +1, -1, +1, -1, -1, -1);
 \end{aligned}$$

Each property is tested following the same methods than previous family, so their results are presented as follows.

★ **BALANCE PROPERTY**

Bal values are presented in Table 4.3 verifying this property.

Table 4.3: Balance: Paley-Hadamard set

$P_n^{(7)}$	Bal	meet?	$P_n^{(7)}$	Bal	meet?	$P_n^{(7)}$	Bal	meet?	$P_n^{(7)}$	Bal	meet?
$P_1^{(7)}$	0	YES	$P_3^{(7)}$	0	YES	$P_5^{(7)}$	0	YES	$P_7^{(7)}$	0	YES
$P_2^{(7)}$	0	YES	$P_4^{(7)}$	0	YES	$P_6^{(7)}$	0	YES			

★ **CORRELATION PROPERTIES**

● **Autocorrelation and cross-correlation**

$\phi = 0$	$P_1^{(7)}$	$P_2^{(7)}$	$P_3^{(7)}$	$P_4^{(7)}$	$P_5^{(7)}$	$P_6^{(7)}$	$P_7^{(7)}$	$\phi = 1$	$P_1^{(7)}$	$P_2^{(7)}$	$P_3^{(7)}$	$P_4^{(7)}$	$P_5^{(7)}$	$P_6^{(7)}$	$P_7^{(7)}$
$P_1^{(7)}$	+1.0	0.0	0.0	0.0	0.0	0.0	0.0	$P_1^{(7)}$	-0.5	-0.5	0.0	-0.5	0.0	0.0	+0.5
$P_2^{(7)}$	0.0	+1.0	0.0	0.0	0.0	0.0	0.0	$P_2^{(7)}$	+0.5	-0.5	0.0	-0.5	0.0	0.0	-0.5
$P_3^{(7)}$	0.0	0.0	+1.0	0.0	0.0	0.0	0.0	$P_3^{(7)}$	-0.5	+0.5	0.0	-0.5	0.0	0.0	-0.5
$P_4^{(7)}$	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$P_4^{(7)}$	0.0	0.0	+1.0	0.0	0.0	0.0	0.0
$P_5^{(7)}$	0.0	0.0	0.0	0.0	+1.0	0.0	0.0	$P_5^{(7)}$	-0.5	-0.5	0.0	+0.5	0.0	0.0	-0.5
$P_6^{(7)}$	0.0	0.0	0.0	0.0	0.0	+1.0	0.0	$P_6^{(7)}$	0.0	0.0	0.0	0.0	+1.0	0.0	0.0
$P_7^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	+1.0	$P_7^{(7)}$	0.0	0.0	0.0	0.0	0.0	+1.0	0.0
$\phi = 2$	$P_1^{(7)}$	$P_2^{(7)}$	$P_3^{(7)}$	$P_4^{(7)}$	$P_5^{(7)}$	$P_6^{(7)}$	$P_7^{(7)}$	$\phi = 3$	$P_1^{(7)}$	$P_2^{(7)}$	$P_3^{(7)}$	$P_4^{(7)}$	$P_5^{(7)}$	$P_6^{(7)}$	$P_7^{(7)}$
$P_1^{(7)}$	0.0	+0.5	-0.5	+0.5	0.0	+0.5	0.0	$P_1^{(7)}$	+0.5	-0.5	+0.5	0.0	+0.5	0.0	0.0
$P_2^{(7)}$	-0.5	0.0	-0.5	0.0	0.0	-0.5	+0.5	$P_2^{(7)}$	+0.5	0.0	0.0	+0.5	-0.5	+0.5	0.0
$P_3^{(7)}$	+0.5	0.0	-0.5	0.0	0.0	-0.5	-0.5	$P_3^{(7)}$	0.0	-0.5	0.0	0.0	-0.5	-0.5	+0.5
$P_4^{(7)}$	-0.5	+0.5	0.0	-0.5	0.0	0.0	-0.5	$P_4^{(7)}$	+0.5	0.0	-0.5	0.0	0.0	-0.5	-0.5
$P_5^{(7)}$	0.0	+0.5	+0.5	+0.5	0.0	-0.5	0.0	$P_5^{(7)}$	0.0	0.0	+0.5	-0.5	-0.5	0.0	-0.5
$P_6^{(7)}$	-0.5	-0.5	0.0	+0.5	0.0	0.0	-0.5	$P_6^{(7)}$	0.0	+0.5	+0.5	+0.5	0.0	-0.5	0.0
$P_7^{(7)}$	0.0	0.0	0.0	0.0	+1.0	0.0	0.0	$P_7^{(7)}$	-0.5	-0.5	0.0	+0.5	0.0	0.0	-0.5
$\phi = 4$	$P_1^{(7)}$	$P_2^{(7)}$	$P_3^{(7)}$	$P_4^{(7)}$	$P_5^{(7)}$	$P_6^{(7)}$	$P_7^{(7)}$	$\phi = 5$	$P_1^{(7)}$	$P_2^{(7)}$	$P_3^{(7)}$	$P_4^{(7)}$	$P_5^{(7)}$	$P_6^{(7)}$	$P_7^{(7)}$
$P_1^{(7)}$	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	$P_1^{(7)}$	+0.5	+0.5	0.0	+0.5	0.0	0.0	-0.5
$P_2^{(7)}$	0.0	0.0	+0.5	-0.5	+0.5	0.0	+0.5	$P_2^{(7)}$	-0.5	0.0	-0.5	0.0	0.0	+0.5	-0.5
$P_3^{(7)}$	0.0	+0.5	0.0	0.0	-0.5	+0.5	+0.5	$P_3^{(7)}$	+0.5	0.0	0.0	-0.5	+0.5	+0.5	0.0
$P_4^{(7)}$	0.0	-0.5	0.0	0.0	-0.5	-0.5	+0.5	$P_4^{(7)}$	0.0	+0.5	0.0	0.0	-0.5	+0.5	+0.5
$P_5^{(7)}$	0.0	+0.5	-0.5	-0.5	0.0	-0.5	0.0	$P_5^{(7)}$	+0.5	-0.5	-0.5	0.0	-0.5	0.0	0.0
$P_6^{(7)}$	0.0	0.0	+0.5	-0.5	-0.5	0.0	-0.5	$P_6^{(7)}$	0.0	+0.5	-0.5	-0.5	0.0	-0.5	0.0
$P_7^{(7)}$	0.0	+0.5	+0.5	+0.5	0.0	-0.5	0.0	$P_7^{(7)}$	0.0	0.0	+0.5	-0.5	-0.5	0.0	-0.5
$\phi = 6$	$P_1^{(7)}$	$P_2^{(7)}$	$P_3^{(7)}$	$P_4^{(7)}$	$P_5^{(7)}$	$P_6^{(7)}$	$P_7^{(7)}$	$\phi = 7$	$P_1^{(7)}$	$P_2^{(7)}$	$P_3^{(7)}$	$P_4^{(7)}$	$P_5^{(7)}$	$P_6^{(7)}$	$P_7^{(7)}$
$P_1^{(7)}$	0.0	-0.5	+0.5	-0.5	0.0	-0.5	0.0	$P_1^{(7)}$	-0.5	+0.5	-0.5	0.0	-0.5	0.0	0.0
$P_2^{(7)}$	+0.5	0.0	0.0	+0.5	+0.5	-0.5	0.0	$P_2^{(7)}$	-0.5	-0.5	+0.5	0.0	-0.5	0.0	0.0
$P_3^{(7)}$	-0.5	-0.5	-0.5	0.0	+0.5	0.0	0.0	$P_3^{(7)}$	0.0	0.0	0.0	+1.0	0.0	0.0	0.0
$P_4^{(7)}$	+0.5	0.0	0.0	-0.5	+0.5	+0.5	0.0	$P_4^{(7)}$	-0.5	-0.5	-0.5	0.0	+0.5	0.0	0.0
$P_5^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	+1.0	$P_5^{(7)}$	0.0	0.0	0.0	0.0	0.0	+1.0	0.0
$P_6^{(7)}$	+0.5	-0.5	-0.5	0.0	-0.5	0.0	0.0	$P_6^{(7)}$	0.0	0.0	0.0	0.0	0.0	0.0	+1.0
$P_7^{(7)}$	0.0	+0.5	-0.5	-0.5	0.0	-0.5	0.0	$P_7^{(7)}$	+0.5	-0.5	-0.5	0.0	-0.5	0.0	0.0

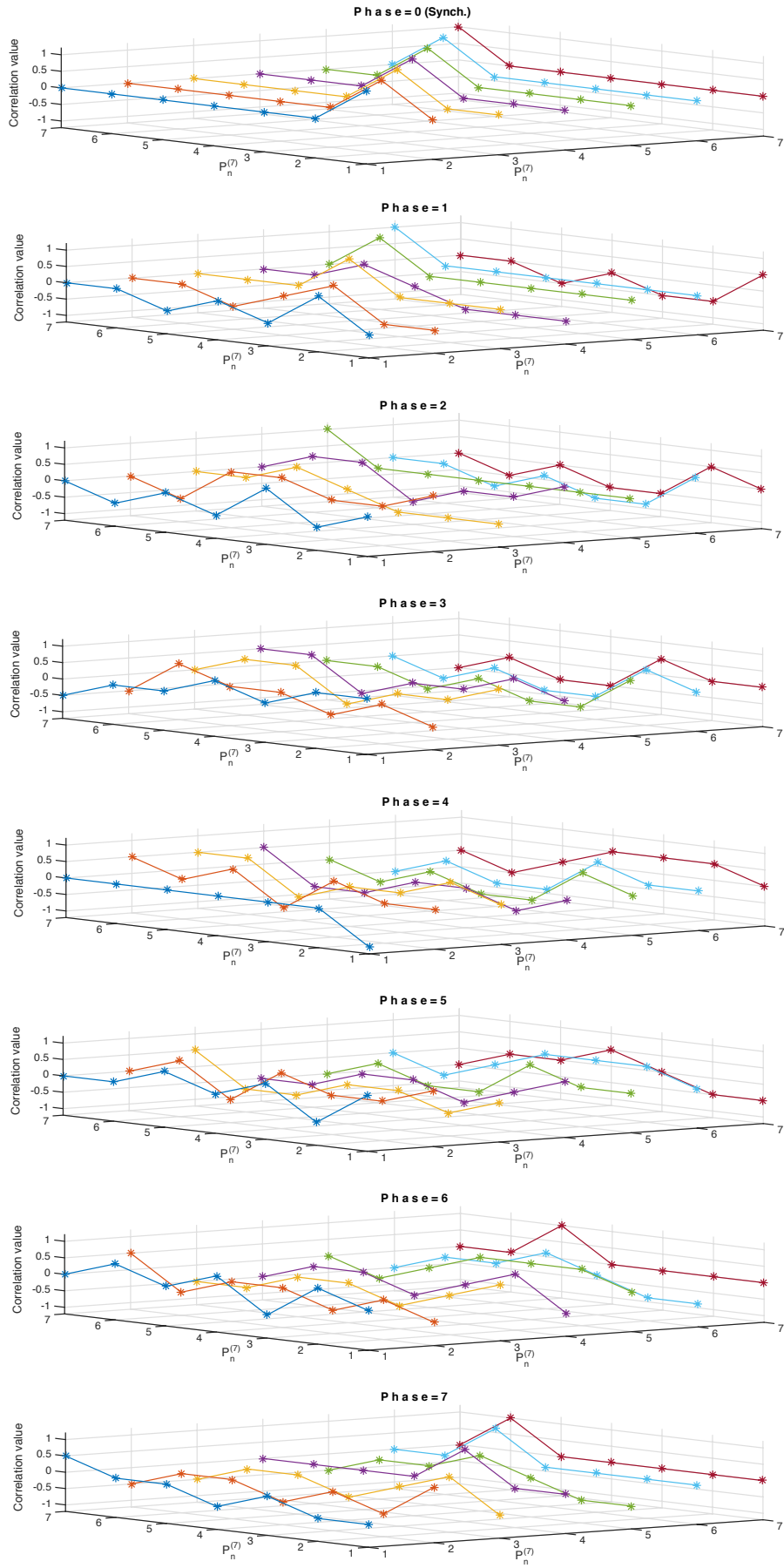


Figure 4.7: Correlation matrices for  $P_n^{(7)}$ : Phases.

• **Double-correlation**

It is verified considering the previous methodology for  $P_n^{(7)}$  set. Again, all the possible subsets are explored and the maximum set size is 4. This way, the indexes for each subset  $P_{\#}^{DC}$  are given as  $P_{\#}^{DC} = \{P_i^{(7)}, P_j^{(7)}, P_k^{(7)}, P_l^{(7)}\}$ .

#	i	j	k	l	#	i	j	k	l	#	i	j	k	l	#	i	j	k	l
1	1	2	3	4	8	1	2	6	7	15	1	4	6	7	22	2	4	5	6
2	1	2	3	6	9	1	3	4	5	16	1	5	6	7	23	2	4	5	7
3	1	2	3	7	10	1	3	4	6	17	2	3	4	5	24	2	4	6	7
4	1	2	4	5	11	1	3	4	7	18	2	3	4	7	25	3	4	5	6
5	1	2	4	6	12	1	3	5	6	19	2	3	5	6	26	3	4	6	7
6	1	2	5	6	13	1	3	5	7	20	2	3	5	7	27	3	5	6	7
7	1	2	5	7	14	1	4	5	7	21	2	3	6	7	28	4	5	6	7

★ **SHIFTING PROPERTY**

The following list show the  $M_{\phi}(i, j)$  for all phases ( $\phi = 0 \dots N - 1$ ) and Figure 4.8 shows the ACF for all codes.

i	j	$\phi$							i	j	$\phi$								
		0	1	2	3	4	5	6			7	0	1	2	3	4	5	6	7
$P_1^{(7)}$	$P_1^{(7)}$	+1.0	-0.5	0.0	+0.5	-1.0	+0.5	0.0	-0.5	$P_3^{(7)}$	$P_4^{(7)}$	0.0	-0.5	0.0	0.0	0.0	-0.5	0.0	+1.0
$P_1^{(7)}$	$P_2^{(7)}$	0.0	-0.5	+0.5	-0.5	0.0	+0.5	-0.5	+0.5	$P_3^{(7)}$	$P_5^{(7)}$	0.0	0.0	0.0	-0.5	-0.5	+0.5	+0.5	0.0
$P_1^{(7)}$	$P_3^{(7)}$	0.0	0.0	-0.5	+0.5	0.0	0.0	+0.5	-0.5	$P_3^{(7)}$	$P_6^{(7)}$	0.0	0.0	-0.5	-0.5	+0.5	+0.5	0.0	0.0
$P_1^{(7)}$	$P_4^{(7)}$	0.0	-0.5	+0.5	0.0	0.0	+0.5	-0.5	0.0	$P_3^{(7)}$	$P_7^{(7)}$	0.0	-0.5	-0.5	+0.5	+0.5	0.0	0.0	0.0
$P_1^{(7)}$	$P_5^{(7)}$	0.0	0.0	0.0	+0.5	0.0	0.0	0.0	-0.5	$P_4^{(7)}$	$P_4^{(7)}$	+1.0	0.0	-0.5	0.0	0.0	0.0	-0.5	0.0
$P_1^{(7)}$	$P_6^{(7)}$	0.0	0.0	+0.5	0.0	0.0	0.0	-0.5	0.0	$P_4^{(7)}$	$P_5^{(7)}$	0.0	0.0	0.0	0.0	-0.5	-0.5	+0.5	+0.5
$P_1^{(7)}$	$P_7^{(7)}$	0.0	+0.5	0.0	0.0	0.0	-0.5	0.0	0.0	$P_4^{(7)}$	$P_6^{(7)}$	0.0	0.0	0.0	-0.5	-0.5	+0.5	+0.5	0.0
$P_2^{(7)}$	$P_2^{(7)}$	+1.0	-0.5	0.0	0.0	0.0	0.0	0.0	-0.5	$P_4^{(7)}$	$P_7^{(7)}$	0.0	0.0	-0.5	-0.5	+0.5	+0.5	0.0	0.0
$P_2^{(7)}$	$P_3^{(7)}$	0.0	0.0	-0.5	0.0	+0.5	-0.5	0.0	+0.5	$P_5^{(7)}$	$P_5^{(7)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$P_2^{(7)}$	$P_4^{(7)}$	0.0	-0.5	0.0	+0.5	-0.5	0.0	+0.5	0.0	$P_5^{(7)}$	$P_6^{(7)}$	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0	+1.0
$P_2^{(7)}$	$P_5^{(7)}$	0.0	0.0	0.0	-0.5	+0.5	0.0	+0.5	-0.5	$P_5^{(7)}$	$P_7^{(7)}$	0.0	-0.5	0.0	-0.5	0.0	0.0	+1.0	0.0
$P_2^{(7)}$	$P_6^{(7)}$	0.0	0.0	-0.5	+0.5	0.0	+0.5	-0.5	0.0	$P_6^{(7)}$	$P_6^{(7)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$P_2^{(7)}$	$P_7^{(7)}$	0.0	-0.5	+0.5	0.0	+0.5	-0.5	0.0	0.0	$P_6^{(7)}$	$P_7^{(7)}$	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0	+1.0
$P_3^{(7)}$	$P_3^{(7)}$	+1.0	0.0	-0.5	0.0	0.0	0.0	-0.5	0.0	$P_7^{(7)}$	$P_7^{(7)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0

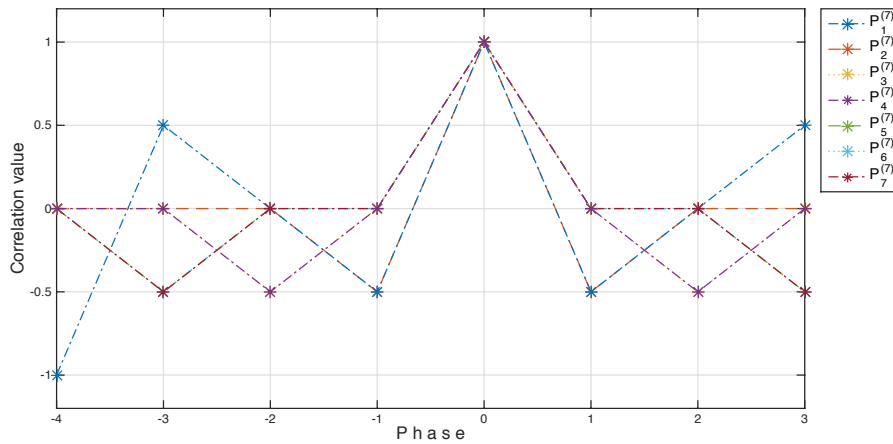


Figure 4.8: 2-D Side-lobe graph: ACF for  $P_n^{(7)}$  set.

**Golay-Hadamard sets**

This set is defined as  $G_n^{(8)}$  with  $M = 8$  elements and length  $N = 8$ .

$$\begin{aligned}
 G_1^{(8)} &= (+1, +1, +1, -1, +1, +1, -1, +1); \\
 G_2^{(8)} &= (+1, -1, +1, +1, +1, -1, -1, -1); \\
 G_3^{(8)} &= (+1, +1, -1, +1, +1, +1, +1, -1); \\
 G_4^{(8)} &= (+1, -1, -1, -1, +1, -1, +1, +1); \\
 G_5^{(8)} &= (+1, +1, +1, -1, -1, -1, +1, -1); \\
 G_6^{(8)} &= (+1, -1, +1, +1, -1, +1, +1, +1); \\
 G_7^{(8)} &= (+1, +1, -1, +1, -1, -1, -1, +1); \\
 G_8^{(8)} &= (+1, -1, -1, -1, -1, +1, -1, -1);
 \end{aligned}$$

**\*BALANCE PROPERTY**

This property is verifying in Table 4.4 which shows  $Bal$  values for  $G^{(8)_n}$  set.

Table 4.4: Balance: Golay-Hadamard set

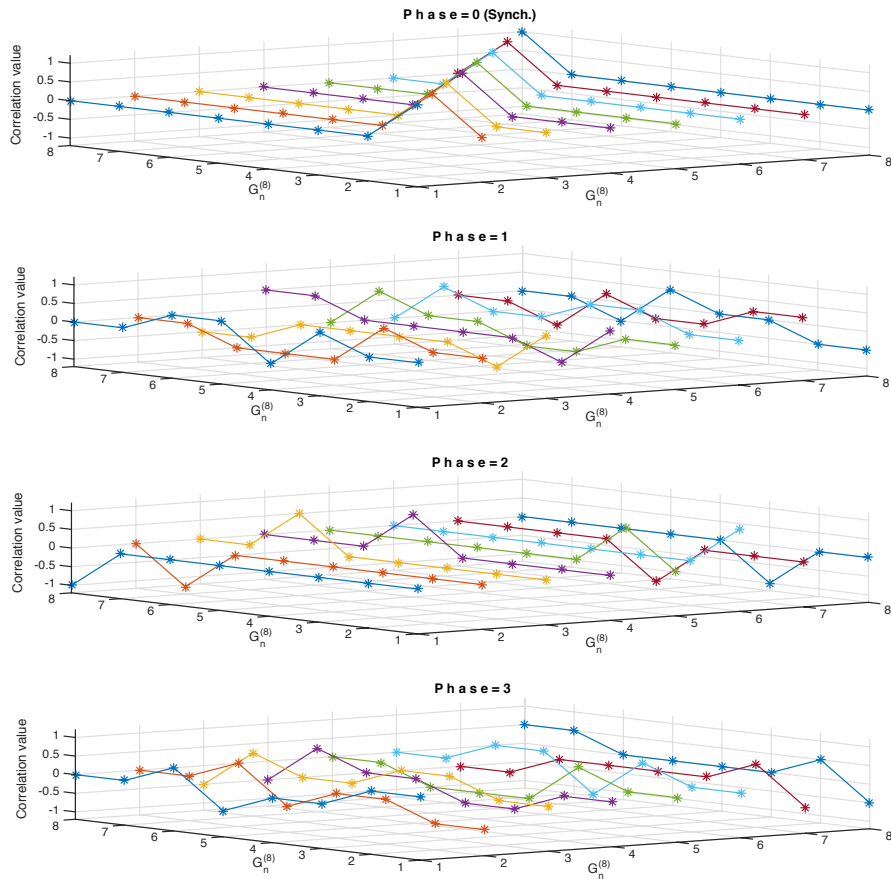
$G_n^{(7)}$	Bal	meet?	$G_n^{(7)}$	Bal	meet?	$G_n^{(7)}$	Bal	meet?	$G_n^{(7)}$	Bal	meet?
$G_1^{(8)}$	4	NO	$G_3^{(8)}$	4	NO	$G_5^{(8)}$	0	YES	$G_7^{(8)}$	0	YES
$G_2^{(8)}$	0	YES	$G_4^{(8)}$	0	YES	$G_6^{(8)}$	4	NO	$G_8^{(8)}$	-4	NO

**\*CORRELATION PROPERTIES**

• **Autocorrelation and cross-correlation**

$\phi = 0$	$G_1^{(8)}$	$G_2^{(8)}$	$G_3^{(8)}$	$G_4^{(8)}$	$G_5^{(8)}$	$G_6^{(8)}$	$G_7^{(8)}$	$G_8^{(8)}$	$\phi = 1$	$G_1^{(8)}$	$G_2^{(8)}$	$G_3^{(8)}$	$G_4^{(8)}$	$G_5^{(8)}$	$G_6^{(8)}$	$G_7^{(8)}$	$G_8^{(8)}$
$G_1^{(8)}$	<b>1.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$G_1^{(8)}$	<b>0.0</b>	0.0	+0.5	+0.5	0.0	0.0	+0.5	-0.5
$G_2^{(8)}$	0.0	<b>1.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	$G_2^{(8)}$	0.0	<b>0.0</b>	-0.5	-0.5	0.0	0.0	+0.5	-0.5
$G_3^{(8)}$	0.0	0.0	<b>1.0</b>	0.0	0.0	0.0	0.0	0.0	$G_3^{(8)}$	+0.5	+0.5	<b>0.0</b>	0.0	-0.5	+0.5	0.0	0.0
$G_4^{(8)}$	0.0	0.0	0.0	<b>1.0</b>	0.0	0.0	0.0	0.0	$G_4^{(8)}$	-0.5	-0.5	0.0	<b>0.0</b>	-0.5	+0.5	0.0	0.0
$G_5^{(8)}$	0.0	0.0	0.0	0.0	<b>1.0</b>	0.0	0.0	0.0	$G_5^{(8)}$	+0.5	-0.5	0.0	0.0	<b>0.0</b>	0.0	+0.5	+0.5
$G_6^{(8)}$	0.0	0.0	0.0	0.0	0.0	<b>1.0</b>	0.0	0.0	$G_6^{(8)}$	+0.5	-0.5	0.0	0.0	0.0	<b>1.0</b>	-0.5	-0.5
$G_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	<b>1.0</b>	0.0	$G_7^{(8)}$	0.0	0.0	-0.5	+0.5	+0.5	+0.5	<b>0.0</b>	0.0
$G_8^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>1.0</b>	$G_8^{(8)}$	0.0	0.0	-0.5	+0.5	-0.5	-0.5	0.0	<b>0.0</b>
$\phi = 2$	$G_1^{(8)}$	$G_2^{(8)}$	$G_3^{(8)}$	$G_4^{(8)}$	$G_5^{(8)}$	$G_6^{(8)}$	$G_7^{(8)}$	$G_8^{(8)}$	$\phi = 3$	$G_1^{(8)}$	$G_2^{(8)}$	$G_3^{(8)}$	$G_4^{(8)}$	$G_5^{(8)}$	$G_6^{(8)}$	$G_7^{(8)}$	$G_8^{(8)}$
$G_1^{(8)}$	<b>0.0</b>	0.0	0.0	0.0	0.0	+1.0	0.0	0.0	$G_1^{(8)}$	<b>+0.5</b>	-0.5	0.0	0.0	0.0	0.0	-0.5	-0.5
$G_2^{(8)}$	0.0	<b>0.0</b>	0.0	0.0	+1.0	0.0	0.0	0.0	$G_2^{(8)}$	+0.5	<b>-0.5</b>	0.0	0.0	0.0	0.0	+0.5	+0.5
$G_3^{(8)}$	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	-1.0	$G_3^{(8)}$	0.0	0.0	<b>+0.5</b>	-0.5	+0.5	+0.5	0.0	0.0
$G_4^{(8)}$	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	-1.0	0.0	$G_4^{(8)}$	0.0	0.0	+0.5	<b>-0.5</b>	-0.5	-0.5	0.0	0.0
$G_5^{(8)}$	0.0	0.0	0.0	+1.0	<b>0.0</b>	0.0	0.0	0.0	$G_5^{(8)}$	-0.5	-0.5	0.0	0.0	<b>-0.5</b>	+0.5	0.0	0.0
$G_6^{(8)}$	0.0	0.0	+1.0	0.0	0.0	<b>0.0</b>	0.0	0.0	$G_6^{(8)}$	+0.5	+0.5	0.0	0.0	-0.5	<b>+0.5</b>	0.0	0.0
$G_7^{(8)}$	0.0	-1.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	$G_7^{(8)}$	0.0	0.0	+0.5	+0.5	0.0	0.0	<b>-0.5</b>	+0.5
$G_8^{(8)}$	-1.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	$G_8^{(8)}$	0.0	0.0	-0.5	-0.5	0.0	0.0	-0.5	<b>+0.5</b>

$\phi = 4$	$G_1^{(8)}$	$G_2^{(8)}$	$G_3^{(8)}$	$G_4^{(8)}$	$G_5^{(8)}$	$G_6^{(8)}$	$G_7^{(8)}$	$G_8^{(8)}$	$\phi = 5$	$G_1^{(8)}$	$G_2^{(8)}$	$G_3^{(8)}$	$G_4^{(8)}$	$G_5^{(8)}$	$G_6^{(8)}$	$G_7^{(8)}$	$G_8^{(8)}$	
$G_1^{(8)}$	<b>0.0</b>	0.0	+1.0	0.0	0.0	0.0	0.0	0.0	$G_1^{(8)}$	<b>+0.5</b>	+0.5	0.0	0.0	-0.5	+0.5	0.0	0.0	
$G_2^{(8)}$	0.0	<b>0.0</b>	0.0	+1.0	0.0	0.0	0.0	0.0	$G_2^{(8)}$	-0.5	<b>-0.5</b>	0.0	0.0	-0.5	+0.5	0.0	0.0	
$G_3^{(8)}$	+1.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	$G_3^{(8)}$	0.0	0.0	<b>+0.5</b>	+0.5	0.0	0.0	+0.5	-0.5	
$G_4^{(8)}$	0.0	+1.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	$G_4^{(8)}$	0.0	0.0	-0.5	<b>-0.5</b>	0.0	0.0	+0.5	-0.5	
$G_5^{(8)}$	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	-1.0	0.0	$G_5^{(8)}$	0.0	0.0	+0.5	-0.5	<b>-0.5</b>	-0.5	0.0	0.0	
$G_6^{(8)}$	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	-1.0	$G_6^{(8)}$	0.0	0.0	+0.5	-0.5	+0.5	<b>+0.5</b>	0.0	0.0	
$G_7^{(8)}$	0.0	0.0	0.0	0.0	-1.0	0.0	<b>0.0</b>	0.0	$G_7^{(8)}$	-0.5	+0.5	0.0	0.0	0.0	0.0	<b>-0.5</b>	-0.5	
$G_8^{(8)}$	0.0	0.0	0.0	0.0	0.0	-1.0	0.0	<b>0.0</b>	$G_8^{(8)}$	-0.5	+0.5	0.0	0.0	0.0	0.0	0.0	<b>+0.5</b>	<b>+0.5</b>
$\phi = 6$	$G_1^{(8)}$	$G_2^{(8)}$	$G_3^{(8)}$	$G_4^{(8)}$	$G_5^{(8)}$	$G_6^{(8)}$	$G_7^{(8)}$	$G_8^{(8)}$	$\phi = 7$	$G_1^{(8)}$	$G_2^{(8)}$	$G_3^{(8)}$	$G_4^{(8)}$	$G_5^{(8)}$	$G_6^{(8)}$	$G_7^{(8)}$	$G_8^{(8)}$	
$G_1^{(8)}$	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	-1.0	$G_1^{(8)}$	<b>0.0</b>	0.0	+0.5	-0.5	+0.5	+0.5	0.0	0.0	
$G_2^{(8)}$	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	-1.0	0.0	$G_2^{(8)}$	0.0	<b>0.0</b>	+0.5	-0.5	-0.5	-0.5	0.0	0.0	
$G_3^{(8)}$	0.0	0.0	<b>0.0</b>	0.0	0.0	+1.0	0.0	0.0	$G_3^{(8)}$	+0.5	-0.5	<b>0.0</b>	0.0	0.0	0.0	-0.5	-0.5	
$G_4^{(8)}$	0.0	0.0	0.0	<b>0.0</b>	+1.0	0.0	0.0	0.0	$G_4^{(8)}$	+0.5	-0.5	0.0	<b>0.0</b>	0.0	0.0	+0.5	+0.5	
$G_5^{(8)}$	0.0	+1.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	$G_5^{(8)}$	0.0	0.0	-0.5	-0.5	<b>0.0</b>	0.0	+0.5	-0.5	
$G_6^{(8)}$	+1.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	$G_6^{(8)}$	0.0	0.0	+0.5	+0.5	0.0	<b>0.0</b>	+0.5	-0.5	
$G_7^{(8)}$	0.0	0.0	0.0	-1.0	0.0	0.0	<b>0.0</b>	0.0	$G_7^{(8)}$	+0.5	+0.5	0.0	0.0	+0.5	-0.5	<b>0.0</b>	0.0	
$G_8^{(8)}$	0.0	0.0	-1.0	0.0	0.0	0.0	0.0	<b>0.0</b>	$G_8^{(8)}$	-0.5	-0.5	0.0	0.0	+0.5	-0.5	0.0	<b>0.0</b>	



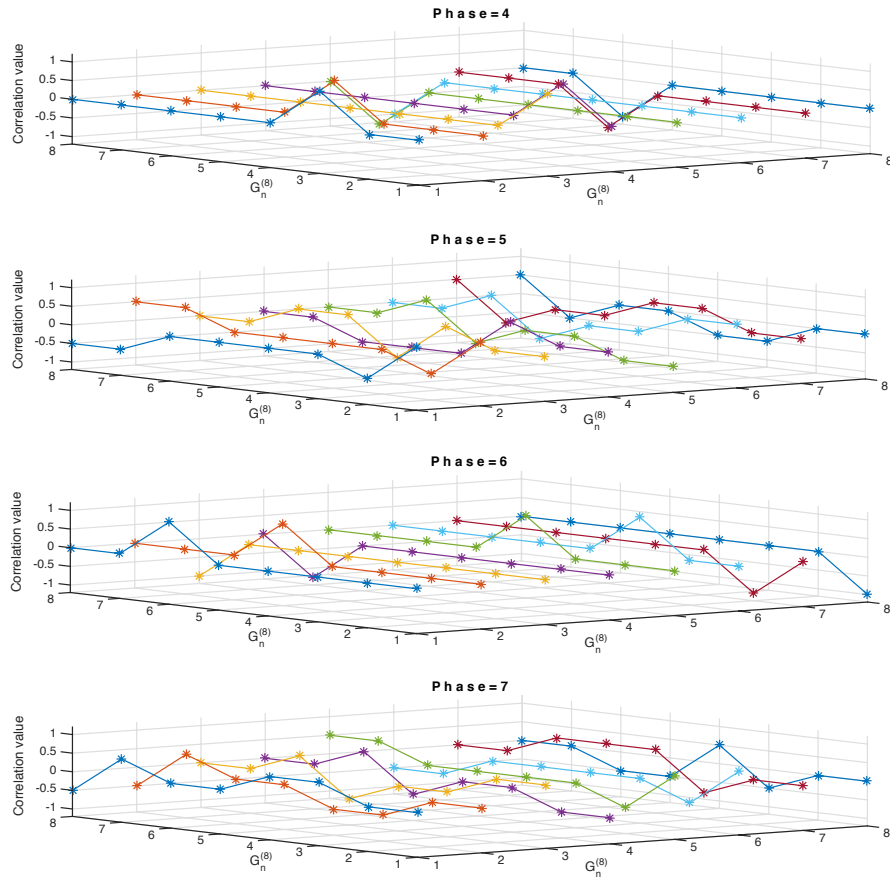


Figure 4.9: Correlation matrices for  $G_n^{(8)}$ : Phases.

• **Double-correlation**

DC subsets derived from  $G_n^{(8)}$  are shown, so the indexes of  $G_{\#}^{DC} = \{G_i^{(8)}, G_j^{(8)}, G_k^{(8)}, G_l^{(8)}\}$  can be

#	$i$	$j$	$k$	$l$	#	$i$	$j$	$k$	$l$	#	$i$	$j$	$k$	$l$	#	$i$	$j$	$k$	$l$
<b>1</b>	1	2	3	5	<b>15</b>	1	3	4	7	<b>29</b>	2	3	4	5	<b>43</b>	2	5	7	8
<b>2</b>	1	2	3	6	<b>16</b>	1	3	4	8	<b>30</b>	2	3	4	6	<b>44</b>	2	6	7	8
<b>3</b>	1	2	3	7	<b>17</b>	1	3	5	6	<b>31</b>	2	3	4	7	<b>45</b>	3	4	5	7
<b>4</b>	1	2	3	8	<b>18</b>	1	3	5	8	<b>32</b>	2	3	4	8	<b>46</b>	3	4	5	8
<b>5</b>	1	2	4	5	<b>19</b>	1	3	6	7	<b>33</b>	2	3	5	6	<b>47</b>	3	4	6	7
<b>6</b>	1	2	4	6	<b>20</b>	1	3	7	8	<b>34</b>	2	3	5	7	<b>48</b>	3	4	6	8
<b>7</b>	1	2	4	7	<b>21</b>	1	4	5	6	<b>35</b>	2	3	6	8	<b>49</b>	3	5	6	7
<b>8</b>	1	2	4	8	<b>22</b>	1	4	5	7	<b>36</b>	2	3	7	8	<b>50</b>	3	5	6	8
<b>9</b>	1	2	5	7	<b>23</b>	1	4	6	8	<b>37</b>	2	4	5	6	<b>51</b>	3	5	7	8
<b>10</b>	1	2	5	8	<b>24</b>	1	4	7	8	<b>38</b>	2	4	5	8	<b>52</b>	3	6	7	8
<b>11</b>	1	2	6	7	<b>25</b>	1	5	6	7	<b>39</b>	2	4	6	7	<b>53</b>	4	5	6	7
<b>12</b>	1	2	6	8	<b>26</b>	1	5	6	8	<b>40</b>	2	4	7	8	<b>54</b>	4	5	6	8
<b>13</b>	1	3	4	5	<b>27</b>	1	5	7	8	<b>41</b>	2	5	6	7	<b>55</b>	4	5	7	8
<b>14</b>	1	3	4	6	<b>28</b>	1	6	7	8	<b>42</b>	2	5	6	8	<b>56</b>	4	6	7	8



★ **SHIFTING PROPERTY**

Golay-Hadamard sets are given in following lists  $M_\phi(i, j)$ , while Figure 4.10 shows the ACF values from all codes.

$i$	$j$	$\phi$							$i$	$j$	$\phi$								
		0	1	2	3	4	5	6			7	0	1	2	3	4	5	6	7
$G_1^{(8)}$	$G_1^{(8)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$G_3^{(8)}$	$G_6^{(8)}$	0.0	+0.5	0.0	+0.5	0.0	0.0	+1.0	0.0
$G_1^{(8)}$	$G_2^{(8)}$	0.0	0.0	0.0	-0.5	0.0	+0.5	0.0	0.0	$G_3^{(8)}$	$G_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	+0.5	0.0	-0.5
$G_1^{(8)}$	$G_3^{(8)}$	0.0	+0.5	0.0	0.0	+1.0	0.0	0.0	+0.5	$G_3^{(8)}$	$G_8^{(8)}$	0.0	0.0	-1.0	0.0	0.0	-0.5	0.0	-0.5
$G_1^{(8)}$	$G_4^{(8)}$	0.0	+0.5	0.0	0.0	0.0	0.0	0.0	-0.5	$G_4^{(8)}$	$G_4^{(8)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$G_1^{(8)}$	$G_5^{(8)}$	0.0	0.0	0.0	0.0	0.0	-0.5	0.0	+0.5	$G_4^{(8)}$	$G_5^{(8)}$	0.0	-0.5	0.0	-0.5	0.0	0.0	+1.0	0.0
$G_1^{(8)}$	$G_6^{(8)}$	0.0	0.0	+1.0	0.0	0.0	+0.5	0.0	+0.5	$G_4^{(8)}$	$G_6^{(8)}$	0.0	+0.5	0.0	-0.5	0.0	0.0	0.0	0.0
$G_1^{(8)}$	$G_7^{(8)}$	0.0	+0.5	0.0	-0.5	0.0	0.0	0.0	0.0	$G_4^{(8)}$	$G_7^{(8)}$	0.0	0.0	-1.0	0.0	0.0	+0.5	0.0	+0.5
$G_1^{(8)}$	$G_8^{(8)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0	$G_4^{(8)}$	$G_8^{(8)}$	0.0	0.0	0.0	0.0	0.0	-0.5	0.0	+0.5
$G_2^{(8)}$	$G_2^{(8)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0	$G_5^{(8)}$	$G_5^{(8)}$	+1.0	0.0	-0.5	0.0	-0.5	0.0	0.0	0.0
$G_2^{(8)}$	$G_3^{(8)}$	0.0	-0.5	0.0	0.0	0.0	0.0	0.0	+0.5	$G_5^{(8)}$	$G_6^{(8)}$	0.0	0.0	0.0	+0.5	0.0	-0.5	0.0	0.0
$G_2^{(8)}$	$G_4^{(8)}$	0.0	-0.5	0.0	0.0	+1.0	0.0	0.0	-0.5	$G_5^{(8)}$	$G_7^{(8)}$	0.0	+0.5	0.0	0.0	-1.0	0.0	0.0	+0.5
$G_2^{(8)}$	$G_5^{(8)}$	0.0	0.0	+1.0	0.0	0.0	-0.5	0.0	-0.5	$G_5^{(8)}$	$G_8^{(8)}$	0.0	+0.5	0.0	0.0	0.0	0.0	0.0	-0.5
$G_2^{(8)}$	$G_6^{(8)}$	0.0	0.0	0.0	0.0	0.0	+0.5	0.0	-0.5	$G_6^{(8)}$	$G_6^{(8)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0
$G_2^{(8)}$	$G_7^{(8)}$	0.0	+0.5	0.0	+0.5	0.0	0.0	-1.0	0.0	$G_6^{(8)}$	$G_7^{(8)}$	0.0	-0.5	0.0	0.0	0.0	0.0	0.0	+0.5
$i$	$j$	$\phi$							$i$	$j$	$\phi$								
$G_2^{(8)}$	$G_8^{(8)}$	0.0	-0.5	+0.5	0.0	0.0	0.0	0.0	0.0	$G_6^{(8)}$	$G_8^{(8)}$	0.0	-0.5	0.0	0.0	-1.0	0.0	0.0	-0.5
$G_3^{(8)}$	$G_3^{(8)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$G_7^{(8)}$	$G_3^{(8)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$G_3^{(8)}$	$G_4^{(8)}$	0.0	0.0	0.0	-0.5	0.0	+0.5	0.0	0.0	$G_7^{(8)}$	$G_5^{(8)}$	0.0	0.0	0.0	+0.5	0.0	-0.5	0.0	0.0
$G_3^{(8)}$	$G_5^{(8)}$	0.0	-0.5	0.0	+0.5	0.0	0.0	0.0	0.0	$G_7^{(8)}$	$G_8^{(8)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0

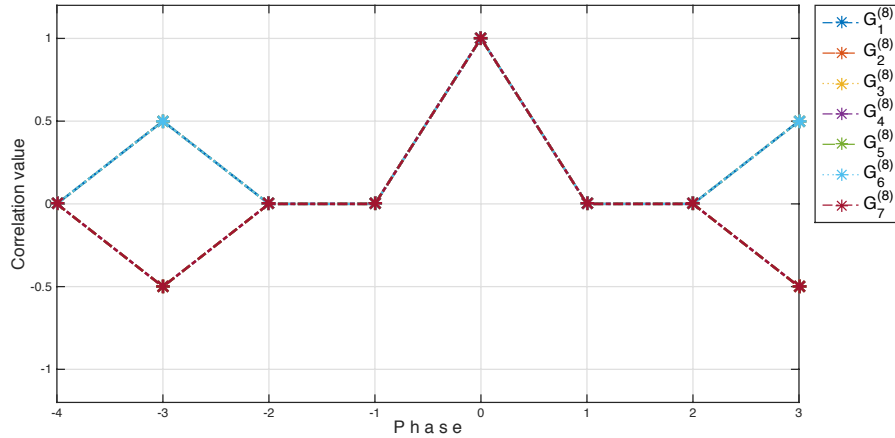


Figure 4.10: 2-D Side-lobe graph: ACF for  $G_n^{(7)}$  set.

### 4.3.2 Complementary sequences sets

This family is analyzed from two kind of sets: Super and Perfect complementary sets. Balance, correlation, and shift properties are tested and their behavior are shown as follows.

#### Super complementary sets

Considering two flocks recovered from *The Next Generation CDMA Technologies* book,[7], Appendix F with  $PG = 32$ ,  $M = 4$  and  $N = 8$ .  $S1_n^{(4)}$  and  $S2_n^{(4)}$  are defined as

$$\begin{aligned}
 S1_1^{(4)} &= (+1, +1, +1, -1, +1, +1, -1, +1); & S2_1^{(4)} &= (+1, -1, +1, +1, +1, -1, -1, -1); \\
 S1_2^{(4)} &= (+1, +1, +1, -1, -1, -1, +1, -1); & S2_2^{(4)} &= (+1, -1, +1, +1, -1, +1, +1, +1); \\
 S1_3^{(4)} &= (+1, +1, +1, -1, +1, +1, -1, +1); & S2_3^{(4)} &= (+1, -1, +1, +1, +1, -1, -1, -1); \\
 S1_4^{(4)} &= (-1, -1, -1, +1, +1, +1, -1, +1); & S2_4^{(4)} &= (-1, +1, -1, -1, +1, -1, -1, -1);
 \end{aligned}$$

★ **BALANCE PROPERTY**

Table 4.5 verifying this property considering the flocks  $S1_n^{(4)}$  and  $S2_n^{(4)}$

Table 4.5: Balance: Super complementary sets

$S1_n^{(4)}$	Bal	meet?	$S1_n^{(4)}$	Bal	meet?	$S2_n^{(4)}$	Bal	meet?	$S2_n^{(4)}$	Bal	meet?
$S1_1^{(4)}$	4	NO	$S1_3^{(4)}$	4	NO	$S2_1^{(4)}$	0	YES	$S2_3^{(4)}$	0	YES
$S1_2^{(4)}$	0	YES	$S1_4^{(4)}$	0	YES	$S2_2^{(4)}$	4	NO	$S2_4^{(4)}$	-4	NO

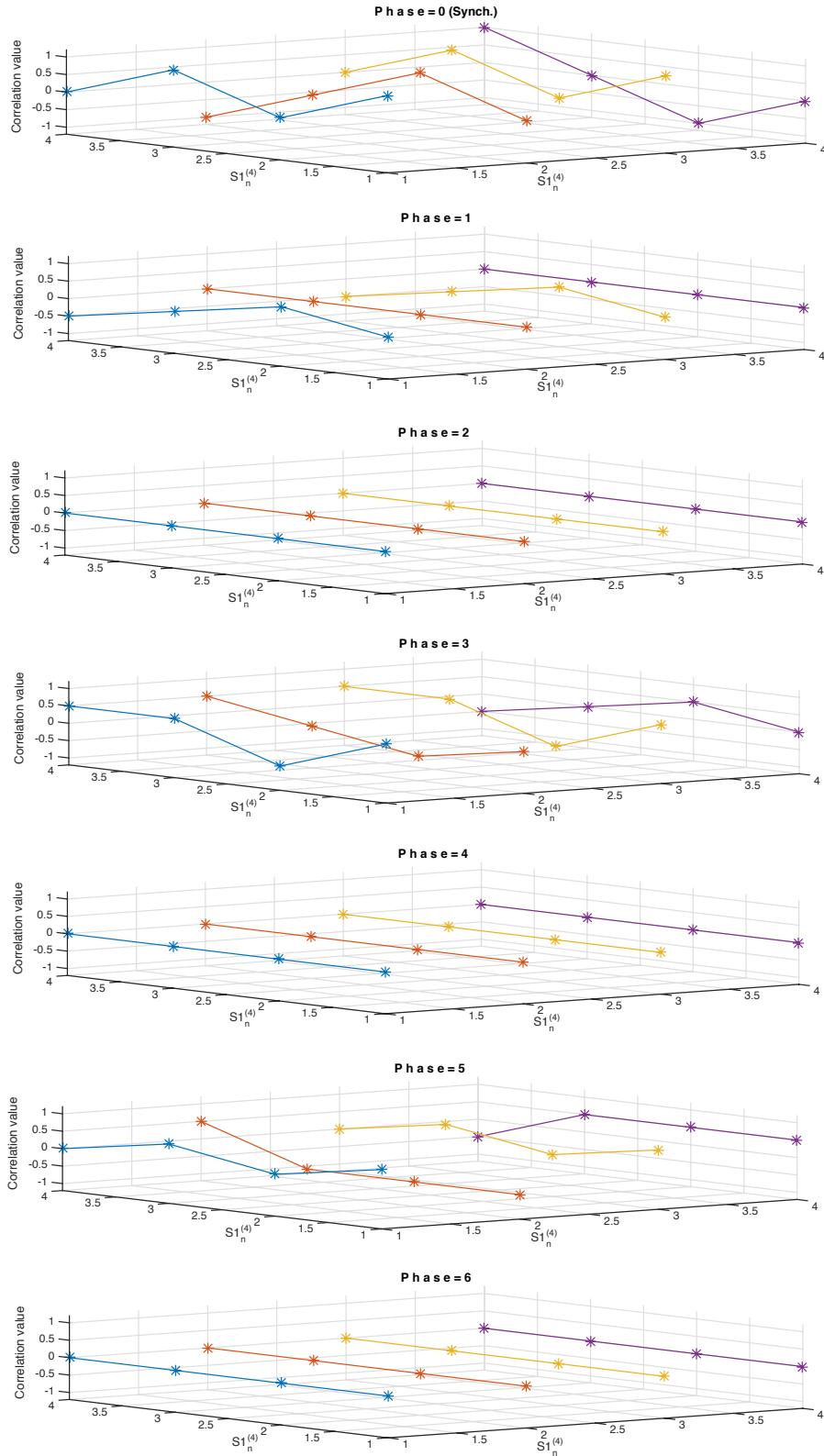
★ **CORRELATION PROPERTIES**

● **Autocorrelation and cross-correlation**

Recall that complementary sets are described through subsets of sequences, which form *flocks*, where the correlation properties are maintained, for this reason they are studied two cases: infra-group, correlation among elements into the same flock, and inter-group, correlation among elements with different flock.

+ Infra-group correlation:  $S1_n^{(4)}$

$\phi = 0$	$S1_1^{(4)}$	$S1_2^{(4)}$	$S1_3^{(4)}$	$S1_4^{(4)}$	$\phi = 1$	$S1_1^{(4)}$	$S1_2^{(4)}$	$S1_3^{(4)}$	$S1_4^{(4)}$	$\phi = 2$	$S1_1^{(4)}$	$S1_2^{(4)}$	$S1_3^{(4)}$	$S1_4^{(4)}$
$S1_1^{(4)}$	+1.0	0.0	+1.0	0.0	$S1_1^{(4)}$	0.0	0.0	0.0	0.0	$S1_1^{(4)}$	0.0	0.0	0.0	0.0
$S1_2^{(4)}$	0.0	+1.0	0.0	-1.0	$S1_2^{(4)}$	+0.5	0.0	+0.5	0.0	$S1_2^{(4)}$	0.0	0.0	0.0	0.0
$S1_3^{(4)}$	+1.0	0.0	+1.0	0.0	$S1_3^{(4)}$	0.0	0.0	0.0	0.0	$S1_3^{(4)}$	0.0	0.0	0.0	0.0
$S1_4^{(4)}$	0.0	-1.0	0.0	+1.0	$S1_4^{(4)}$	-0.5	0.0	-0.5	0.0	$S1_4^{(4)}$	0.0	0.0	0.0	0.0
$\phi = 3$	$S1_1^{(4)}$	$S1_2^{(4)}$	$S1_3^{(4)}$	$S1_4^{(4)}$	$\phi = 4$	$S1_1^{(4)}$	$S1_2^{(4)}$	$S1_3^{(4)}$	$S1_4^{(4)}$	$\phi = 5$	$S1_1^{(4)}$	$S1_2^{(4)}$	$S1_3^{(4)}$	$S1_4^{(4)}$
$S1_1^{(4)}$	+0.5	0.0	+0.5	0.0	$S1_1^{(4)}$	0.0	0.0	0.0	0.0	$S1_1^{(4)}$	+0.5	-0.5	+0.5	+0.5
$S1_2^{(4)}$	-0.5	-0.5	-0.5	+0.5	$S1_2^{(4)}$	0.0	0.0	0.0	0.0	$S1_2^{(4)}$	0.0	-0.5	0.0	+0.5
$S1_3^{(4)}$	+0.5	0.0	+0.5	0.0	$S1_3^{(4)}$	0.0	0.0	0.0	0.0	$S1_3^{(4)}$	+0.5	-0.5	+0.5	+0.5
$S1_4^{(4)}$	+0.5	+0.5	+0.5	-0.5	$S1_4^{(4)}$	0.0	0.0	0.0	0.0	$S1_4^{(4)}$	0.0	+0.5	0.0	-0.5
$\phi = 6$	$S1_1^{(4)}$	$S1_2^{(4)}$	$S1_3^{(4)}$	$S1_4^{(4)}$	$\phi = 7$	$S1_1^{(4)}$	$S1_2^{(4)}$	$S1_3^{(4)}$	$S1_4^{(4)}$					
$S1_1^{(4)}$	0.0	0.0	0.0	0.0	$S1_1^{(4)}$	0.0	+0.5	0.0	-0.5					
$S1_2^{(4)}$	0.0	0.0	0.0	0.0	$S1_2^{(4)}$	0.0	0.0	0.0	0.0					
$S1_3^{(4)}$	0.0	0.0	0.0	0.0	$S1_3^{(4)}$	0.0	+0.5	0.0	-0.5					
$S1_4^{(4)}$	0.0	0.0	0.0	0.0	$S1_4^{(4)}$	0.0	0.0	0.0	0.0					



+ Infra-group correlation:  $S_2^{(4)}$

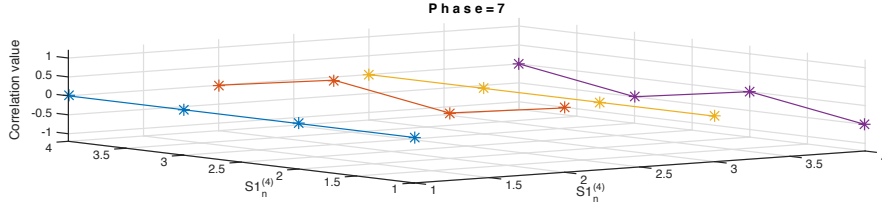
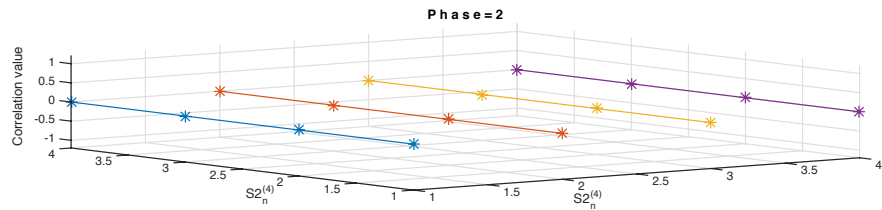
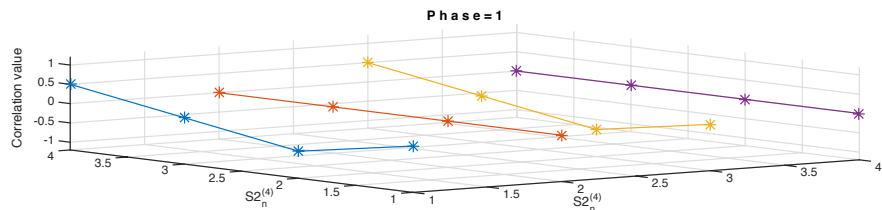
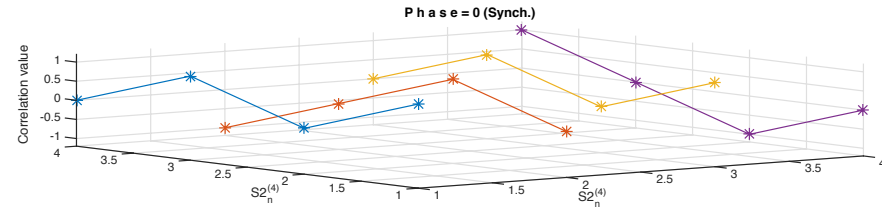


Figure 4.11: Correlation matrices for  $S1_n^{(4)}$ : Phases.

$\phi = 0$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_1^{(3)}$	$S2_4^{(4)}$	$\phi = 1$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$	$\phi = 2$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$
$S2_1^{(4)}$	+1.0	0.0	+1.0	0.0	$S2_1^{(4)}$	0.0	0.0	0.0	0.0	$S2_1^{(4)}$	0.0	0.0	0.0	0.0
$S2_2^{(4)}$	0.0	+1.0	0.0	-1.0	$S2_2^{(4)}$	-0.5	0.0	-0.5	0.0	$S2_2^{(4)}$	0.0	0.0	0.0	0.0
$S2_3^{(4)}$	+1.0	0.0	+1.0	0.0	$S2_3^{(4)}$	0.0	0.0	0.0	0.0	$S2_3^{(4)}$	0.0	0.0	0.0	0.0
$S2_4^{(4)}$	0.0	-1.0	0.0	+1.0	$S2_4^{(4)}$	+0.5	0.0	+0.5	0.0	$S2_4^{(4)}$	0.0	0.0	0.0	0.0
$\phi = 3$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_1^{(3)}$	$S2_4^{(4)}$	$\phi = 4$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$	$\phi = 5$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$
$S2_1^{(4)}$	-0.5	0.0	-0.5	0.0	$S2_1^{(4)}$	0.0	0.0	0.0	0.0	$S2_1^{(4)}$	-0.5	+0.5	-0.5	-0.5
$S2_2^{(4)}$	+0.5	+0.5	+0.5	-0.5	$S2_2^{(4)}$	0.0	0.0	0.0	0.0	$S2_2^{(4)}$	0.0	+0.5	0.0	-0.5
$S2_3^{(4)}$	-0.5	0.0	-0.5	0.0	$S2_3^{(4)}$	0.0	0.0	0.0	0.0	$S2_3^{(4)}$	-0.5	+0.5	-0.5	-0.5
$S2_4^{(4)}$	-0.5	-0.5	-0.5	+0.5	$S2_4^{(4)}$	0.0	-0.5	0.0	0.0	$S2_4^{(4)}$	0.0	-0.5	0.0	+0.5
$\phi = 6$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_1^{(3)}$	$S2_4^{(4)}$	$\phi = 7$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$					
$S2_1^{(4)}$	0.0	0.0	0.0	0.0	$S2_1^{(4)}$	0.0	-0.5	0.0	+0.5					
$S2_2^{(4)}$	0.0	0.0	0.0	0.0	$S2_2^{(4)}$	0.0	0.0	0.0	0.0					
$S2_3^{(4)}$	0.0	0.0	0.0	0.0	$S2_3^{(4)}$	0.0	-0.5	0.0	+0.5					
$S2_4^{(4)}$	0.0	0.0	0.0	0.0	$S2_4^{(4)}$	0.0	0.0	0.0	0.0					



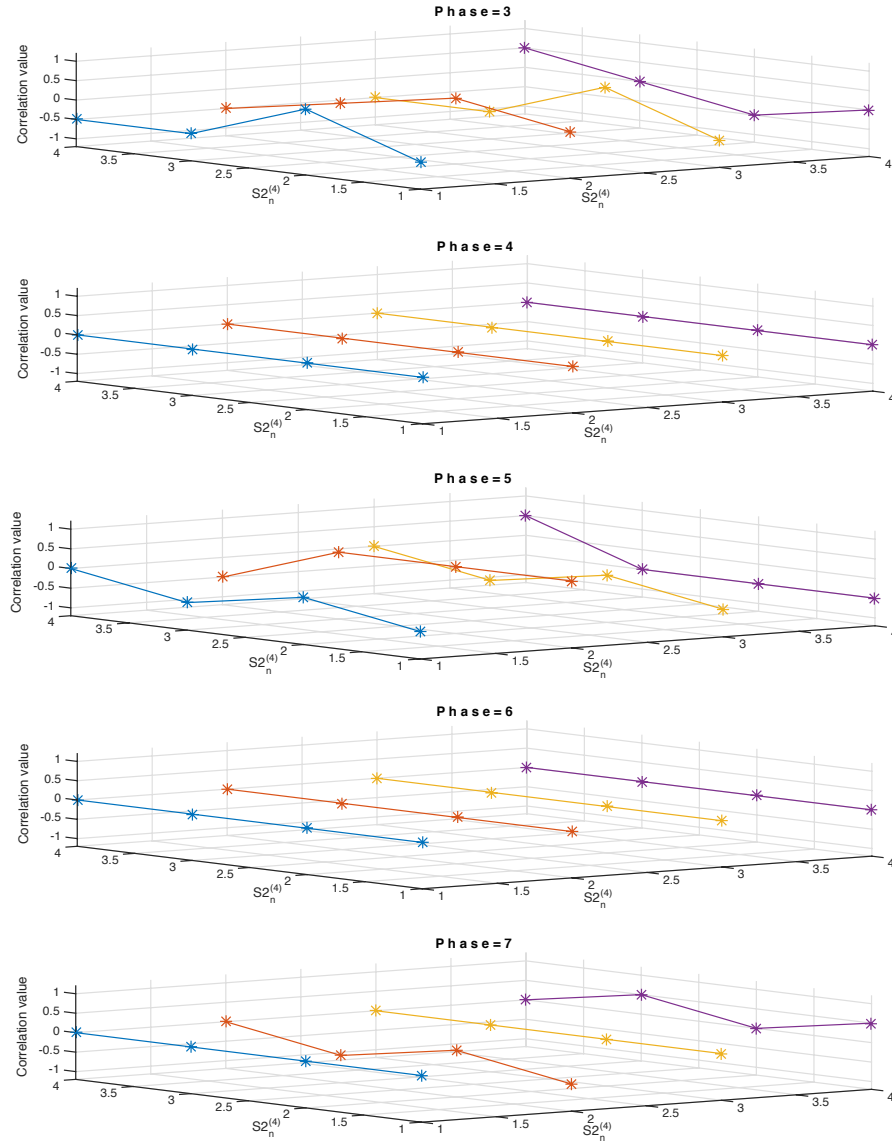
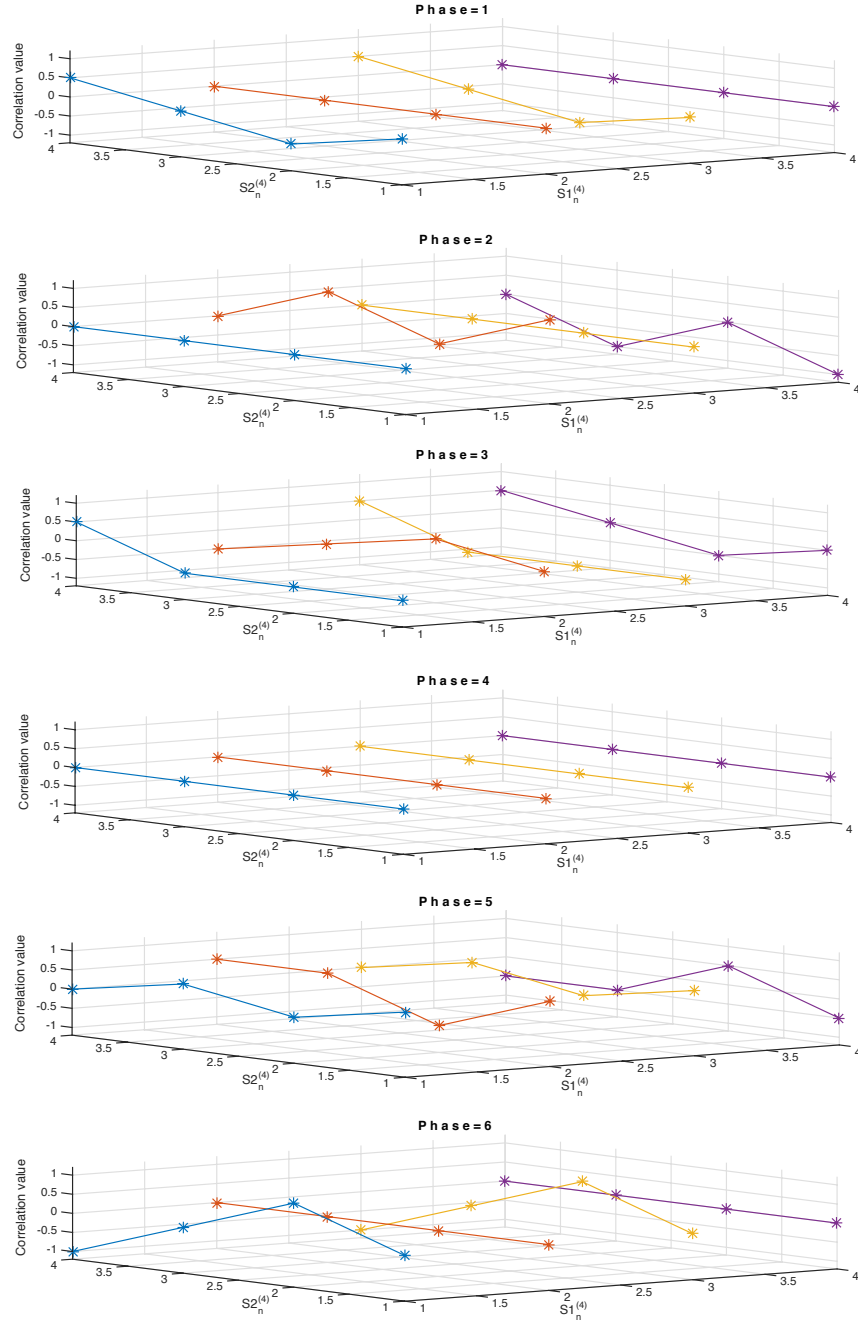


Figure 4.12: Correlation matrices for  $S2_n^{(4)}$ : Phases.

+ *Inter-group correlation:  $S1_n^{(4)}$  vs  $S2_n^{(4)}$*

$\phi = 0$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_1^{(3)}$	$S2_4^{(4)}$	$\phi = 1$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$	$\phi = 2$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$
$S1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$S1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$S1_1^{(4)}$	<b>0.0</b>	+1.0	0.0	-1.0
$S1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$S1_2^{(4)}$	-0.5	<b>0.0</b>	-0.5	0.0	$S1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0
$S1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$S1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$S1_3^{(4)}$	0.0	+1.0	<b>0.0</b>	-1.0
$S1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$S1_4^{(4)}$	+0.5	0.0	+0.5	<b>0.0</b>	$S1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>
$\phi = 3$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_1^{(3)}$	$S2_4^{(4)}$	$\phi = 4$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$	$\phi = 5$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$
$S1_1^{(4)}$	-0.5	0.0	-0.5	0.0	$S1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$S1_1^{(4)}$	+0.5	+0.5	+0.5	-0.5
$S1_2^{(4)}$	-0.5	+0.5	-0.5	-0.5	$S1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$S1_2^{(4)}$	0.0	-0.5	0.0	+0.5
$S1_3^{(4)}$	-0.5	0.0	-0.5	0.0	$S1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$S1_3^{(4)}$	+0.5	+0.5	+0.5	-0.5
$S1_4^{(4)}$	+0.5	-0.5	+0.5	+0.5	$S1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$S1_4^{(4)}$	0.0	+0.5	0.0	-0.5

$\phi = 6$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_1^{(3)}$	$S2_4^{(4)}$	$\phi = 7$	$S2_1^{(4)}$	$S2_2^{(4)}$	$S2_3^{(4)}$	$S2_4^{(4)}$
$S1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$S1_1^{(4)}$	<b>0.0</b>	+0.5	0.0	-0.5
$S1_2^{(4)}$	+1.0	<b>0.0</b>	+1.0	0.0	$S1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0
$S1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$S1_3^{(4)}$	0.0	+0.5	<b>0.0</b>	-0.5
$S1_4^{(4)}$	-1.0	0.0	-1.0	<b>0.0</b>	$S1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>



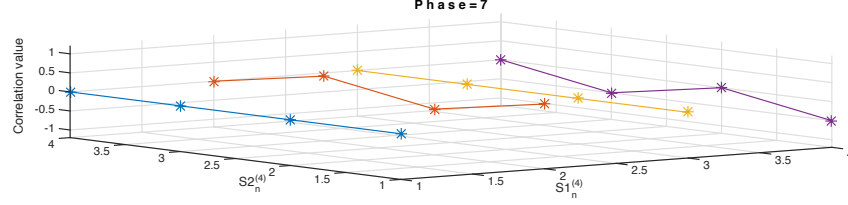


Figure 4.13: Correlation matrices for  $S1_n^{(4)}$  vs  $S2_n^{(4)}$ : Phases.

• **Double-correlation**

Note that  $S1_1^{(4)} = S1_3^{(4)}$ ,  $S1_2^{(4)} = -S1_4^{(4)}$ ,  $S2_1^{(4)} = S2_3^{(4)}$ , and  $S2_2^{(4)} = -S2_4^{(4)}$  which reduce the availability of codes. However, combining these sets it is possible to get the set  $S = \{S1_1^{(4)}, S1_2^{(4)}, S2_1^{(4)}, S2_2^{(4)}\}$  as the unique that meets the DC property.

★ **SHIFTING PROPERTY**

Considering  $S1_n^{(4)}$  and  $S2_n^{(4)}$ , shifting property is evaluated through following lists  $M_\phi(i, j)$ . Besides, Figure 4.14 shows the ACF values for Infra group from both flocks and CCF for Inter group between flocks.

$i$	$j$	$\phi$							$i$	$j$	$\phi$								
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$S1_1^{(4)}$	$S1_1^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$S1_2^{(4)}$	$S1_3^{(4)}$	0.0	+0.5	0.0	-0.5	0.0	0.0	0.0	0.0
$S1_1^{(4)}$	$S1_2^{(4)}$	0.0	0.0	0.0	0.0	0.0	-0.5	0.0	+0.5	$S1_2^{(4)}$	$S1_4^{(4)}$	-1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0
$S1_1^{(4)}$	$S1_3^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$S1_3^{(4)}$	$S1_3^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0
$S1_1^{(4)}$	$S1_4^{(4)}$	0.0	0.0	0.0	0.0	0.0	+0.5	0.0	-0.5	$S1_3^{(4)}$	$S1_4^{(4)}$	0.0	0.0	0.0	0.0	0.0	+0.5	0.0	-0.5
$S1_2^{(4)}$	$S1_2^{(4)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0	$S1_4^{(4)}$	$S1_4^{(4)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0

$i$	$j$	$\phi$							$i$	$j$	$\phi$								
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$S2_1^{(4)}$	$S2_1^{(4)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0	$S2_2^{(4)}$	$S2_3^{(4)}$	0.0	-0.5	0.0	+0.5	0.0	0.0	0.0	0.0
$S2_1^{(4)}$	$S2_2^{(4)}$	0.0	0.0	0.0	0.0	0.0	+0.5	0.0	-0.5	$S2_2^{(4)}$	$S2_4^{(4)}$	-1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$S2_1^{(4)}$	$S2_3^{(4)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0	$S2_3^{(4)}$	$S2_3^{(4)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$S2_1^{(4)}$	$S2_4^{(4)}$	0.0	0.0	0.0	0.0	0.0	-0.5	0.0	+0.5	$S2_3^{(4)}$	$S2_4^{(4)}$	0.0	0.0	0.0	0.0	0.0	-0.5	0.0	+0.5
$S2_2^{(4)}$	$S2_2^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$S2_4^{(4)}$	$S2_4^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0

$i$	$j$	$\phi$							$i$	$j$	$\phi$								
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$S1_1^{(4)}$	$S2_1^{(4)}$	0.0	0.0	0.0	-0.5	0.0	+0.5	0.0	0.0	$S1_2^{(4)}$	$S2_3^{(4)}$	0.0	-0.5	0.0	-0.5	0.0	0.0	+1.0	0.0
$S1_1^{(4)}$	$S2_2^{(4)}$	0.0	0.0	+1.0	0.0	0.0	+0.5	0.0	+0.5	$S1_2^{(4)}$	$S2_4^{(4)}$	0.0	0.0	0.0	-0.5	0.0	+0.5	0.0	0.0
$S1_1^{(4)}$	$S2_3^{(4)}$	0.0	0.0	0.0	-0.5	0.0	+0.5	0.0	0.0	$S1_3^{(4)}$	$S2_3^{(4)}$	0.0	0.0	0.0	-0.5	0.0	+0.5	0.0	0.0
$S1_1^{(4)}$	$S2_4^{(4)}$	0.0	0.0	-1.0	0.0	0.0	-0.5	0.0	-0.5	$S1_3^{(4)}$	$S2_4^{(4)}$	0.0	0.0	-1.0	0.0	0.0	-0.5	0.0	-0.5
$S1_2^{(4)}$	$S2_2^{(4)}$	0.0	0.0	0.0	+0.5	0.0	-0.5	0.0	0.0	$S1_4^{(4)}$	$S2_4^{(4)}$	+0.0	0.0	0.0	+0.5	0.0	-0.5	0.0	0.0

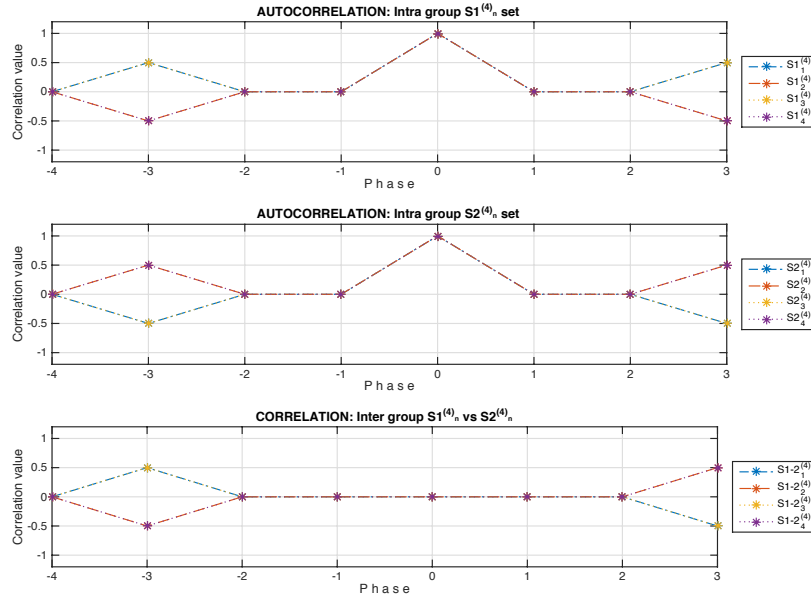


Figure 4.14: 2-D Side-lobe graph: ACF Intra group for  $S1_n^{(4)}$  -  $S2_n^{(4)}$ , and CCF Inter group for  $S1_n^{(4)}$  vs  $S2_n^{(4)}$ .

### Perfect complementary sets

These sets are in *An algebraic approach to generate super-set of perfect complementary codes for interference-free CDMA* paper, [9], so  $R1^{(8)n}$  and  $R2^{(8)n}$  are the first and second flocks.

$$\begin{aligned}
 R1_1^{(8)} &= (+1, +1, +1, -1, +1, +1, -1, +1); & R2_1^{(8)} &= (+1, +1, +1, -1, +1, +1, -1, +1); \\
 R1_2^{(8)} &= (+1, -1, +1, +1, +1, -1, -1, -1); & R2_2^{(8)} &= (-1, +1, -1, -1, -1, +1, +1, +1); \\
 R1_3^{(8)} &= (+1, +1, +1, -1, -1, -1, +1, -1); & R2_3^{(8)} &= (+1, +1, +1, -1, -1, -1, +1, -1); \\
 R1_4^{(8)} &= (+1, -1, +1, +1, -1, +1, +1, +1); & R2_4^{(8)} &= (-1, +1, -1, -1, +1, -1, -1, -1); \\
 R1_5^{(8)} &= (+1, +1, +1, -1, +1, +1, -1, +1); & R2_5^{(8)} &= (+1, +1, +1, -1, +1, +1, -1, +1); \\
 R1_6^{(8)} &= (+1, -1, +1, +1, +1, -1, -1, -1); & R2_6^{(8)} &= (-1, +1, -1, -1, -1, +1, +1, +1); \\
 R1_7^{(8)} &= (-1, -1, -1, +1, +1, +1, -1, +1); & R2_7^{(8)} &= (-1, -1, -1, +1, +1, +1, -1, +1); \\
 R1_8^{(8)} &= (-1, +1, -1, -1, +1, -1, -1, -1); & R2_8^{(8)} &= (+1, -1, +1, +1, -1, +1, +1, +1);
 \end{aligned}$$

### ★ BALANCE PROPERTY

This property is verifying in Table 4.6 which shows *Bal* values for both sets.



Table 4.6: Balance: Perfect complementary sets

$R1_n^{(8)}$	Bal	meet?	$R1_n^{(8)}$	Bal	meet?	$R2_n^{(8)}$	Bal	meet?	$R2_n^{(8)}$	Bal	meet?
$R1_1^{(8)}$	4	NO	$R1_5^{(8)}$	4	NO	$R2_1^{(8)}$	4	NO	$R2_5^{(8)}$	4	NO
$R1_2^{(8)}$	0	YES	$R1_6^{(8)}$	0	YES	$R2_2^{(8)}$	0	YES	$R2_6^{(8)}$	0	YES
$R1_3^{(8)}$	0	YES	$R1_7^{(8)}$	0	YES	$R2_3^{(8)}$	0	YES	$R2_7^{(8)}$	0	YES
$R1_4^{(8)}$	4	NO	$R1_8^{(8)}$	-4	NO	$R2_4^{(8)}$	-4	NO	$R2_8^{(8)}$	4	NO

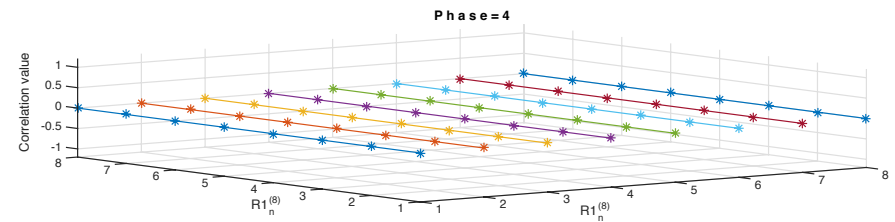
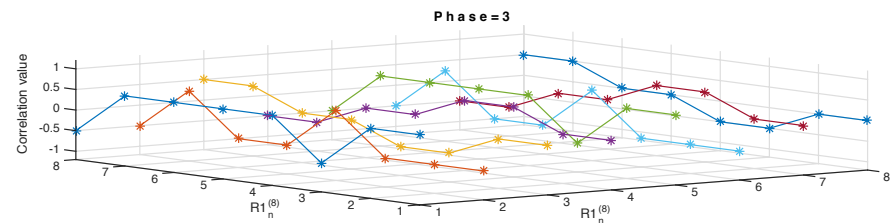
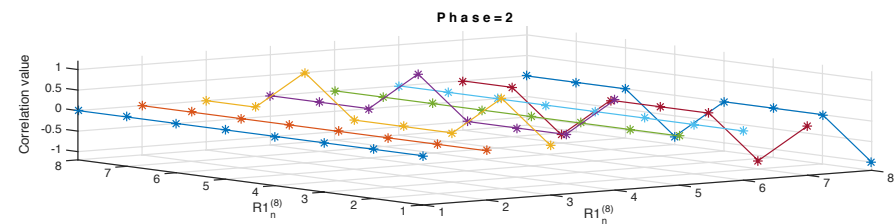
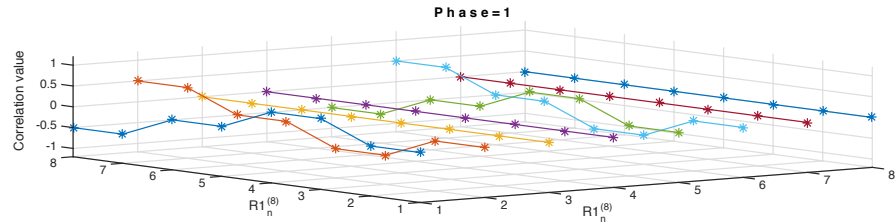
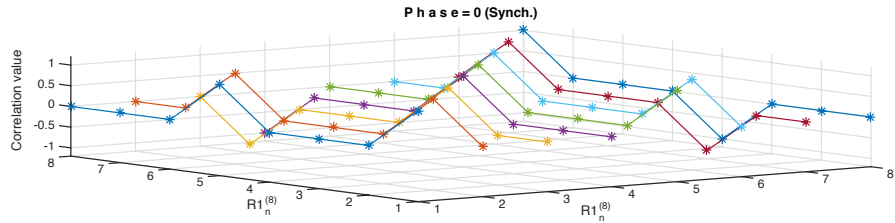
**★CORRELATION PROPERTIES**

• **Autocorrelation and cross-correlation**

+ **Infra-group correlation:**  $R1_n^{(4)}$

$\phi = 0$	$R1_1^{(8)}$	$R1_2^{(8)}$	$R1_3^{(8)}$	$R1_4^{(8)}$	$R1_5^{(8)}$	$R1_6^{(8)}$	$R1_7^{(8)}$	$R1_8^{(8)}$	$\phi = 1$	$R1_1^{(8)}$	$R1_2^{(8)}$	$R1_3^{(8)}$	$R1_4^{(8)}$	$R1_5^{(8)}$	$R1_6^{(8)}$	$R1_7^{(8)}$	$R1_8^{(8)}$
$R1_1^{(8)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$R1_1^{(8)}$	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$R1_2^{(8)}$	0.0	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	$R1_2^{(8)}$	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0
$R1_3^{(8)}$	0.0	0.0	+1.0	0.0	0.0	0.0	-1.0	0.0	$R1_3^{(8)}$	+0.5	-0.5	<b>0.0</b>	0.0	+0.5	-0.5	0.0	0.0
$R1_4^{(8)}$	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	-1.0	$R1_4^{(8)}$	+0.5	-0.5	0.0	<b>0.0</b>	+0.5	-0.5	0.0	0.0
$R1_5^{(8)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$R1_5^{(8)}$	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0
$R1_6^{(8)}$	0.0	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	$R1_6^{(8)}$	0.0	0.0	0.0	0.0	0.0	<b>1.0</b>	0.0	0.0
$R1_7^{(8)}$	0.0	0.0	-1.0	0.0	0.0	0.0	+1.0	0.0	$R1_7^{(8)}$	-0.5	+0.5	0.0	0.0	-0.5	+0.5	<b>0.0</b>	0.0
$R1_8^{(8)}$	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	+1.0	$R1_8^{(8)}$	-0.5	+0.5	0.0	0.0	-0.5	+0.5	0.0	<b>0.0</b>
$\phi = 2$	$R1_1^{(8)}$	$R1_2^{(8)}$	$R1_3^{(8)}$	$R1_4^{(8)}$	$R1_5^{(8)}$	$R1_6^{(8)}$	$R1_7^{(8)}$	$R1_8^{(8)}$	$\phi = 3$	$R1_1^{(8)}$	$R1_2^{(8)}$	$R1_3^{(8)}$	$R1_4^{(8)}$	$R1_5^{(8)}$	$R1_6^{(8)}$	$R1_7^{(8)}$	$R1_8^{(8)}$
$R1_1^{(8)}$	<b>0.0</b>	0.0	0.0	+1.0	0.0	0.0	0.0	-1.0	$R1_1^{(8)}$	+0.5	-0.5	0.0	0.0	+0.5	-0.5	0.0	0.0
$R1_2^{(8)}$	0.0	<b>0.0</b>	+1.0	0.0	0.0	0.0	-1.0	0.0	$R1_2^{(8)}$	+0.5	-0.5	0.0	0.0	+0.5	-0.5	0.0	0.0
$R1_3^{(8)}$	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	$R1_3^{(8)}$	-0.5	-0.5	-0.5	+0.5	-0.5	-0.5	+0.5	-0.5
$R1_4^{(8)}$	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	$R1_4^{(8)}$	+0.5	+0.5	-0.5	+0.5	+0.5	+0.5	+0.5	-0.5
$R1_5^{(8)}$	0.0	0.0	0.0	+1.0	<b>0.0</b>	0.0	0.0	-1.0	$R1_5^{(8)}$	+0.5	-0.5	0.0	0.0	+0.5	-0.5	0.0	0.0
$R1_6^{(8)}$	0.0	0.0	+1.0	0.0	0.0	<b>0.0</b>	-1.0	0.0	$R1_6^{(8)}$	+0.5	-0.5	0.0	0.0	+0.5	-0.5	0.0	0.0
$R1_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	$R1_7^{(8)}$	+0.5	+0.5	+0.5	-0.5	+0.5	+0.5	-0.5	+0.5
$R1_8^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	$R1_8^{(8)}$	-0.5	-0.5	+0.5	-0.5	-0.5	-0.5	-0.5	+0.5
$\phi = 4$	$R1_1^{(8)}$	$R1_2^{(8)}$	$R1_3^{(8)}$	$R1_4^{(8)}$	$R1_5^{(8)}$	$R1_6^{(8)}$	$R1_7^{(8)}$	$R1_8^{(8)}$	$\phi = 5$	$R1_1^{(8)}$	$R1_2^{(8)}$	$R1_3^{(8)}$	$R1_4^{(8)}$	$R1_5^{(8)}$	$R1_6^{(8)}$	$R1_7^{(8)}$	$R1_8^{(8)}$
$R1_1^{(8)}$	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$R1_1^{(8)}$	+0.5	+0.5	-0.5	+0.5	+0.5	+0.5	+0.5	-0.5
$R1_2^{(8)}$	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	$R1_2^{(8)}$	-0.5	-0.5	-0.5	+0.5	-0.5	-0.5	+0.5	-0.5
$R1_3^{(8)}$	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	$R1_3^{(8)}$	0.0	0.0	-0.5	-0.5	0.0	0.0	+0.5	+0.5
$R1_4^{(8)}$	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	$R1_4^{(8)}$	0.0	0.0	+0.5	+0.5	0.0	0.0	-0.5	-0.5
$R1_5^{(8)}$	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	$R1_5^{(8)}$	+0.5	+0.5	-0.5	+0.5	+0.5	+0.5	+0.5	-0.5
$R1_6^{(8)}$	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	$R1_6^{(8)}$	-0.5	-0.5	-0.5	+0.5	-0.5	-0.5	+0.5	-0.5
$R1_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	$R1_7^{(8)}$	0.0	0.0	+0.5	+0.5	0.0	0.0	-0.5	-0.5
$R1_8^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	$R1_8^{(8)}$	0.0	0.0	-0.5	-0.5	0.0	0.0	+0.5	+0.5

$\phi = 6$	$R1_1^{(8)}$	$R1_2^{(8)}$	$R1_3^{(8)}$	$R1_4^{(8)}$	$R1_5^{(8)}$	$R1_6^{(8)}$	$R1_7^{(8)}$	$R1_8^{(8)}$	$\phi = 7$	$R1_1^{(8)}$	$R1_2^{(8)}$	$R1_3^{(8)}$	$R1_4^{(8)}$	$R1_5^{(8)}$	$R1_6^{(8)}$	$R1_7^{(8)}$	$R1_8^{(8)}$
$R1_1^{(8)}$	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$R1_1^{(8)}$	<b>0.0</b>	0.0	+0.5	+0.5	0.0	0.0	-0.5	-0.5
$R1_2^{(8)}$	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	$R1_2^{(8)}$	0.0	<b>0.0</b>	-0.5	-0.5	0.0	0.0	+0.5	+0.5
$R1_3^{(8)}$	0.0	+1.0	<b>0.0</b>	0.0	0.0	+1.0	0.0	0.0	$R1_3^{(8)}$	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0
$R1_4^{(8)}$	+1.0	0.0	0.0	<b>0.0</b>	+1.0	0.0	0.0	0.0	$R1_4^{(8)}$	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0
$R1_5^{(8)}$	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	$R1_5^{(8)}$	0.0	0.0	+0.5	+0.5	<b>0.0</b>	0.0	-0.5	-0.5
$R1_6^{(8)}$	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	$R1_6^{(8)}$	0.0	0.0	-0.5	-0.5	0.0	<b>0.0</b>	+0.5	+0.5
$R1_7^{(8)}$	0.0	-1.0	0.0	0.0	0.0	-1.0	<b>0.0</b>	0.0	$R1_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0
$R1_8^{(8)}$	-1.0	0.0	0.0	0.0	-1.0	0.0	0.0	<b>0.0</b>	$R1_8^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>



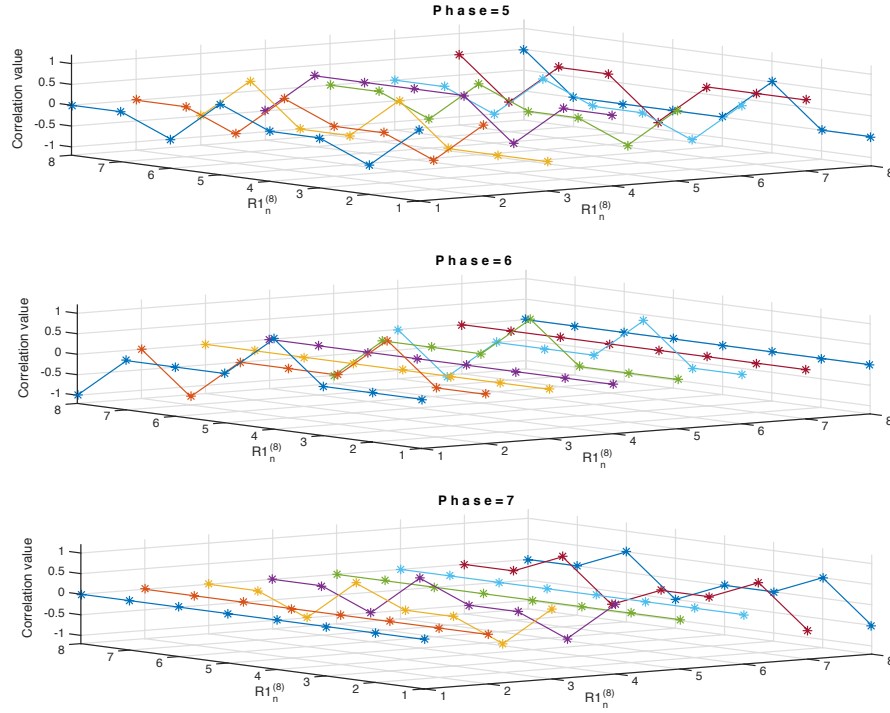
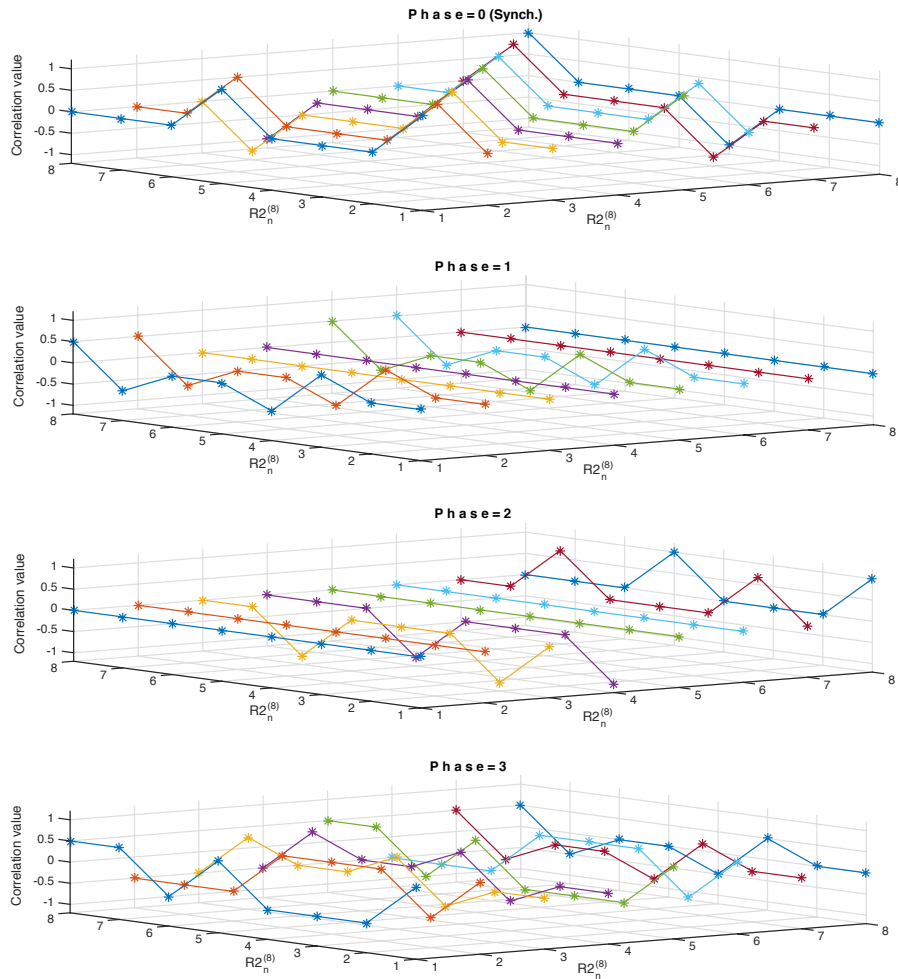


Figure 4.15: Correlation matrices for  $R1_n^{(8)}$ : Phases.

+ Infra-group correlation:  $R2_n^{(4)}$

$\phi = 0$	$R2_1^{(8)}$ $R2_2^{(8)}$ $R2_3^{(8)}$ $R2_4^{(8)}$ $R2_5^{(8)}$ $R2_6^{(8)}$ $R2_7^{(8)}$ $R2_8^{(8)}$	$\phi = 1$	$R2_1^{(8)}$ $R2_2^{(8)}$ $R2_3^{(8)}$ $R2_4^{(8)}$ $R2_5^{(8)}$ $R2_6^{(8)}$ $R2_7^{(8)}$ $R2_8^{(8)}$
$R2_1^{(8)}$	<b>+1.0</b> 0.0 0.0 0.0 +1.0 0.0 0.0 0.0	$R2_1^{(8)}$	<b>0.0</b> 0.0 0.0 0.0 0.0 0.0 0.0 0.0
$R2_2^{(8)}$	0.0 <b>+1.0</b> 0.0 0.0 0.0 +1.0 0.0 0.0	$R2_2^{(8)}$	0.0 <b>0.0</b> 0.0 0.0 0.0 0.0 0.0 0.0
$R2_3^{(8)}$	0.0 0.0 <b>+1.0</b> 0.0 0.0 0.0 -1.0 0.0	$R2_3^{(8)}$	+0.5 +0.5 <b>0.0</b> 0.0 +0.5 +0.5 0.0 0.0
$R2_4^{(8)}$	0.0 0.0 0.0 <b>+1.0</b> 0.0 0.0 0.0 -1.0	$R2_4^{(8)}$	-0.5 -0.5 0.0 <b>0.0</b> -0.5 -0.5 0.0 0.0
$R2_5^{(8)}$	+1.0 0.0 0.0 0.0 <b>+1.0</b> 0.0 0.0 0.0	$R2_5^{(8)}$	0.0 0.0 0.0 0.0 <b>0.0</b> 0.0 0.0 0.0
$R2_6^{(8)}$	0.0 +1.0 0.0 0.0 0.0 <b>+1.0</b> 0.0 0.0	$R2_6^{(8)}$	0.0 0.0 0.0 0.0 0.0 <b>0.0</b> 0.0 0.0
$R2_7^{(8)}$	0.0 0.0 -1.0 0.0 0.0 0.0 <b>+1.0</b> 0.0	$R2_7^{(8)}$	-0.5 -0.5 0.0 0.0 -0.5 -0.5 <b>0.0</b> 0.0
$R2_8^{(8)}$	0.0 0.0 0.0 -1.0 0.0 0.0 0.0 <b>+1.0</b>	$R2_8^{(8)}$	+0.5 +0.5 0.0 0.0 +0.5 +0.5 0.0 <b>0.0</b>
$\phi = 2$	$R2_1^{(8)}$ $R2_2^{(8)}$ $R2_3^{(8)}$ $R2_4^{(8)}$ $R2_5^{(8)}$ $R2_6^{(8)}$ $R2_7^{(8)}$ $R2_8^{(8)}$	$\phi = 3$	$R2_1^{(8)}$ $R2_2^{(8)}$ $R2_3^{(8)}$ $R2_4^{(8)}$ $R2_5^{(8)}$ $R2_6^{(8)}$ $R2_7^{(8)}$ $R2_8^{(8)}$
$R2_1^{(8)}$	<b>0.0</b> 0.0 0.0 -1.0 0.0 0.0 0.0 +1.0	$R2_1^{(8)}$	<b>+0.5</b> +0.5 0.0 0.0 +0.5 +0.5 0.0 0.0
$R2_2^{(8)}$	0.0 <b>0.0</b> -1.0 0.0 0.0 0.0 +1.0 0.0	$R2_2^{(8)}$	-0.5 <b>-0.5</b> 0.0 0.0 -0.5 -0.5 0.0 0.0
$R2_3^{(8)}$	0.0 0.0 <b>0.0</b> 0.0 0.0 0.0 0.0 0.0	$R2_3^{(8)}$	-0.5 +0.5 <b>-0.5</b> -0.5 -0.5 +0.5 +0.5 +0.5
$R2_4^{(8)}$	0.0 0.0 0.0 <b>0.0</b> 0.0 0.0 0.0 0.0	$R2_4^{(8)}$	-0.5 +0.5 +0.5 <b>+0.5</b> -0.5 +0.5 -0.5 -0.5
$R2_5^{(8)}$	0.0 0.0 0.0 -1.0 <b>0.0</b> 0.0 0.0 +1.0	$R2_5^{(8)}$	+0.5 +0.5 0.0 0.0 <b>+0.5</b> +0.5 0.0 0.0
$R2_6^{(8)}$	0.0 0.0 -1.0 0.0 0.0 <b>0.0</b> +1.0 0.0	$R2_6^{(8)}$	-0.5 -0.5 0.0 0.0 -0.5 <b>-0.5</b> 0.0 0.0
$R2_7^{(8)}$	0.0 0.0 0.0 0.0 0.0 0.0 <b>0.0</b> 0.0	$R2_7^{(8)}$	+0.5 -0.5 +0.5 +0.5 +0.5 -0.5 <b>-0.5</b> -0.5
$R2_8^{(8)}$	0.0 0.0 0.0 0.0 0.0 0.0 0.0 <b>0.0</b>	$R2_8^{(8)}$	+0.5 -0.5 -0.5 -0.5 +0.5 -0.5 +0.5 <b>+0.5</b>

$\phi = 4$	$R2_1^{(8)}$	$R2_2^{(8)}$	$R2_3^{(8)}$	$R2_4^{(8)}$	$R2_5^{(8)}$	$R2_6^{(8)}$	$R2_7^{(8)}$	$R2_8^{(8)}$	$\phi = 5$	$R2_1^{(8)}$	$R2_2^{(8)}$	$R2_3^{(8)}$	$R2_4^{(8)}$	$R2_5^{(8)}$	$R2_6^{(8)}$	$R2_7^{(8)}$	$R2_8^{(8)}$
$R2_1^{(8)}$	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$R2_1^{(8)}$	<b>+0.5</b>	-0.5	-0.5	-0.5	+0.5	-0.5	+0.5	+0.5
$R2_2^{(8)}$	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	$R2_2^{(8)}$	+0.5	<b>-0.5</b>	+0.5	+0.5	+0.5	-0.5	-0.5	-0.5
$R2_3^{(8)}$	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	$R2_3^{(8)}$	0.0	0.0	<b>-0.5</b>	+0.5	0.0	0.0	+0.5	-0.5
$R2_4^{(8)}$	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	$R2_4^{(8)}$	0.0	0.0	-0.5	<b>+0.5</b>	0.0	0.0	+0.5	-0.5
$R2_5^{(8)}$	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	$R2_5^{(8)}$	+0.5	-0.5	-0.5	-0.5	<b>+0.5</b>	-0.5	+0.5	+0.5
$R2_6^{(8)}$	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	$R2_6^{(8)}$	+0.5	-0.5	+0.5	+0.5	+0.5	<b>-0.5</b>	-0.5	-0.5
$R2_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	$R2_7^{(8)}$	0.0	0.0	+0.5	-0.5	0.0	0.0	<b>-0.5</b>	+0.5
$R2_8^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	$R2_8^{(8)}$	0.0	0.0	+0.5	-0.5	0.0	0.0	-0.5	<b>+0.5</b>
$\phi = 6$	$R2_1^{(8)}$	$R2_2^{(8)}$	$R2_3^{(8)}$	$R2_4^{(8)}$	$R2_5^{(8)}$	$R2_6^{(8)}$	$R2_7^{(8)}$	$R2_8^{(8)}$	$\phi = 7$	$R2_1^{(8)}$	$R2_2^{(8)}$	$R2_3^{(8)}$	$R2_4^{(8)}$	$R2_5^{(8)}$	$R2_6^{(8)}$	$R2_7^{(8)}$	$R2_8^{(8)}$
$R2_1^{(8)}$	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$R2_1^{(8)}$	<b>0.0</b>	0.0	+0.5	-0.5	0.0	0.0	-0.5	+0.5
$R2_2^{(8)}$	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0	0.0	$R2_2^{(8)}$	0.0	<b>0.0</b>	+0.5	-0.5	0.0	0.0	-0.5	+0.5
$R2_3^{(8)}$	0.0	-1.0	<b>0.0</b>	0.0	0.0	-1.0	0.0	0.0	$R2_3^{(8)}$	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0	0.0
$R2_4^{(8)}$	-1.0	0.0	0.0	<b>0.0</b>	-1.0	0.0	0.0	0.0	$R2_4^{(8)}$	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	0.0
$R2_5^{(8)}$	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	0.0	$R2_5^{(8)}$	0.0	0.0	+0.5	-0.5	<b>0.0</b>	0.0	-0.5	+0.5
$R2_6^{(8)}$	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0	0.0	$R2_6^{(8)}$	0.0	0.0	+0.5	-0.5	0.0	<b>0.0</b>	-0.5	+0.5
$R2_7^{(8)}$	0.0	+1.0	0.0	0.0	0.0	+1.0	<b>0.0</b>	0.0	$R2_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>	0.0
$R2_8^{(8)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	<b>0.0</b>	$R2_8^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	<b>0.0</b>



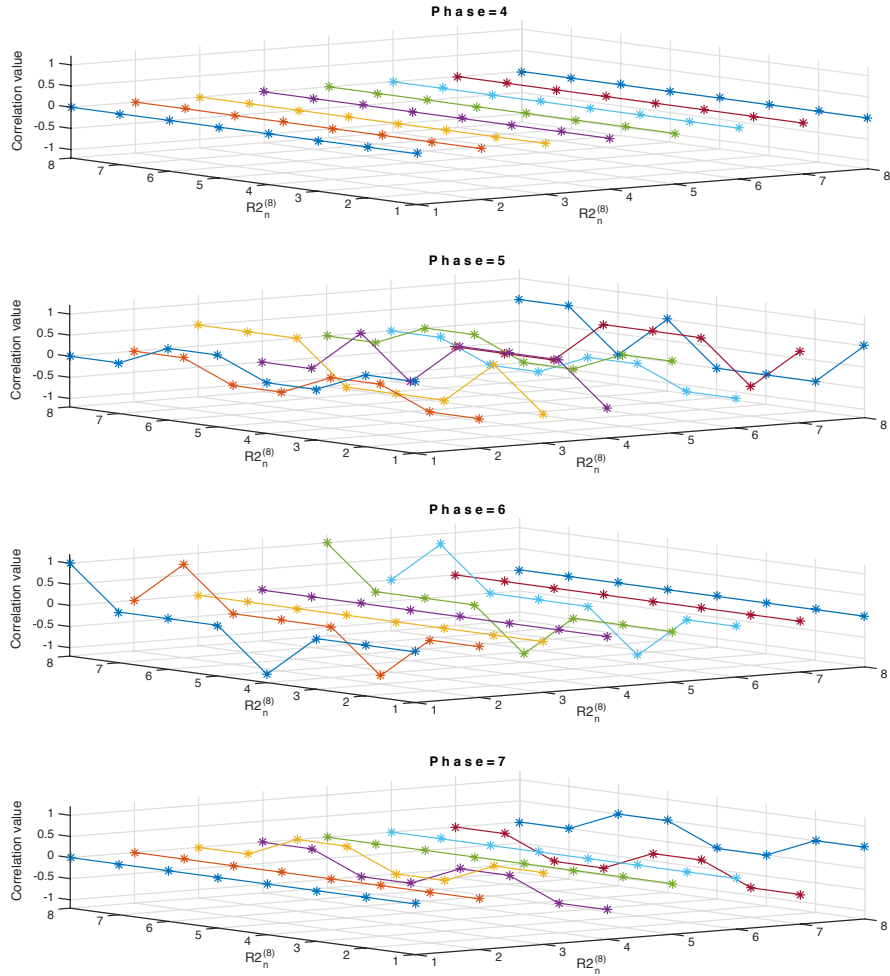
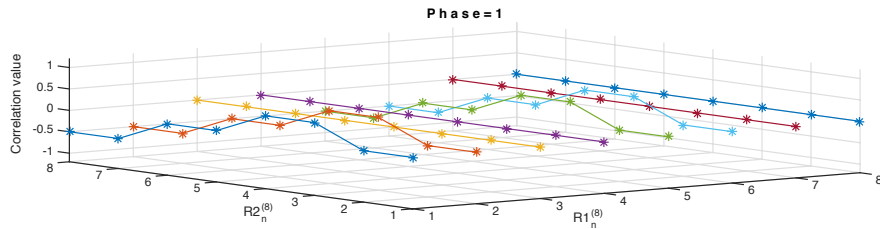
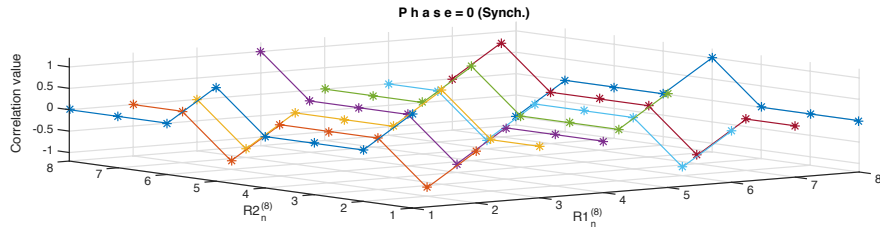


Figure 4.16: Correlation matrices for  $R2_n^{(8)}$ : Phases.

+ *Inter-group correlation:  $R1_n^{(4)}$  vs  $R2_n^{(4)}$*

$\phi = 0$	$R2_1^{(8)}$	$R2_2^{(8)}$	$R2_3^{(8)}$	$R2_4^{(8)}$	$R2_5^{(8)}$	$R2_6^{(8)}$	$R2_7^{(8)}$	$R2_8^{(8)}$	$\phi = 1$	$R2_1^{(8)}$	$R2_2^{(8)}$	$R2_3^{(8)}$	$R2_4^{(8)}$	$R2_5^{(8)}$	$R2_6^{(8)}$	$R2_7^{(8)}$	$R2_8^{(8)}$
$R1_1^{(8)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$R1_1^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$R1_2^{(8)}$	0.0	-1.0	0.0	0.0	0.0	-1.0	0.0	0.0	$R1_2^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$R1_3^{(8)}$	0.0	0.0	+1.0	0.0	0.0	0.0	-1.0	0.0	$R1_3^{(8)}$	+0.5	+0.5	0.0	0.0	+0.5	+0.5	0.0	0.0
$R1_4^{(8)}$	0.0	0.0	0.0	-1.0	0.0	0.0	0.0	+1.0	$R1_4^{(8)}$	+0.5	+0.5	0.0	0.0	+0.5	+0.5	0.0	0.0
$R1_5^{(8)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$R1_5^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$R1_6^{(8)}$	0.0	-1.0	0.0	0.0	0.0	-1.0	0.0	0.0	$R1_6^{(8)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$R1_7^{(8)}$	0.0	0.0	-1.0	0.0	0.0	0.0	+1.0	0.0	$R1_7^{(8)}$	-0.5	-0.5	0.0	0.0	-0.5	-0.5	0.0	0.0
$R1_8^{(8)}$	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	-1.0	$R1_8^{(8)}$	-0.5	-0.5	0.0	0.0	-0.5	-0.5	0.0	0.0

$\phi = 2$	$R2_1^{(8)} R2_2^{(8)} R2_3^{(8)} R2_4^{(8)} R2_5^{(8)} R2_6^{(8)} R2_7^{(8)} R2_8^{(8)}$	$\phi = 3$	$R2_1^{(8)} R2_2^{(8)} R2_3^{(8)} R2_4^{(8)} R2_5^{(8)} R2_6^{(8)} R2_7^{(8)} R2_8^{(8)}$
$R1_1^{(8)}$	<b>0.0</b> 0.0 0.0 -1.0 0.0 0.0 0.0 +1.0	$R1_1^{(8)}$	<b>+0.5</b> +0.5 0.0 0.0 +0.5 +0.5 0.0 0.0
$R1_2^{(8)}$	0.0 <b>0.0</b> +1.0 0.0 0.0 0.0 -1.0 0.0	$R1_2^{(8)}$	+0.5 <b>+0.5</b> 0.0 0.0 +0.5 +0.5 0.0 0.0
$R1_3^{(8)}$	0.0 0.0 <b>0.0</b> 0.0 0.0 0.0 0.0 0.0	$R1_3^{(8)}$	-0.5 +0.5 <b>-0.5</b> -0.5 -0.5 +0.5 +0.5 +0.5
$R1_4^{(8)}$	0.0 0.0 0.0 <b>0.0</b> 0.0 0.0 0.0 0.0	$R1_4^{(8)}$	+0.5 -0.5 -0.5 <b>-0.5</b> +0.5 -0.5 +0.5 +0.5
$R1_5^{(8)}$	0.0 0.0 0.0 -1.0 <b>0.0</b> 0.0 0.0 +1.0	$R1_5^{(8)}$	+0.5 +0.5 0.0 0.0 <b>+0.5</b> +0.5 0.0 0.0
$R1_6^{(8)}$	0.0 0.0 +1.0 0.0 0.0 <b>0.0</b> -1.0 0.0	$R1_6^{(8)}$	+0.5 +0.5 0.0 0.0 +0.5 <b>+0.5</b> 0.0 0.0
$R1_7^{(8)}$	0.0 0.0 0.0 0.0 0.0 0.0 <b>0.0</b> 0.0	$R1_7^{(8)}$	+0.5 -0.5 +0.5 +0.5 +0.5 -0.5 <b>-0.5</b> -0.5
$R1_8^{(8)}$	0.0 0.0 0.0 0.0 0.0 0.0 0.0 <b>0.0</b>	$R1_8^{(8)}$	-0.5 +0.5 +0.5 +0.5 -0.5 +0.5 -0.5 <b>-0.5</b>
$\phi = 4$	$R2_1^{(8)} R2_2^{(8)} R2_3^{(8)} R2_4^{(8)} R2_5^{(8)} R2_6^{(8)} R2_7^{(8)} R2_8^{(8)}$	$\phi = 5$	$R2_1^{(8)} R2_2^{(8)} R2_3^{(8)} R2_4^{(8)} R2_5^{(8)} R2_6^{(8)} R2_7^{(8)} R2_8^{(8)}$
$R1_1^{(8)}$	<b>0.0</b> 0.0 0.0 0.0 0.0 0.0 0.0 0.0	$R1_1^{(8)}$	<b>+0.5</b> -0.5 -0.5 -0.5 +0.5 -0.5 +0.5 +0.5
$R1_2^{(8)}$	0.0 <b>0.0</b> 0.0 0.0 0.0 0.0 0.0 0.0	$R1_2^{(8)}$	-0.5 <b>+0.5</b> -0.5 -0.5 -0.5 +0.5 +0.5 +0.5
$R1_3^{(8)}$	0.0 0.0 <b>0.0</b> 0.0 0.0 0.0 0.0 0.0	$R1_3^{(8)}$	0.0 0.0 <b>-0.5</b> +0.5 0.0 0.0 +0.5 -0.5
$R1_4^{(8)}$	0.0 0.0 0.0 <b>0.0</b> 0.0 0.0 0.0 0.0	$R1_4^{(8)}$	0.0 0.0 +0.5 <b>-0.5</b> 0.0 0.0 -0.5 +0.5
$R1_5^{(8)}$	0.0 0.0 0.0 0.0 <b>0.0</b> 0.0 0.0 0.0	$R1_5^{(8)}$	+0.5 -0.5 -0.5 -0.5 <b>+0.5</b> -0.5 +0.5 +0.5
$R1_6^{(8)}$	0.0 0.0 0.0 0.0 0.0 <b>0.0</b> 0.0 0.0	$R1_6^{(8)}$	-0.5 +0.5 -0.5 -0.5 -0.5 <b>+0.5</b> +0.5 +0.5
$R1_7^{(8)}$	0.0 0.0 0.0 0.0 0.0 0.0 <b>0.0</b> 0.0	$R1_7^{(8)}$	0.0 0.0 +0.5 -0.5 0.0 0.0 <b>-0.5</b> +0.5
$R1_8^{(8)}$	0.0 0.0 0.0 0.0 0.0 0.0 0.0 <b>0.0</b>	$R1_8^{(8)}$	0.0 0.0 -0.5 +0.5 0.0 0.0 +0.5 <b>-0.5</b>
$\phi = 6$	$R2_1^{(8)} R2_2^{(8)} R2_3^{(8)} R2_4^{(8)} R2_5^{(8)} R2_6^{(8)} R2_7^{(8)} R2_8^{(8)}$	$\phi = 7$	$R2_1^{(8)} R2_2^{(8)} R2_3^{(8)} R2_4^{(8)} R2_5^{(8)} R2_6^{(8)} R2_7^{(8)} R2_8^{(8)}$
$R1_1^{(8)}$	<b>0.0</b> 0.0 0.0 0.0 0.0 0.0 0.0 0.0	$R1_1^{(8)}$	<b>0.0</b> 0.0 +0.5 -0.5 0.0 0.0 -0.5 +0.5
$R1_2^{(8)}$	0.0 <b>0.0</b> 0.0 0.0 0.0 0.0 0.0 0.0	$R1_2^{(8)}$	0.0 <b>0.0</b> -0.5 +0.5 0.0 0.0 +0.5 -0.5
$R1_3^{(8)}$	0.0 -1.0 <b>0.0</b> 0.0 0.0 -1.0 0.0 0.0	$R1_3^{(8)}$	0.0 0.0 <b>0.0</b> 0.0 0.0 0.0 0.0 0.0
$R1_4^{(8)}$	+1.0 0.0 0.0 <b>0.0</b> +1.0 0.0 0.0 0.0	$R1_4^{(8)}$	0.0 0.0 0.0 <b>0.0</b> 0.0 0.0 0.0 0.0
$R1_5^{(8)}$	0.0 0.0 0.0 0.0 <b>0.0</b> 0.0 0.0 0.0	$R1_5^{(8)}$	0.0 0.0 +0.5 -0.5 <b>0.0</b> 0.0 -0.5 +0.5
$R1_6^{(8)}$	0.0 0.0 0.0 0.0 0.0 <b>0.0</b> 0.0 0.0	$R1_6^{(8)}$	0.0 0.0 -0.5 +0.5 0.0 <b>0.0</b> +0.5 -0.5
$R1_7^{(8)}$	0.0 +1.0 0.0 0.0 0.0 +1.0 <b>0.0</b> 0.0	$R1_7^{(8)}$	0.0 0.0 0.0 0.0 0.0 0.0 <b>0.0</b> 0.0
$R1_8^{(8)}$	-1.0 0.0 0.0 0.0 -1.0 0.0 0.0 <b>0.0</b>	$R1_8^{(8)}$	0.0 0.0 0.0 0.0 0.0 0.0 0.0 <b>0.0</b>



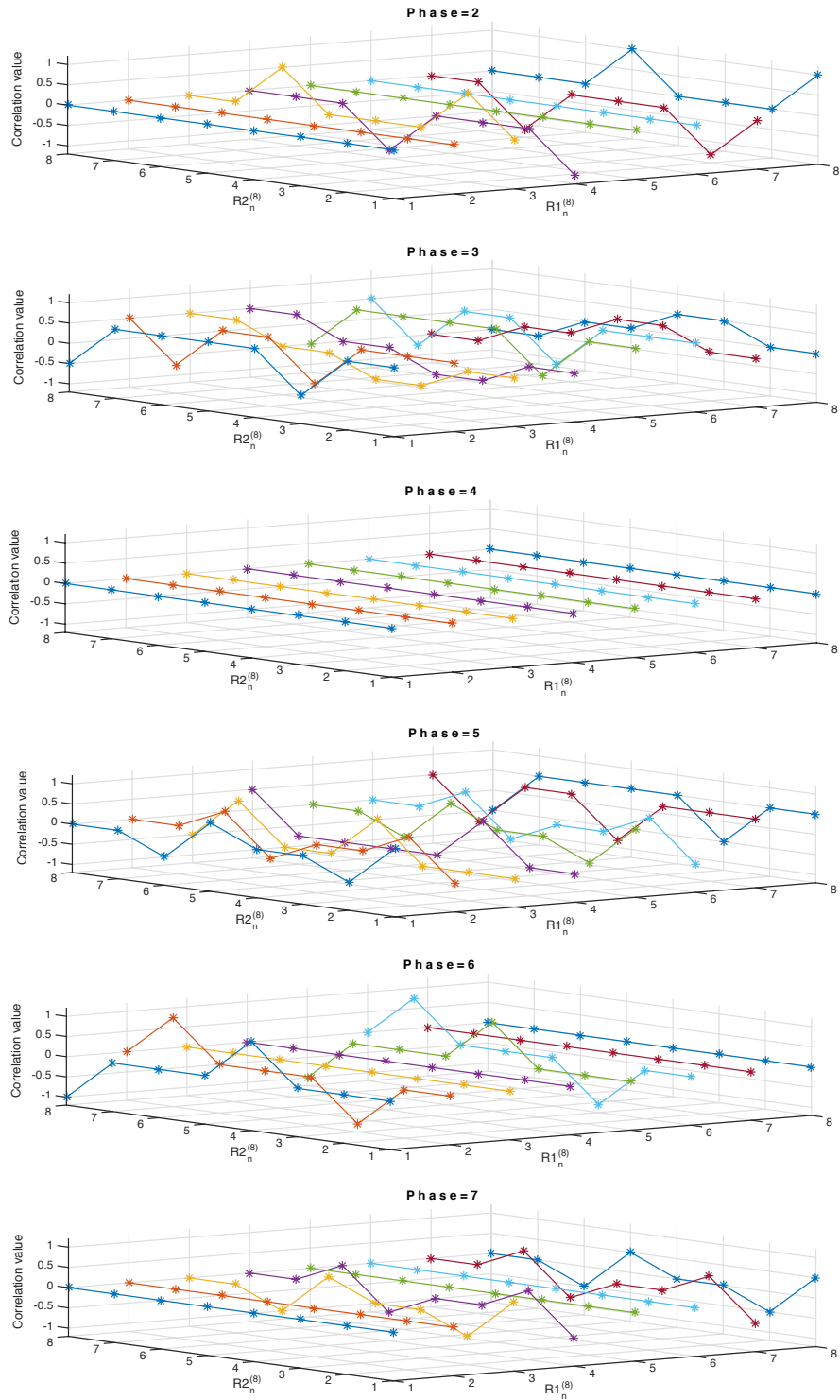


Figure 4.17: Correlation matrices for  $R_1_n^{(8)}$  vs  $R_2_n^{(8)}$ : Phases.





$i$	$j$	$\phi$								$i$	$j$	$\phi$							
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$R1_1^{(8)}$	$R2_1^{(8)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$R1_3^{(8)}$	$R2_5^{(8)}$	0.0	+0.5	0.0	+0.5	0.0	0.0	-1.0	0.0
$R1_1^{(8)}$	$R2_2^{(8)}$	0.0	0.0	0.0	-0.5	0.0	+0.5	0.0	0.0	$R1_3^{(8)}$	$R2_6^{(8)}$	-1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0
$R1_1^{(8)}$	$R2_3^{(8)}$	0.0	0.0	0.0	0.0	0.0	-0.5	0.0	+0.5	$R1_3^{(8)}$	$R2_7^{(8)}$	0.0	0.0	0.0	+0.5	0.0	-0.5	0.0	0.0
$R1_1^{(8)}$	$R2_4^{(8)}$	0.0	0.0	-1.0	0.0	0.0	-0.5	0.0	-0.5	$R1_4^{(8)}$	$R2_8^{(8)}$	-1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$R1_1^{(8)}$	$R2_5^{(8)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$R1_4^{(8)}$	$R2_5^{(8)}$	0.0	+0.5	0.0	+0.5	0.0	0.0	+1.0	0.0
$R1_1^{(8)}$	$R2_6^{(8)}$	0.0	0.0	0.0	+0.5	0.0	-0.5	0.0	0.0	$R1_4^{(8)}$	$R2_6^{(8)}$	0.0	+0.5	0.0	-0.5	0.0	0.0	0.0	0.0
$R1_1^{(8)}$	$R2_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	+0.5	0.0	-0.5	$R1_4^{(8)}$	$R2_7^{(8)}$	0.0	0.0	0.0	+0.5	0.0	-0.5	0.0	0.0
$R1_1^{(8)}$	$R2_8^{(8)}$	0.0	0.0	+1.0	0.0	0.0	+0.5	0.0	+0.5	$R1_4^{(8)}$	$R2_8^{(8)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0
$i$	$j$	$\phi$								$i$	$j$	$\phi$							
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$R1_2^{(8)}$	$R2_2^{(8)}$	-1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$R1_5^{(8)}$	$R2_5^{(8)}$	+1.0	0.0	+0.5	0.0	+0.5	0.0	0.0	0.0
$R1_2^{(8)}$	$R2_3^{(8)}$	0.0	0.0	+1.0	0.0	0.0	-0.5	0.0	-0.5	$R1_5^{(8)}$	$R2_6^{(8)}$	0.0	0.0	0.0	+0.5	0.0	-0.5	0.0	0.0
$R1_2^{(8)}$	$R2_4^{(8)}$	0.0	0.0	0.0	0.0	0.0	-0.5	0.0	+0.5	$R1_5^{(8)}$	$R2_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	+0.5	0.0	-0.5
$R1_2^{(8)}$	$R2_5^{(8)}$	0.0	0.0	0.0	+0.5	0.0	-0.5	0.0	0.0	$R1_5^{(8)}$	$R2_8^{(8)}$	0.0	0.0	+1.0	0.0	0.0	+0.5	0.0	+0.5
$R1_2^{(8)}$	$R2_6^{(8)}$	-1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$R1_6^{(8)}$	$R2_5^{(8)}$	-1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0
$R1_2^{(8)}$	$R2_7^{(8)}$	0.0	0.0	-1.0	0.0	0.0	+0.5	0.0	+0.5	$R1_6^{(8)}$	$R2_6^{(8)}$	0.0	0.0	-1.0	0.0	0.0	+0.5	0.0	+0.5
$R1_2^{(8)}$	$R2_8^{(8)}$	0.0	0.0	0.0	0.0	0.0	+0.5	0.0	-0.5	$R1_6^{(8)}$	$R2_7^{(8)}$	0.0	0.0	0.0	0.0	0.0	+0.5	0.0	-0.5
$R1_3^{(8)}$	$R2_3^{(8)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0	$R1_7^{(8)}$	$R2_8^{(8)}$	+1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$R1_3^{(8)}$	$R2_4^{(8)}$	0.0	0.0	0.0	-0.5	0.0	+0.5	0.0	0.0	$R1_7^{(8)}$	$R2_5^{(8)}$	0.0	0.0	0.0	-0.5	0.0	+0.5	0.0	0.0
$R1_3^{(8)}$	$R2_5^{(8)}$	0.0	+0.5	0.0	-0.5	0.0	0.0	0.0	0.0	$R1_8^{(8)}$	$R2_8^{(8)}$	-1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0

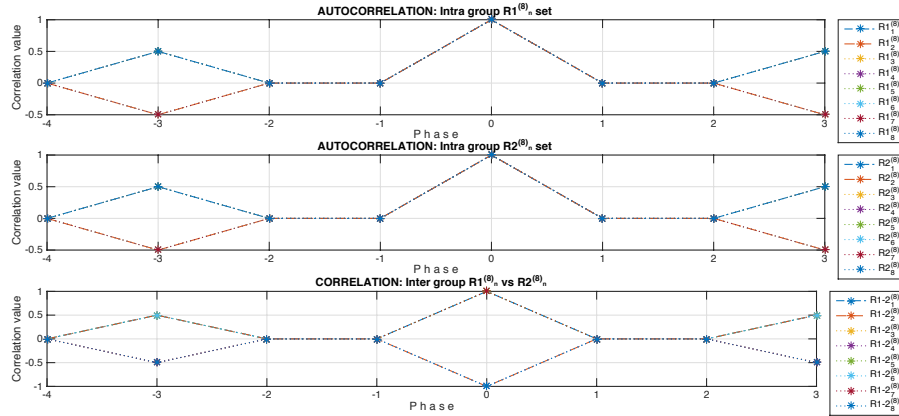


Figure 4.18: 2-D Side-lobe graph: ACF Intra group for  $R1_n^{(8)} - R2_n^{(8)}$ , and CCF Inter group for  $R1_n^{(8)}$  vs  $R2_n^{(8)}$ .

### 4.3.3 Interference-Free Window sets

They are considered Inter Group codes (IGC) and Zero Correlation Zone (ZCZ) sets to explore their properties.

#### Inter Group code sets

Considering two flocks with  $M = 4$ ,  $N = 8$ ,  $W_{min} = 6$  from [31], so  $Ix_n^{(4)}$  sets are defined as

$$\begin{aligned}
 I1_1^{(4)} &= (+1, +1, +1, -1, +1, +1, +1, -1); & I2_1^{(4)} &= (+1, +1, +1, -1, -1, -1, -1, +1); \\
 I1_2^{(4)} &= (+1, +1, -1, +1, +1, +1, -1, +1); & I2_2^{(4)} &= (+1, +1, -1, +1, -1, -1, +1, -1); \\
 I1_3^{(4)} &= (+1, +1, +1, -1, +1, +1, +1, -1); & I2_3^{(4)} &= (+1, +1, +1, -1, -1, -1, -1, +1); \\
 I1_4^{(4)} &= (-1, -1, +1, -1, -1, -1, +1, -1); & I2_4^{(4)} &= (-1, -1, +1, -1, +1, +1, -1, +1);
 \end{aligned}$$

★ **BALANCE PROPERTY**

Table 4.7 verifies this property considering the flocks  $I1_n^{(4)}$  and  $I2_n^{(4)}$

Table 4.7: Balance: Inter Group code sets

$I1_n^{(4)}$	Bal	meet?	$I1_n^{(4)}$	Bal	meet?	$I2_n^{(4)}$	Bal	meet?	$I2_n^{(4)}$	Bal	meet?
$I1_1^{(4)}$	4	NO	$I1_3^{(4)}$	4	NO	$I2_1^{(4)}$	0	YES	$I2_3^{(4)}$	0	YES
$I1_2^{(4)}$	4	NO	$I1_4^{(4)}$	-4	NO	$I2_2^{(4)}$	0	YES	$I2_4^{(4)}$	0	YES

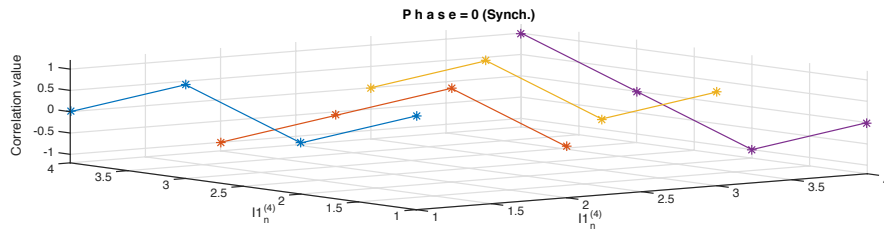
★ **CORRELATION PROPERTIES**

● **Autocorrelation and cross-correlation**

Recall that Inter Group codes are complementary sets, so they are also described through *flocks*. Hence, they are studied considering inter-group and infra-group correlation.

+ Infra-group correlation:  $I1_n^{(4)}$

$\phi = 0$	$I1_1^{(4)}$	$I1_2^{(4)}$	$I1_3^{(4)}$	$I1_4^{(4)}$	$\phi = 1$	$I1_1^{(4)}$	$I1_2^{(4)}$	$I1_3^{(4)}$	$I1_4^{(4)}$	$\phi = 2$	$I1_1^{(4)}$	$I1_2^{(4)}$	$I1_3^{(4)}$	$I1_4^{(4)}$
$I1_1^{(4)}$	+1.0	0.0	+1.0	0.0	$I1_1^{(4)}$	0.0	+1.0	0.0	-1.0	$I1_1^{(4)}$	0.0	0.0	0.0	0.0
$I1_2^{(4)}$	0.0	+1.0	0.0	-1.0	$I1_2^{(4)}$	0.0	0.0	0.0	0.0	$I1_2^{(4)}$	0.0	0.0	0.0	0.0
$I1_3^{(4)}$	+1.0	0.0	+1.0	0.0	$I1_3^{(4)}$	0.0	+1.0	0.0	-1.0	$I1_3^{(4)}$	0.0	0.0	0.0	0.0
$I1_4^{(4)}$	0.0	-1.0	0.0	+1.0	$I1_4^{(4)}$	0.0	0.0	0.0	0.0	$I1_4^{(4)}$	0.0	0.0	0.0	0.0
$\phi = 3$	$I1_1^{(4)}$	$I1_2^{(4)}$	$I1_3^{(4)}$	$I1_4^{(4)}$	$\phi = 4$	$I1_1^{(4)}$	$I1_2^{(4)}$	$I1_3^{(4)}$	$I1_4^{(4)}$	$\phi = 5$	$I1_1^{(4)}$	$I1_2^{(4)}$	$I1_3^{(4)}$	$I1_4^{(4)}$
$I1_1^{(4)}$	0.0	0.0	0.0	0.0	$I1_1^{(4)}$	+1.0	0.0	+1.0	0.0	$I1_1^{(4)}$	0.0	+1.0	0.0	-1.0
$I1_2^{(4)}$	+1.0	0.0	+1.0	0.0	$I1_2^{(4)}$	0.0	+1.0	0.0	-1.0	$I1_2^{(4)}$	0.0	0.0	0.0	0.0
$I1_3^{(4)}$	0.0	0.0	0.0	0.0	$I1_3^{(4)}$	+1.0	0.0	+1.0	0.0	$I1_3^{(4)}$	0.0	+1.0	0.0	-1.0
$I1_4^{(4)}$	-1.0	0.0	-1.0	0.0	$I1_4^{(4)}$	0.0	-1.0	0.0	+1.0	$I1_4^{(4)}$	0.0	0.0	0.0	0.0
$\phi = 6$	$I1_1^{(4)}$	$I1_2^{(4)}$	$I1_3^{(4)}$	$I1_4^{(4)}$	$\phi = 7$	$I1_1^{(4)}$	$I1_2^{(4)}$	$I1_3^{(4)}$	$I1_4^{(4)}$					
$I1_1^{(4)}$	0.0	0.0	0.0	0.0	$I1_1^{(4)}$	0.0	0.0	0.0	0.0					
$I1_2^{(4)}$	0.0	0.0	0.0	0.0	$I1_2^{(4)}$	+1.0	0.0	+1.0	0.0					
$I1_3^{(4)}$	0.0	0.0	0.0	0.0	$I1_3^{(4)}$	0.0	0.0	0.0	0.0					
$I1_4^{(4)}$	0.0	0.0	0.0	0.0	$I1_4^{(4)}$	-1.0	0.0	-1.0	0.0					



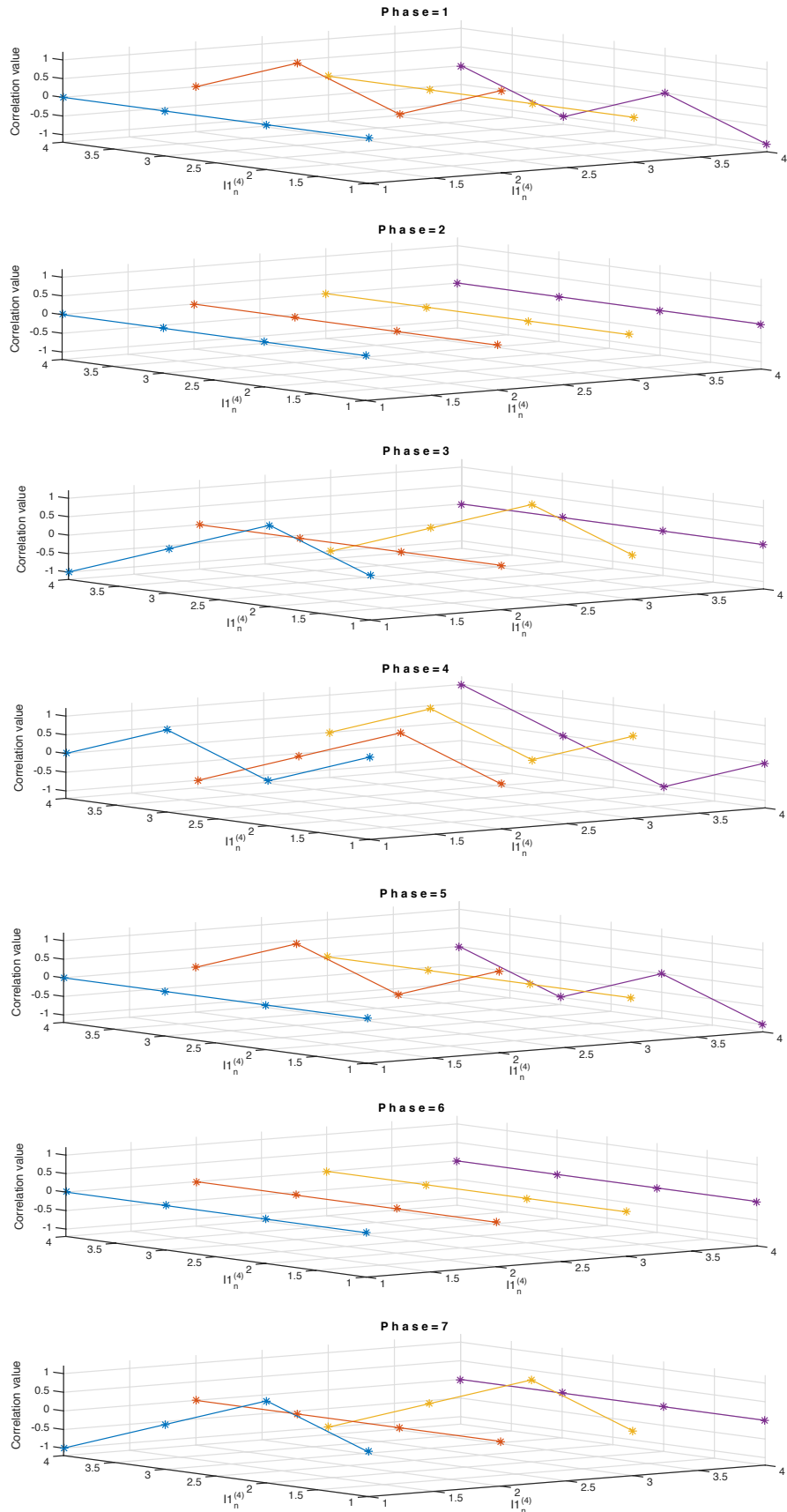
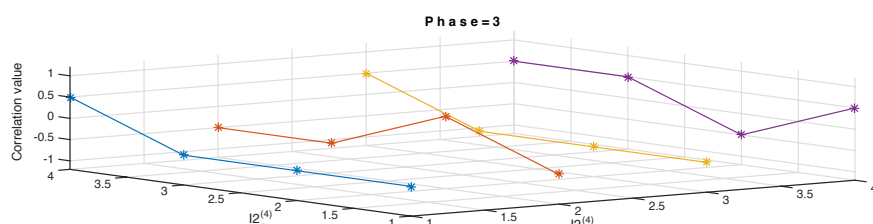
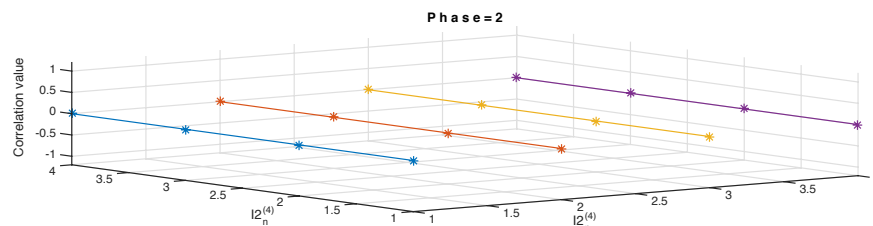
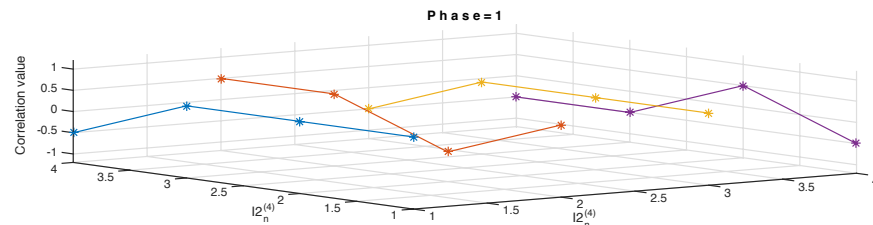
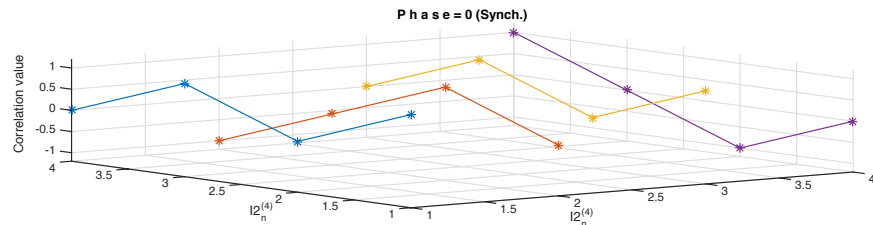


Figure 4.19: Correlation matrices for  $I1_n^{(4)}$ : Phases.

+ Infra-group correlation:  $I2_n^{(4)}$

$\phi = 0$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_1^{(3)}$	$I2_4^{(4)}$	$\phi = 1$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$	$\phi = 2$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$
$I2_1^{(4)}$	+1.0	0.0	+1.0	0.0	$I2_1^{(4)}$	+0.5	+0.5	+0.5	-0.5	$I2_1^{(4)}$	0.0	0.0	0.0	0.0
$I2_2^{(4)}$	0.0	+1.0	0.0	-1.0	$I2_2^{(4)}$	+0.5	-0.5	+0.5	+0.5	$I2_2^{(4)}$	0.0	0.0	0.0	0.0
$I2_3^{(4)}$	+1.0	0.0	+1.0	0.0	$I2_3^{(4)}$	+0.5	+0.5	+0.5	-0.5	$I2_3^{(4)}$	0.0	0.0	0.0	0.0
$I2_4^{(4)}$	0.0	-1.0	0.0	+1.0	$I2_4^{(4)}$	-0.5	+0.5	-0.5	-0.5	$I2_4^{(4)}$	0.0	0.0	0.0	0.0
$\phi = 3$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_1^{(3)}$	$I2_4^{(4)}$	$\phi = 4$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$	$\phi = 5$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$
$I2_1^{(4)}$	-0.5	-0.5	-0.5	+0.5	$I2_1^{(4)}$	-1.0	0.0	-1.0	0.0	$I2_1^{(4)}$	-0.5	-0.5	-0.5	+0.5
$I2_2^{(4)}$	-0.5	+0.5	-0.5	-0.5	$I2_2^{(4)}$	0.0	-1.0	0.0	+1.0	$I2_2^{(4)}$	-0.5	+0.5	-0.5	-0.5
$I2_3^{(4)}$	-0.5	-0.5	-0.5	+0.5	$I2_3^{(4)}$	-1.0	0.0	-1.0	0.0	$I2_3^{(4)}$	-0.5	-0.5	-0.5	+0.5
$I2_4^{(4)}$	+0.5	-0.5	+0.5	+0.5	$I2_4^{(4)}$	0.0	+1.0	0.0	-1.0	$I2_4^{(4)}$	+0.5	-0.5	+0.5	+0.5
$\phi = 6$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_1^{(3)}$	$I2_4^{(4)}$	$\phi = 7$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$					
$I2_1^{(4)}$	0.0	0.0	0.0	0.0	$I2_1^{(4)}$	+0.5	+0.5	+0.5	-0.5					
$I2_2^{(4)}$	0.0	0.0	0.0	0.0	$I2_2^{(4)}$	+0.5	-0.5	+0.5	+0.5					
$I2_3^{(4)}$	0.0	0.0	0.0	0.0	$I2_3^{(4)}$	+0.5	+0.5	+0.5	-0.5					
$I2_4^{(4)}$	0.0	0.0	0.0	0.0	$I2_4^{(4)}$	-0.5	+0.5	-0.5	-0.5					



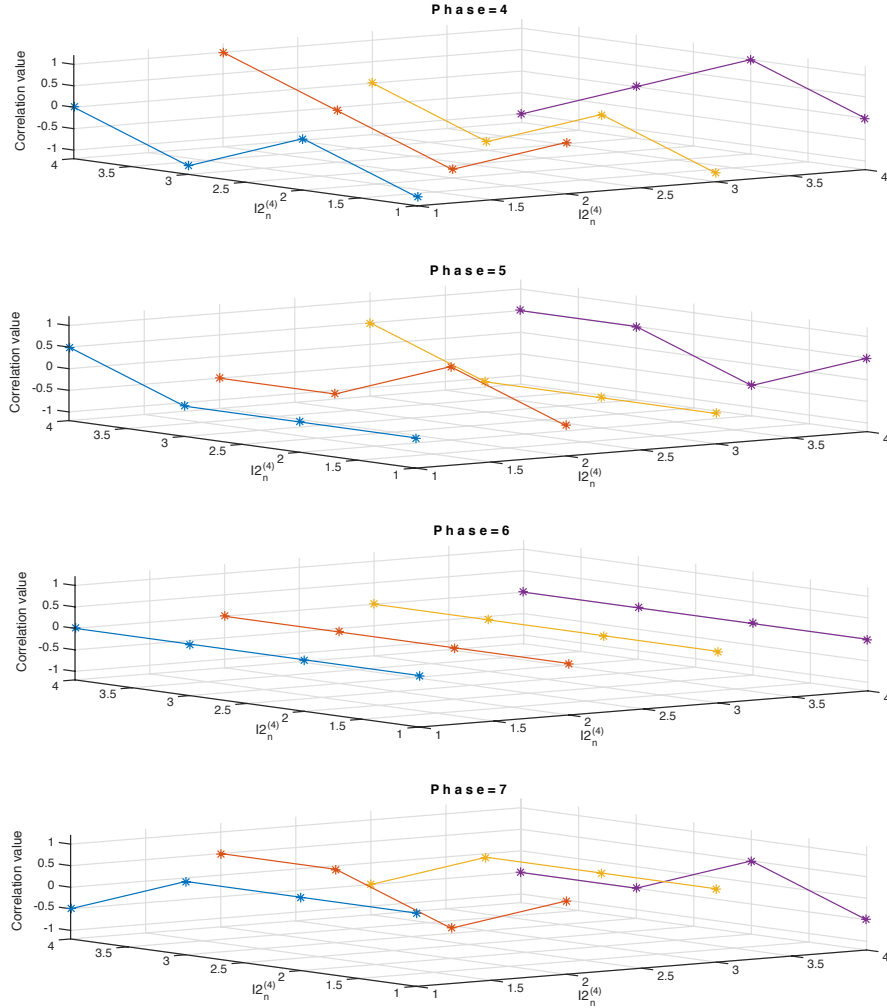
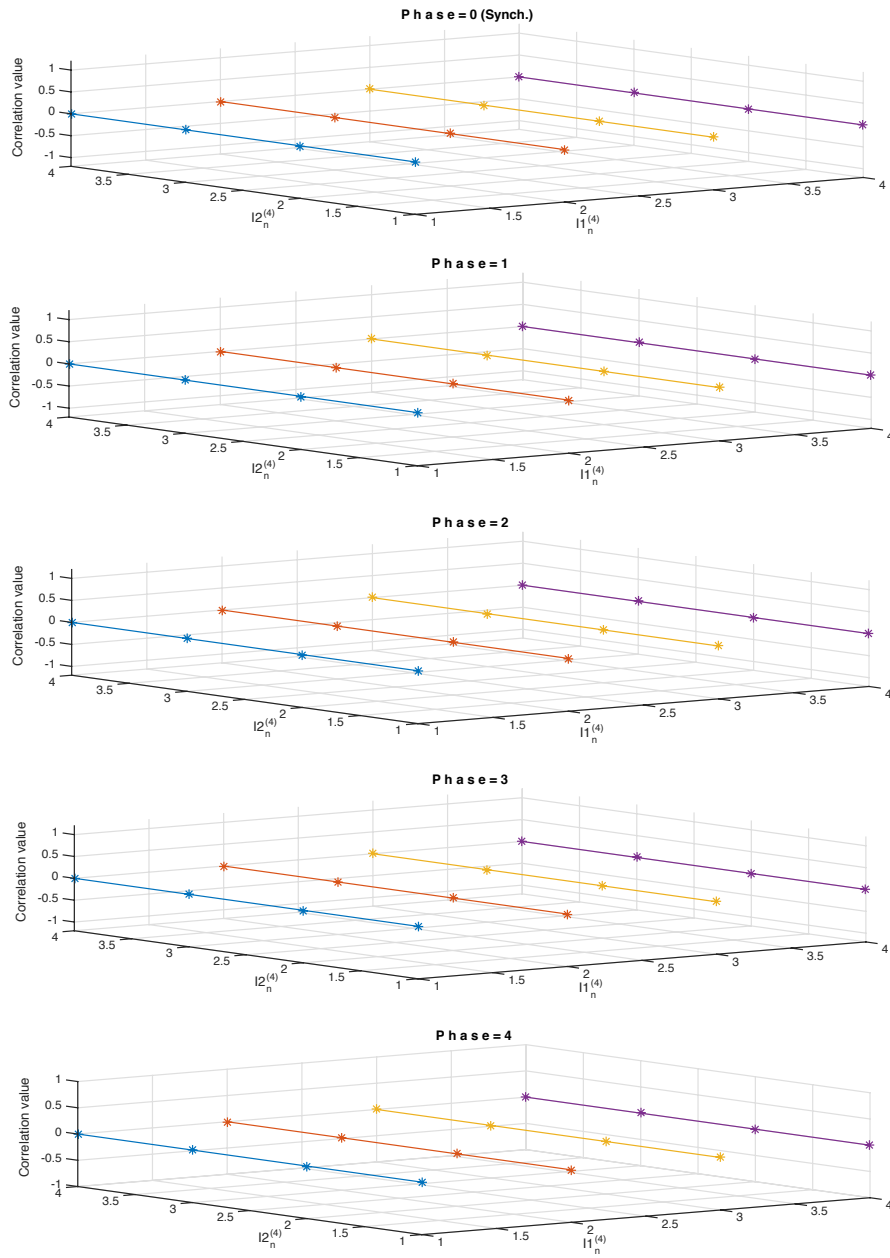


Figure 4.20: Correlation matrices for  $I2_n^{(4)}$ : Phases.

+ Inter-group correlation:  $I1_n^{(4)}$  vs  $I2_n^{(4)}$

$\phi = 0$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_1^{(3)}$	$I2_4^{(4)}$	$\phi = 1$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$	$\phi = 2$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$
$I1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$I1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$I1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0
$I1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$I1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$I1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0
$I1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$I1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$I1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0
$I1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$I1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$I1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>
$\phi = 3$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_1^{(3)}$	$I2_4^{(4)}$	$\phi = 4$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$	$\phi = 5$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$
$I1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$I1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$I1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0
$I1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$I1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$I1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0
$I1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$I1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$I1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0
$I1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$I1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$I1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>

$\phi = 6$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_1^{(3)}$	$I2_4^{(4)}$	$\phi = 7$	$I2_1^{(4)}$	$I2_2^{(4)}$	$I2_3^{(4)}$	$I2_4^{(4)}$
$I1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$I1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0
$I1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$I1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0
$I1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$I1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0
$I1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$I1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>



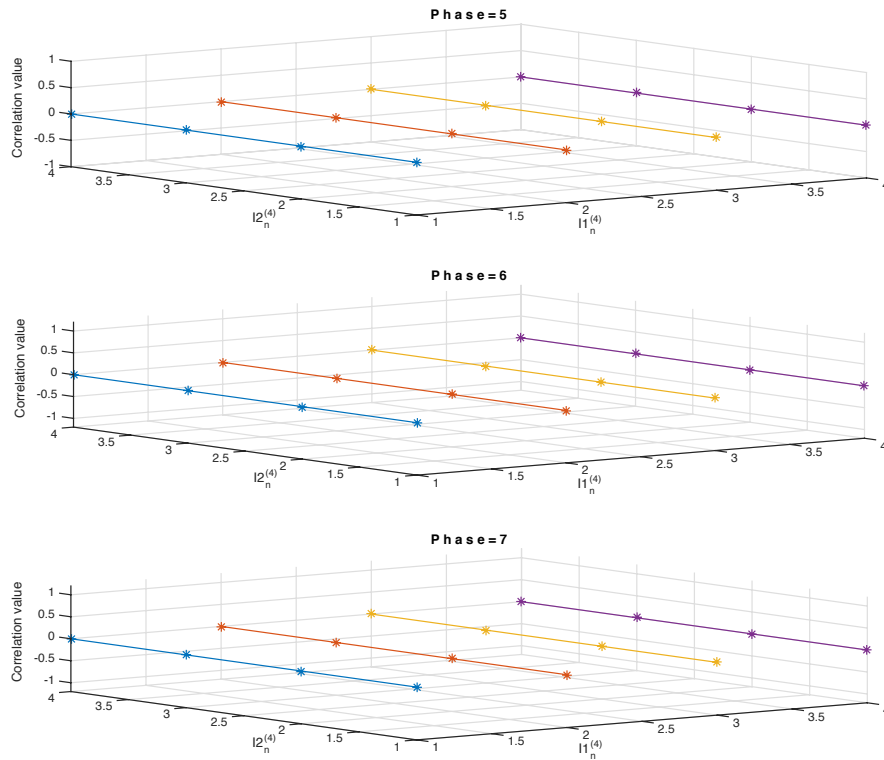


Figure 4.21: Correlation matrices for  $I1_n^{(4)}$  vs  $I2_n^{(4)}$ : Phases.

• **Double-correlation**

The codes of these sets are also related as follows  $Ix_1^{(4)} = Ix_3^{(4)}$  and  $Ix_2^{(4)} = -Ix_4^{(4)}$  which do not exhibit the DC property.

★ **SHIFTING PROPERTY**

$I1_n^{(4)}$  and  $I2_n^{(4)}$  are considered to evaluate shifting property, so  $M_\phi(i, j)$  The lists are giving as follows. Besides, Figure 4.22 shows the ACF values from both flocks.

$i$	$j$	$\phi$							$i$	$j$	$\phi$								
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$I1_1^{(4)}$	$I1_1^{(4)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$I1_2^{(4)}$	$I1_3^{(4)}$	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	+1.0
$I1_1^{(4)}$	$I1_2^{(4)}$	0.0	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	$I1_2^{(4)}$	$I1_4^{(4)}$	-1.0	0.0	0.0	0.0	-1.0	0.0	0.0	0.0
$I1_1^{(4)}$	$I1_3^{(4)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$I1_3^{(4)}$	$I1_3^{(4)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	0.0
$I1_1^{(4)}$	$I1_4^{(4)}$	0.0	-1.0	0.0	0.0	0.0	-1.0	0.0	0.0	$I1_3^{(4)}$	$I1_4^{(4)}$	0.0	-1.0	0.0	0.0	0.0	-1.0	0.0	0.0
$I1_2^{(4)}$	$I1_2^{(4)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	0.0	$I1_4^{(4)}$	$I1_4^{(4)}$	+1.0	0.0	0.0	0.0	+1.0	0.0	0.0	0.0

$i$	$j$	$\phi$							$i$	$j$	$\phi$								
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$I2_1^{(4)}$	$I2_1^{(4)}$	+1.0	+0.5	0.0	-0.5	-1.0	-0.5	0.0	+0.5	$I2_2^{(4)}$	$I2_3^{(4)}$	0.0	+0.5	0.0	-0.5	0.0	-0.5	0.0	+0.5
$I2_1^{(4)}$	$I2_2^{(4)}$	0.0	+0.5	0.0	-0.5	0.0	-0.5	0.0	+0.5	$I2_2^{(4)}$	$I2_4^{(4)}$	-1.0	+0.5	0.0	-0.5	+1.0	-0.5	0.0	+0.5
$I2_1^{(4)}$	$I2_3^{(4)}$	+1.0	+0.5	0.0	-0.5	-1.0	-0.5	0.0	+0.5	$I2_3^{(4)}$	$I2_3^{(4)}$	+1.0	+0.5	0.0	-0.5	-1.0	-0.5	0.0	+0.5
$I2_1^{(4)}$	$I2_4^{(4)}$	0.0	-0.5	0.0	+0.5	0.0	+0.5	0.0	-0.5	$I2_3^{(4)}$	$I2_4^{(4)}$	0.0	-0.5	0.0	+0.5	0.0	+0.5	0.0	-0.5
$I2_2^{(4)}$	$I2_2^{(4)}$	+1.0	-0.5	0.0	+0.5	-1.0	+0.5	0.0	-0.5	$I2_4^{(4)}$	$I2_4^{(4)}$	+1.0	-0.5	0.0	+0.5	-1.0	+0.5	0.0	-0.5

$i$	$j$	$\phi$								$i$	$j$	$\phi$							
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$I1_1^{(4)}$	$I2_1^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$I1_2^{(4)}$	$I2_3^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$I1_1^{(4)}$	$I2_2^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$I1_2^{(4)}$	$I2_4^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$I1_1^{(4)}$	$I2_3^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$I1_3^{(4)}$	$I2_1^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$I1_1^{(4)}$	$I2_4^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$I1_3^{(4)}$	$I2_2^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
$I1_2^{(4)}$	$I2_2^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	$I1_4^{(4)}$	$I2_4^{(4)}$	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

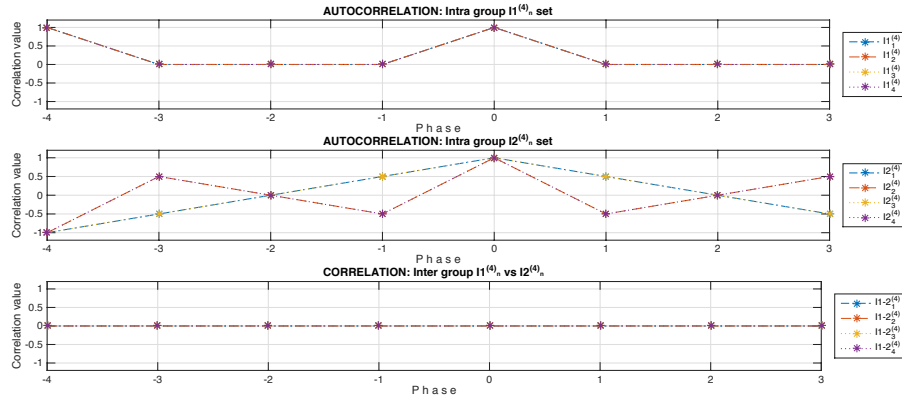


Figure 4.22: 2-D Side-lobe graph: ACF Infra group for  $I1_n^{(4)} - I2_n^{(4)}$ , and CCF Inter group for  $I1_n^{(4)}$  vs  $I2_n^{(4)}$ .

**Zero Correlation Zone sets**

In order to verify this class of codes, they are analyzed the following flocks defined as  $M = 4$  and  $N = 8$ .

$$\begin{aligned}
 Z1_1^{(4)} &= (+1, +1, +1, -1, +1, +1, -1, +1); & Z2_1^{(4)} &= (+1, +1, -1, +1, +1, +1, +1, -1); \\
 Z1_2^{(4)} &= (+1, -1, +1, +1, -1, +1, +1, +1); & Z2_2^{(4)} &= (+1, -1, -1, -1, -1, +1, -1, -1); \\
 Z1_3^{(4)} &= (+1, +1, +1, -1, +1, +1, -1, +1); & Z2_3^{(4)} &= (+1, +1, -1, +1, +1, +1, +1, -1); \\
 Z1_4^{(4)} &= (-1, +1, -1, -1, +1, -1, -1, -1); & Z2_4^{(4)} &= (-1, +1, +1, +1, +1, -1, +1, +1);
 \end{aligned}$$

**\*BALANCE PROPERTY**

Considering the flocks  $Z1_n^{(4)}$  and  $Z2_n^{(4)}$  and parameters  $M = 4$  and  $N = 8$ , this property is verified in Table 4.8.

Table 4.8: Balance: Zero Correlation Zone sets

$Z1_n^{(4)}$	Bal	meet?	$Z1_n^{(4)}$	Bal	meet?	$Z2_n^{(4)}$	Bal	meet?	$Z2_n^{(4)}$	Bal	meet?
$Z1_1^{(4)}$	4	NO	$Z1_3^{(4)}$	4	NO	$Z2_1^{(4)}$	4	NO	$Z2_3^{(4)}$	4	NO
$Z1_2^{(4)}$	4	NO	$Z1_4^{(4)}$	-4	NO	$Z2_2^{(4)}$	-4	NO	$Z2_4^{(4)}$	4	NO

**\*CORRELATION PROPERTIES**

**• Autocorrelation and cross-correlation**

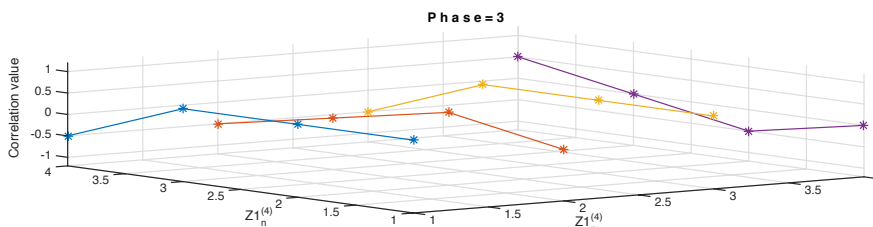
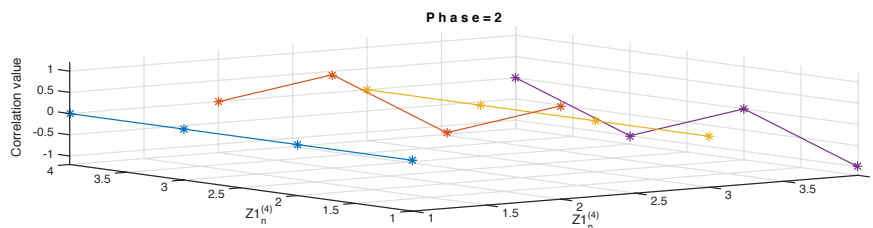
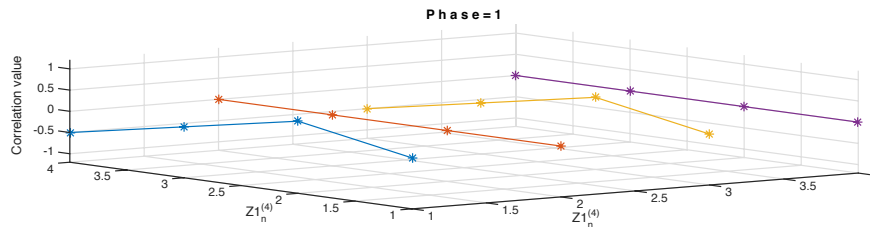
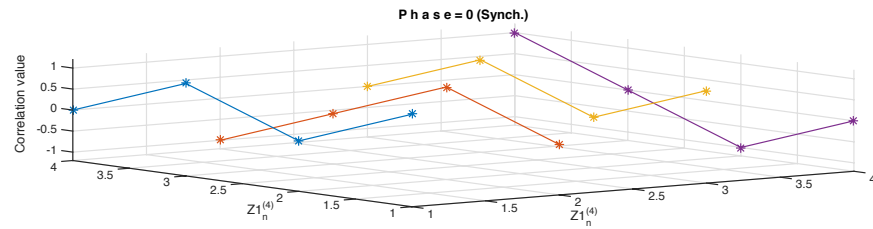
This family of sets are also described through subsets or *flocks*, so they are also studied two



cases: inter-group and infra-group, correlation.

+ Infra-group correlation:  $Z1_n^{(4)}$

$\phi = 0$	$Z1_1^{(4)}$	$Z1_2^{(4)}$	$Z1_1^{(3)}$	$Z1_4^{(4)}$	$\phi = 1$	$Z1_1^{(4)}$	$Z1_2^{(4)}$	$Z1_3^{(4)}$	$Z1_4^{(4)}$	$\phi = 2$	$Z1_1^{(4)}$	$Z1_2^{(4)}$	$Z1_3^{(4)}$	$Z1_4^{(4)}$
$Z1_1^{(4)}$	+1.0	0.0	+1.0	0.0	$Z1_1^{(4)}$	0.0	0.0	0.0	0.0	$Z1_1^{(4)}$	0.0	+1.0	0.0	-1.0
$Z1_2^{(4)}$	0.0	+1.0	0.0	-1.0	$Z1_2^{(4)}$	+0.5	0.0	+0.5	0.0	$Z1_2^{(4)}$	0.0	0.0	0.0	0.0
$Z1_3^{(4)}$	+1.0	0.0	+1.0	0.0	$Z1_3^{(4)}$	0.0	0.0	0.0	0.0	$Z1_3^{(4)}$	0.0	+1.0	0.0	-1.0
$Z1_4^{(4)}$	0.0	-1.0	0.0	+1.0	$Z1_4^{(4)}$	-0.5	0.0	-0.5	0.0	$Z1_4^{(4)}$	0.0	0.0	0.0	0.0
$\phi = 3$	$Z1_1^{(4)}$	$Z1_2^{(4)}$	$Z1_1^{(3)}$	$Z1_4^{(4)}$	$\phi = 4$	$Z1_1^{(4)}$	$Z1_2^{(4)}$	$Z1_3^{(4)}$	$Z1_4^{(4)}$	$\phi = 5$	$Z1_1^{(4)}$	$Z1_2^{(4)}$	$Z1_3^{(4)}$	$Z1_4^{(4)}$
$Z1_1^{(4)}$	+0.5	0.0	+0.5	0.0	$Z1_1^{(4)}$	0.0	0.0	0.0	0.0	$Z1_1^{(4)}$	+0.5	+0.5	+0.5	-0.5
$Z1_2^{(4)}$	+0.5	+0.5	+0.5	-0.5	$Z1_2^{(4)}$	0.0	0.0	0.0	0.0	$Z1_2^{(4)}$	0.0	+0.5	0.0	-0.5
$Z1_3^{(4)}$	+0.5	0.0	+0.5	0.0	$Z1_3^{(4)}$	0.0	0.0	0.0	0.0	$Z1_3^{(4)}$	+0.5	+0.5	+0.5	-0.5
$Z1_4^{(4)}$	-0.5	-0.5	-0.5	+0.5	$Z1_4^{(4)}$	0.0	0.0	0.0	0.0	$Z1_4^{(4)}$	0.0	-0.5	0.0	+0.5
$\phi = 6$	$Z1_1^{(4)}$	$Z1_2^{(4)}$	$Z1_1^{(3)}$	$Z1_4^{(4)}$	$\phi = 7$	$Z1_1^{(4)}$	$Z1_2^{(4)}$	$Z1_3^{(4)}$	$Z1_4^{(4)}$					
$Z1_1^{(4)}$	0.0	0.0	0.0	0.0	$Z1_1^{(4)}$	0.0	+0.5	0.0	-0.5					
$Z1_2^{(4)}$	+1.0	0.0	+1.0	0.0	$Z1_2^{(4)}$	0.0	0.0	0.0	0.0					
$Z1_3^{(4)}$	0.0	0.0	0.0	0.0	$Z1_3^{(4)}$	0.0	+0.5	0.0	-0.5					
$Z1_4^{(4)}$	-1.0	0.0	-1.0	0.0	$Z1_4^{(4)}$	0.0	0.0	0.0	0.0					



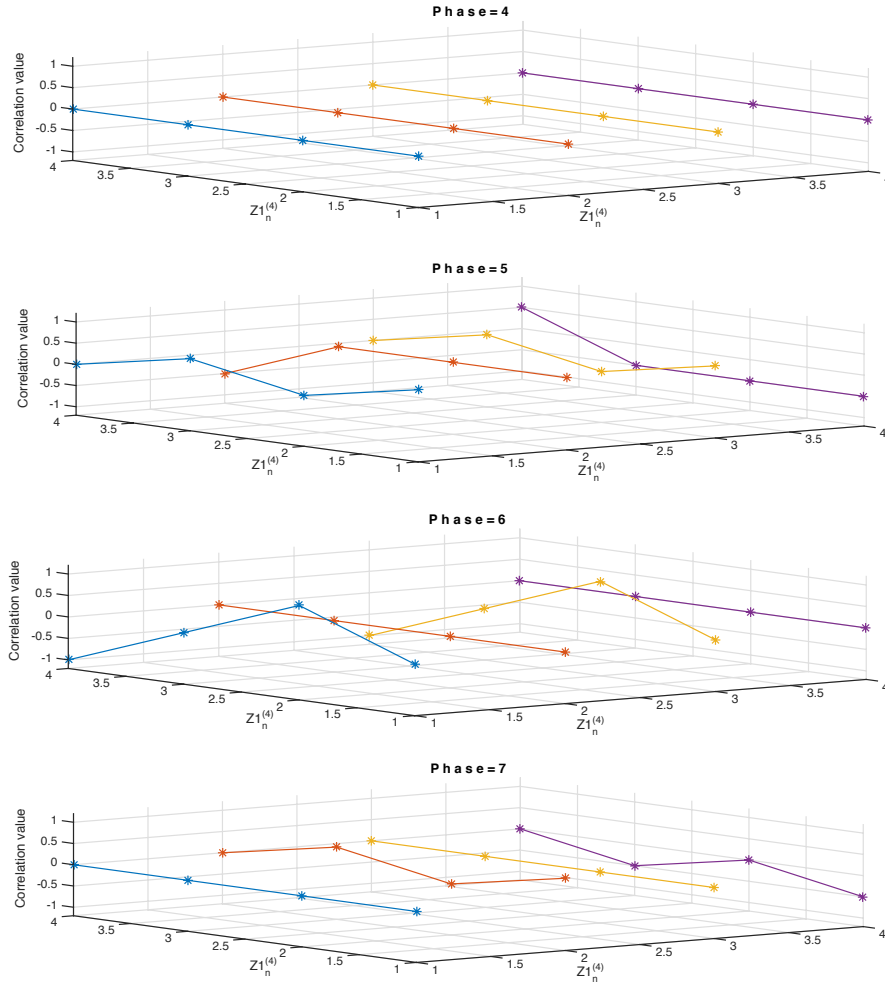
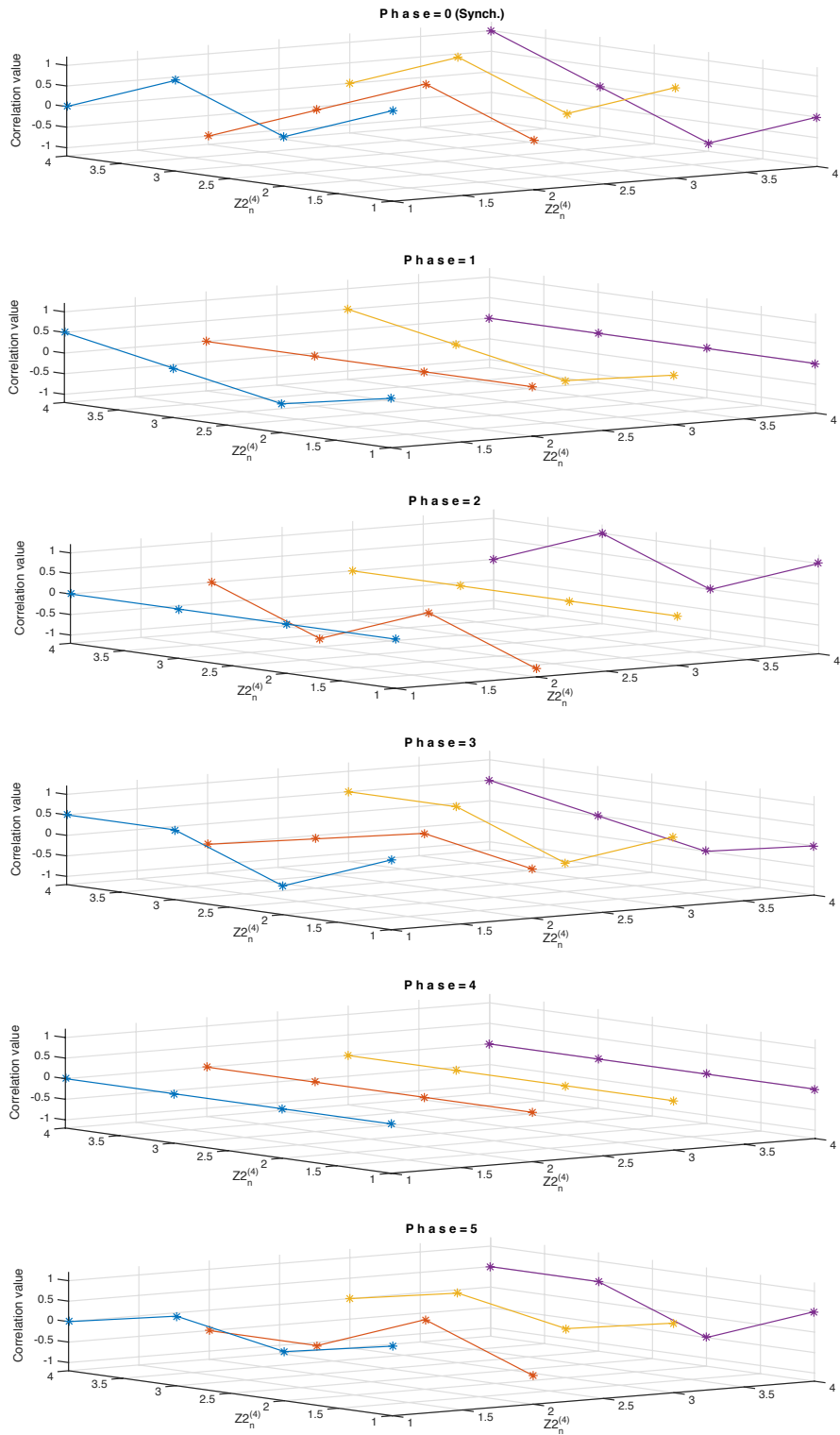


Figure 4.23: Correlation matrices for  $Z1_n^{(4)}$ : Phases.

+ *Infra-group correlation:  $Z2_n^{(4)}$*

$\phi = 0$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_1^{(3)}$	$Z2_4^{(4)}$	$\phi = 1$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$	$\phi = 2$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$
$Z2_1^{(4)}$	<b>+1.0</b>	0.0	+1.0	0.0	$Z2_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$Z2_1^{(4)}$	<b>0.0</b>	-1.0	0.0	+1.0
$Z2_2^{(4)}$	0.0	<b>+1.0</b>	0.0	-1.0	$Z2_2^{(4)}$	-0.5	<b>0.0</b>	-0.5	0.0	$Z2_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0
$Z2_3^{(4)}$	+1.0	0.0	<b>+1.0</b>	0.0	$Z2_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$Z2_3^{(4)}$	0.0	-1.0	<b>0.0</b>	+1.0
$Z2_4^{(4)}$	0.0	-1.0	0.0	<b>+1.0</b>	$Z2_4^{(4)}$	+0.5	0.0	+0.5	<b>0.0</b>	$Z2_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>
$\phi = 3$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_1^{(3)}$	$Z2_4^{(4)}$	$\phi = 4$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$	$\phi = 5$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$
$Z2_1^{(4)}$	<b>+0.5</b>	0.0	+0.5	0.0	$Z2_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$Z2_1^{(4)}$	<b>+0.5</b>	-0.5	+0.5	+0.5
$Z2_2^{(4)}$	-0.5	<b>+0.5</b>	-0.5	-0.5	$Z2_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$Z2_2^{(4)}$	0.0	<b>+0.5</b>	0.0	-0.5
$Z2_3^{(4)}$	+0.5	0.0	<b>+0.5</b>	0.0	$Z2_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$Z2_3^{(4)}$	+0.5	-0.5	<b>+0.5</b>	+0.5
$Z2_4^{(4)}$	+0.5	-0.5	+0.5	<b>+0.5</b>	$Z2_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$Z2_4^{(4)}$	0.0	-0.5	0.0	<b>+0.5</b>
$\phi = 6$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_1^{(3)}$	$Z2_4^{(4)}$	$\phi = 7$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$					
$Z2_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$Z2_1^{(4)}$	<b>0.0</b>	-0.5	0.0	+0.5					
$Z2_2^{(4)}$	-1.0	<b>0.0</b>	-1.0	0.0	$Z2_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0					
$Z2_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$Z2_3^{(4)}$	0.0	-0.5	<b>0.0</b>	+0.5					
$Z2_4^{(4)}$	+1.0	0.0	+1.0	<b>0.0</b>	$Z2_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>					



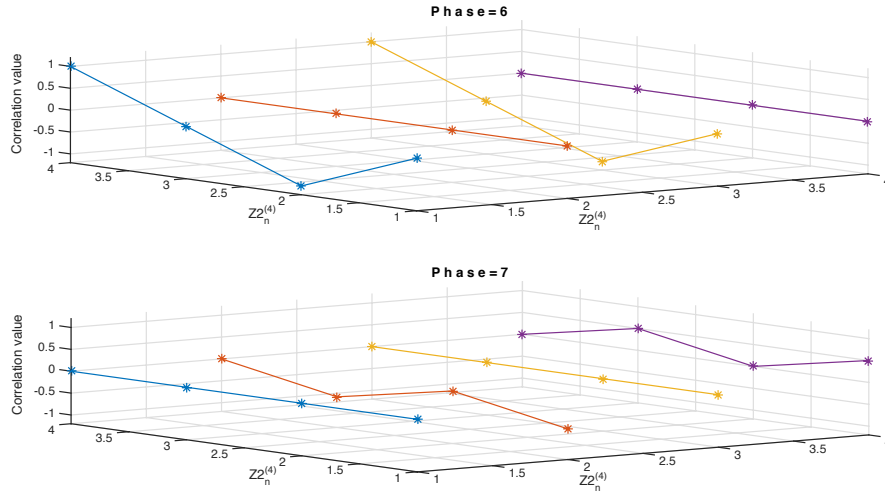
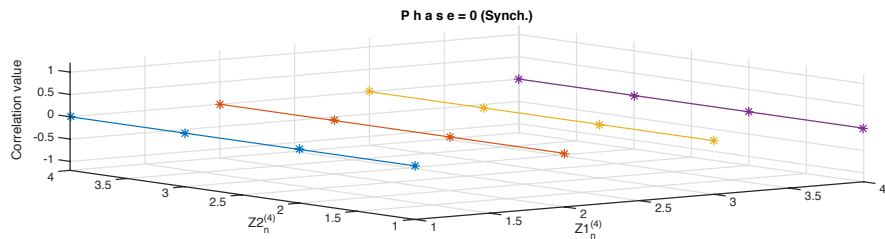


Figure 4.24: Correlation matrices for  $Z2_n^{(4)}$ : Phases.

+ *Inter-group correlation:  $Z1_n^{(4)}$  vs  $Z2_n^{(4)}$*

$\phi = 0$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_1^{(3)}$	$Z2_4^{(4)}$	$\phi = 1$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$	$\phi = 2$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$
$Z1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0	$Z1_1^{(4)}$	<b>+0.5</b>	-0.5	+0.5	+0.5	$Z1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0
$Z1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$Z1_2^{(4)}$	0.0	<b>-0.5</b>	0.0	+0.5	$Z1_2^{(4)}$	+1.0	<b>0.0</b>	+1.0	0.0
$Z1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0	$Z1_3^{(4)}$	+0.5	-0.5	<b>+0.5</b>	+0.5	$Z1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0
$Z1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$Z1_4^{(4)}$	0.0	+0.5	0.0	<b>-0.5</b>	$Z1_4^{(4)}$	-1.0	0.0	-1.0	<b>0.0</b>
$\phi = 3$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_1^{(3)}$	$Z2_4^{(4)}$	$\phi = 4$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$	$\phi = 5$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$
$Z1_1^{(4)}$	<b>0.0</b>	-0.5	0.0	+0.5	$Z1_1^{(4)}$	<b>+1.0</b>	0.0	+1.0	0.0	$Z1_1^{(4)}$	<b>0.0</b>	0.0	0.0	0.0
$Z1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$Z1_2^{(4)}$	0.0	<b>-1.0</b>	0.0	+1.0	$Z1_2^{(4)}$	+0.5	<b>0.0</b>	+0.5	0.0
$Z1_3^{(4)}$	0.0	-0.5	<b>0.0</b>	+0.5	$Z1_3^{(4)}$	+1.0	0.0	<b>+1.0</b>	0.0	$Z1_3^{(4)}$	0.0	0.0	<b>0.0</b>	0.0
$Z1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$Z1_4^{(4)}$	0.0	+1.0	0.0	<b>-1.0</b>	$Z1_4^{(4)}$	-0.5	0.0	-0.5	<b>0.0</b>
$\phi = 6$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_1^{(3)}$	$Z2_4^{(4)}$	$\phi = 7$	$Z2_1^{(4)}$	$Z2_2^{(4)}$	$Z2_3^{(4)}$	$Z2_4^{(4)}$					
$Z1_1^{(4)}$	<b>0.0</b>	-1.0	0.0	+1.0	$Z1_1^{(4)}$	<b>+0.5</b>	0.0	+0.5	0.0					
$Z1_2^{(4)}$	0.0	<b>0.0</b>	0.0	0.0	$Z1_2^{(4)}$	+0.5	<b>-0.5</b>	+0.5	+0.5					
$Z1_3^{(4)}$	0.0	-1.0	<b>0.0</b>	+1.0	$Z1_3^{(4)}$	+0.5	0.0	<b>+0.5</b>	0.0					
$Z1_4^{(4)}$	0.0	0.0	0.0	<b>0.0</b>	$Z1_4^{(4)}$	-0.5	+0.5	-0.5	<b>-0.5</b>					



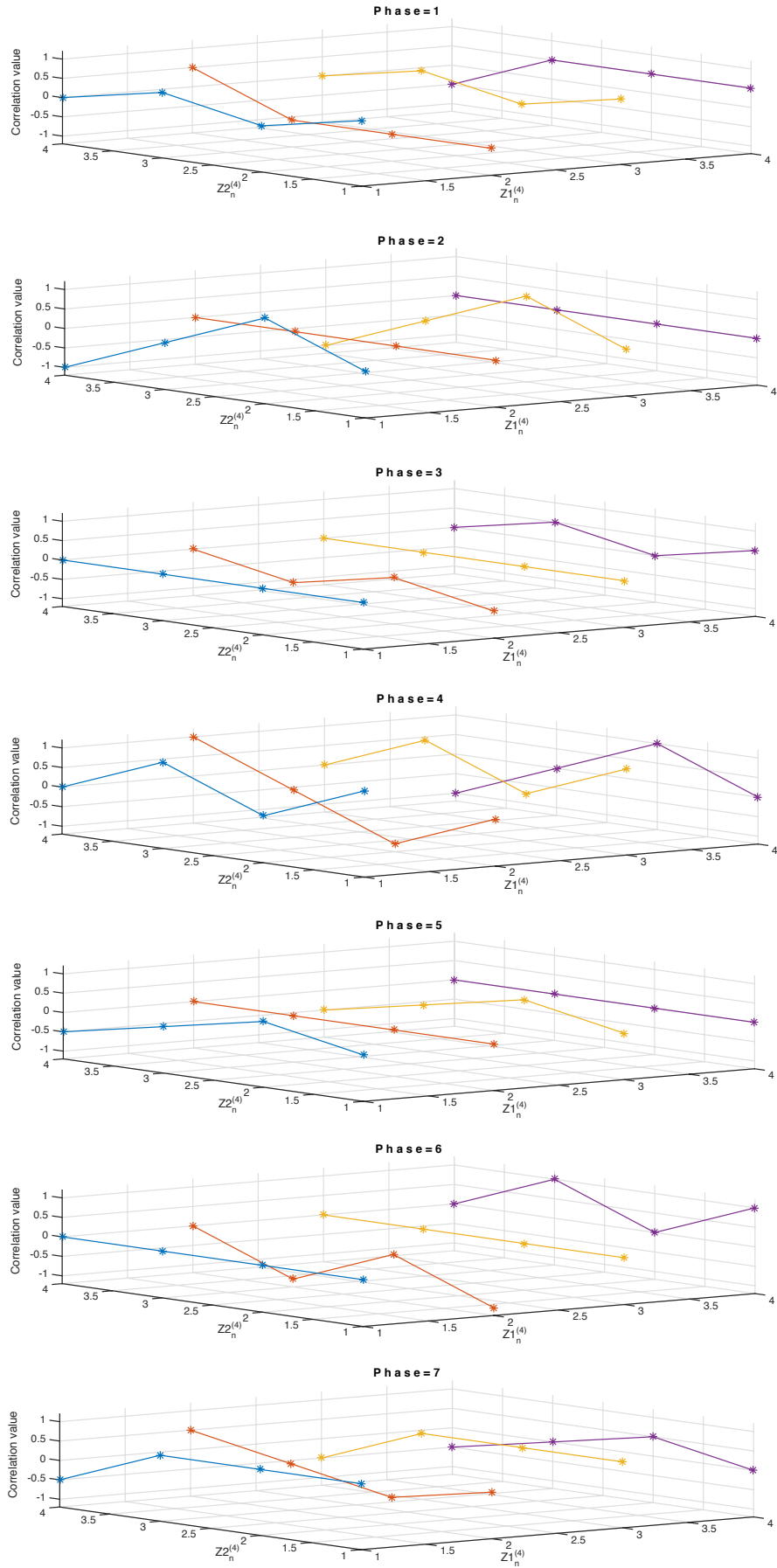


Figure 4.25: Correlation matrices for  $Z_1_n^{(4)}$  vs  $Z_2_n^{(4)}$ : Phases.

• **Double-correlation**

Aforementioned, this kind of code sets have particularities which are observed. In particular, this ZCZ sets show that  $Zx_1^{(4)} = Zx_3^{(4)}$  and  $Zx_2^{(4)} = -Zx_4^{(4)}$  which do not show DC property.

★ **SHIFTING PROPERTY**

Considering  $Z1_n^{(4)}$  and  $Z2_n^{(4)}$ , shifting property is evaluated through following lists  $M_\phi(i, j)$ . Besides, Figure 4.26 shows the ACF values from both flocks.

$i$	$j$	$\phi$							$i$	$j$	$\phi$								
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$Z1_1^{(4)}$	$Z1_1^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$Z1_2^{(4)}$	$Z1_3^{(4)}$	0.0	+0.5	0.0	+0.5	0.0	0.0	+1.0	0.0
$Z1_1^{(4)}$	$Z1_2^{(4)}$	0.0	0.0	+1.0	0.0	0.0	+0.5	0.0	+0.5	$Z1_2^{(4)}$	$Z1_4^{(4)}$	-1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$Z1_1^{(4)}$	$Z1_3^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$Z1_3^{(4)}$	$Z1_3^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0
$Z1_1^{(4)}$	$Z1_4^{(4)}$	0.0	0.0	-1.0	0.0	0.0	-0.5	0.0	-0.5	$Z1_3^{(4)}$	$Z1_4^{(4)}$	0.0	0.0	-1.0	0.0	0.0	-0.5	0.0	-0.5
$Z1_2^{(4)}$	$Z1_2^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$Z1_4^{(4)}$	$Z1_4^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0

$i$	$j$	$\phi$							$i$	$j$	$\phi$								
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$Z2_1^{(4)}$	$Z2_1^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$Z2_2^{(4)}$	$Z2_3^{(4)}$	0.0	-0.5	0.0	-0.5	0.0	0.0	-1.0	0.0
$Z2_1^{(4)}$	$Z2_2^{(4)}$	0.0	0.0	-1.0	0.0	0.0	-0.5	0.0	-0.5	$Z2_2^{(4)}$	$Z2_4^{(4)}$	-1.0	0.0	0.0	-0.5	0.0	-0.5	0.0	0.0
$Z2_1^{(4)}$	$Z2_3^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$Z2_3^{(4)}$	$Z2_3^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0
$Z2_1^{(4)}$	$Z2_4^{(4)}$	0.0	0.0	+1.0	0.0	0.0	+0.5	0.0	+0.5	$Z2_3^{(4)}$	$Z2_4^{(4)}$	0.0	0.0	+1.0	0.0	0.0	+0.5	0.0	+0.5
$Z2_2^{(4)}$	$Z2_2^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0	$Z2_4^{(4)}$	$Z2_4^{(4)}$	+1.0	0.0	0.0	+0.5	0.0	+0.5	0.0	0.0

$i$	$j$	$\phi$							$i$	$j$	$\phi$								
		0	1	2	3	4	5	6	7			0	1	2	3	4	5	6	7
$Z1_1^{(4)}$	$Z2_1^{(4)}$	0.0	+0.5	0.0	0.0	+1.0	0.0	0.0	+0.5	$Z1_2^{(4)}$	$Z2_3^{(4)}$	0.0	0.0	+1.0	0.0	0.0	+0.5	0.0	+0.5
$Z1_1^{(4)}$	$Z2_2^{(4)}$	0.0	-0.5	0.0	-0.5	0.0	0.0	-1.0	0.0	$Z1_2^{(4)}$	$Z2_4^{(4)}$	0.0	+0.5	0.0	0.0	+1.0	0.0	0.0	+0.5
$Z1_1^{(4)}$	$Z2_3^{(4)}$	0.0	+0.5	0.0	0.0	+1.0	0.0	0.0	+0.5	$Z1_3^{(4)}$	$Z2_3^{(4)}$	0.0	+0.5	0.0	0.0	+1.0	0.0	0.0	+0.5
$Z1_1^{(4)}$	$Z2_4^{(4)}$	0.0	+0.5	0.0	+0.5	0.0	0.0	+1.0	0.0	$Z1_3^{(4)}$	$Z2_4^{(4)}$	0.0	+0.5	0.0	+0.5	0.0	0.0	+1.0	0.0
$Z1_2^{(4)}$	$Z2_2^{(4)}$	0.0	-0.5	0.0	0.0	-1.0	0.0	0.0	-0.5	$Z1_4^{(4)}$	$Z2_4^{(4)}$	0.0	-0.5	0.0	0.0	-1.0	0.0	0.0	-0.5

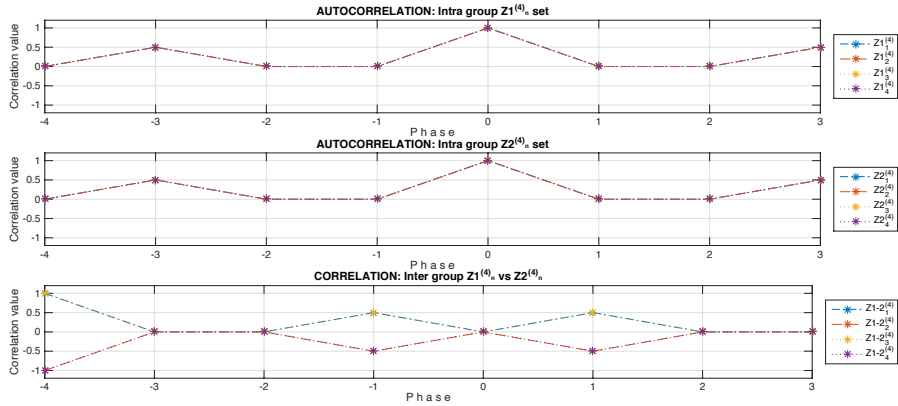


Figure 4.26: 2-D Side-lobe graph: ACF Infra group for  $Z1_n^{(4)}$  -  $Z2_n^{(4)}$ , and CCF Inter group for  $Z1_n^{(4)}$  vs  $Z2_n^{(4)}$ .

#### 4.3.4 Observations

In this section, families of sequences were examined to balance, correlation and shifting properties which show interesting features to consider in the study of codes. For example, it is possible to distinguish that balance sequences are desirable but not necessary as Golay-Hadamard sets shows, this way the number of potential sequences increases. On the other hand, correlation properties and their crucial role can be interpreted in order to provide interesting features in CDMA systems. This way, the categorization of correlation values as ideal, good or bad are useful in sequences study, e.g. Hadamard sets exhibit ideal correlation values in synchronous reception ( $Phase = 0$ ) but their values are *irregular* when the synchrony is broken which make them no attractive in this case. Complementary sets try to reduce the irregularity and IFW sets provide zones (or shifts) with desirable values. All these concepts are keys in next chapter that allow to describe criteria to search codes.

# Chapter 5

## Exhaustive search of codes

In this chapter, an exhaustive search method is explored in order to select sequences that satisfy some conditions and properties by forming potential code sets useful to CDMA systems. The selected properties are no negative, balance, correlation (AC and CC), and shifting that add robustness to different interference presented in RWN. This set of properties is not usually used jointly in codes design. All the binary sequences with length  $N = 8$  and some with length  $N = 16$  are considered.

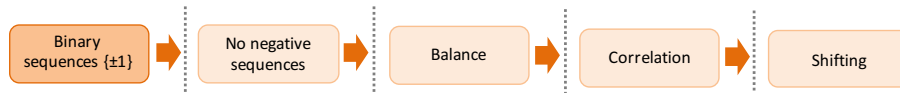


Figure 5.1: Selecting method.

### 5.1 Sequence selection method

The method shown in Figure 5.1 has five steps which are described as follows. This can be easily implemented for any group of sequences. Note that the desired features are tested by following a serial order what ensure their incorporation and the sequences with mutual properties are grouped to form mutual code sets or flocks.

1. **Binary sequences.** It is the set of all the binary sequences with elements  $\{\pm 1\}$  and length  $N$ . Sequences with length  $N = 8$  are considered in this work due to computational complexity issues.
2. **No negative sequences.** This property considers that the energy per bit is  $\mathcal{E}_b = \pm 1$ , so antipodal sequences must be discarded in order to avoid cancellation of signals and information loss, i.e. the negative combination of sequences is not considered.
3. **Balance.** This property establishes the relationship between the number of  $+1s$  and  $-1s$  in the sequences through value of  $Bal$  parameter. Four cases are considered, the first one (C-I) contemplates the perfect balance of sequences or  $Bal = 0$  value, while second and third cases consider soft balance values, so  $Bal = 0, \pm 2$  and so  $Bal = 0, \pm 2, \pm 4$  define C-II and C-III respectively.



4. **Correlation.** It refers to evaluate the values of ACF, CCF, and DC. The first two values are considered for DS-CDMA model while DC values are included for C2DMA.
5. **Shifting.** It considers sequences that maintain their correlation values within a *windows* of shifts. This property allows asynchronous reception in CDMA systems by attenuating multipath signals that arrive within the window.

## 5.2 Selection of sequences with $N = 8$

In this subsection the process to search binary sequences subsets with length  $N = 8$  are developed, every subset is presented step by step as previous section described.

### 5.2.1 Binary sequences

The number of binary sequences is given as  $S_{All} = 2^N$ , so considering  $N = 8$  there are 256 sequences.

Num.	Binary sequence	Num.	Binary sequence	Num.	Binary sequence
1	+1, +1, +1, +1, +1, +1, +1, +1	87	+1, -1, +1, -1, +1, -1, -1, +1	173	-1, +1, -1, +1, -1, -1, +1, +1
2	+1, +1, +1, +1, +1, +1, +1, -1	88	+1, -1, +1, -1, +1, -1, -1, -1	174	-1, +1, -1, +1, -1, -1, +1, -1
3	+1, +1, +1, +1, +1, +1, -1, +1	89	+1, -1, +1, -1, -1, +1, +1, +1	175	-1, +1, -1, +1, -1, -1, -1, +1
4	+1, +1, +1, +1, +1, +1, -1, -1	90	+1, -1, +1, -1, -1, +1, +1, -1	176	-1, +1, -1, +1, -1, -1, -1, -1
5	+1, +1, +1, +1, +1, -1, +1, +1	91	+1, -1, +1, -1, -1, +1, -1, +1	177	-1, +1, -1, -1, +1, +1, +1, +1
6	+1, +1, +1, +1, +1, -1, +1, -1	92	+1, -1, +1, -1, -1, +1, -1, -1	178	-1, +1, -1, -1, +1, +1, +1, -1
7	+1, +1, +1, +1, +1, -1, -1, +1	93	+1, -1, +1, -1, -1, -1, +1, +1	179	-1, +1, -1, -1, +1, +1, -1, +1
8	+1, +1, +1, +1, +1, -1, -1, -1	94	+1, -1, +1, -1, -1, -1, +1, -1	180	-1, +1, -1, -1, +1, +1, -1, -1
9	+1, +1, +1, +1, -1, +1, +1, +1	95	+1, -1, +1, -1, -1, -1, -1, +1	181	-1, +1, -1, -1, +1, -1, +1, +1
10	+1, +1, +1, +1, -1, +1, +1, -1	96	+1, -1, +1, -1, -1, -1, -1, -1	182	-1, +1, -1, -1, +1, -1, +1, -1
11	+1, +1, +1, +1, -1, +1, -1, +1	97	+1, -1, -1, +1, +1, +1, +1, +1	183	-1, +1, -1, -1, +1, -1, -1, +1
12	+1, +1, +1, +1, -1, +1, -1, -1	98	+1, -1, -1, +1, +1, +1, +1, -1	184	-1, +1, -1, -1, +1, -1, -1, -1
13	+1, +1, +1, +1, -1, -1, +1, +1	99	+1, -1, -1, +1, +1, +1, -1, +1	185	-1, +1, -1, -1, -1, +1, +1, +1
14	+1, +1, +1, +1, -1, -1, +1, -1	100	+1, -1, -1, +1, +1, +1, -1, -1	186	-1, +1, -1, -1, -1, +1, +1, -1
15	+1, +1, +1, +1, -1, -1, -1, +1	101	+1, -1, -1, +1, +1, -1, +1, +1	187	-1, +1, -1, -1, -1, +1, -1, +1
16	+1, +1, +1, +1, -1, -1, -1, -1	102	+1, -1, -1, +1, +1, -1, +1, -1	188	-1, +1, -1, -1, -1, +1, -1, -1
17	+1, +1, +1, -1, +1, +1, +1, +1	103	+1, -1, -1, +1, +1, -1, -1, +1	189	-1, +1, -1, -1, -1, -1, +1, +1
18	+1, +1, +1, -1, +1, +1, +1, -1	104	+1, -1, -1, +1, +1, -1, -1, -1	190	-1, +1, -1, -1, -1, -1, +1, -1
19	+1, +1, +1, -1, +1, +1, -1, +1	105	+1, -1, -1, +1, -1, +1, +1, +1	191	-1, +1, -1, -1, -1, -1, -1, +1
20	+1, +1, +1, -1, +1, +1, -1, -1	106	+1, -1, -1, +1, -1, +1, +1, -1	192	-1, +1, -1, -1, -1, -1, -1, -1
21	+1, +1, +1, -1, +1, -1, +1, +1	107	+1, -1, -1, +1, -1, +1, -1, +1	193	-1, -1, +1, +1, +1, +1, +1, +1
22	+1, +1, +1, -1, +1, -1, +1, -1	108	+1, -1, -1, +1, -1, +1, -1, -1	194	-1, -1, +1, +1, +1, +1, +1, -1
23	+1, +1, +1, -1, +1, -1, -1, +1	109	+1, -1, -1, +1, -1, -1, +1, +1	195	-1, -1, +1, +1, +1, +1, -1, +1
24	+1, +1, +1, -1, +1, -1, -1, -1	110	+1, -1, -1, +1, -1, -1, +1, -1	196	-1, -1, +1, +1, +1, +1, -1, -1
25	+1, +1, +1, -1, -1, +1, +1, +1	111	+1, -1, -1, +1, -1, -1, -1, +1	197	-1, -1, +1, +1, +1, -1, +1, +1
26	+1, +1, +1, -1, -1, +1, +1, -1	112	+1, -1, -1, +1, -1, -1, -1, -1	198	-1, -1, +1, +1, +1, -1, +1, -1
27	+1, +1, +1, -1, -1, +1, -1, +1	113	+1, -1, -1, -1, +1, +1, +1, +1	199	-1, -1, +1, +1, +1, -1, -1, +1
28	+1, +1, +1, -1, -1, +1, -1, -1	114	+1, -1, -1, -1, +1, +1, +1, -1	200	-1, -1, +1, +1, +1, -1, -1, -1
29	+1, +1, +1, -1, -1, -1, +1, +1	115	+1, -1, -1, -1, +1, +1, -1, +1	201	-1, -1, +1, +1, -1, +1, +1, +1
30	+1, +1, +1, -1, -1, -1, +1, -1	116	+1, -1, -1, -1, +1, +1, -1, -1	202	-1, -1, +1, +1, -1, +1, +1, -1
31	+1, +1, +1, -1, -1, -1, -1, +1	117	+1, -1, -1, -1, +1, -1, +1, +1	203	-1, -1, +1, +1, -1, +1, -1, +1
32	+1, +1, +1, -1, -1, -1, -1, -1	118	+1, -1, -1, -1, +1, -1, +1, -1	204	-1, -1, +1, +1, -1, +1, -1, -1
33	+1, +1, -1, +1, +1, +1, +1, +1	119	+1, -1, -1, -1, +1, -1, -1, +1	205	-1, -1, +1, +1, -1, -1, +1, +1
34	+1, +1, -1, +1, +1, +1, +1, -1	120	+1, -1, -1, -1, +1, -1, -1, -1	206	-1, -1, +1, +1, -1, -1, +1, -1
35	+1, +1, -1, +1, +1, +1, -1, +1	121	+1, -1, -1, -1, -1, +1, +1, +1	207	-1, -1, +1, +1, -1, -1, -1, +1

Num.	Binary sequence	Num.	Binary sequence	Num.	Binary sequence
36	+1, +1, -1, +1, +1, +1, -1, -1	122	+1, -1, -1, -1, -1, +1, +1, -1	208	-1, -1, +1, +1, -1, -1, -1, -1
37	+1, +1, -1, +1, +1, -1, +1, +1	123	+1, -1, -1, -1, -1, +1, -1, +1	209	-1, -1, +1, -1, +1, +1, +1, +1
38	+1, +1, -1, +1, +1, -1, +1, -1	124	+1, -1, -1, -1, -1, +1, -1, -1	210	-1, -1, +1, -1, +1, +1, +1, -1
39	+1, +1, -1, +1, +1, -1, -1, +1	125	+1, -1, -1, -1, -1, -1, +1, +1	211	-1, -1, +1, -1, +1, +1, -1, +1
40	+1, +1, -1, +1, +1, -1, -1, -1	126	+1, -1, -1, -1, -1, -1, +1, -1	212	-1, -1, +1, -1, +1, +1, -1, -1
41	+1, +1, -1, +1, -1, +1, +1, +1	127	+1, -1, -1, -1, -1, -1, -1, +1	213	-1, -1, +1, -1, +1, -1, +1, +1
42	+1, +1, -1, +1, -1, +1, +1, -1	128	+1, -1, -1, -1, -1, -1, -1, -1	214	-1, -1, +1, -1, +1, -1, +1, -1
43	+1, +1, -1, +1, -1, +1, -1, +1	129	-1, +1, +1, +1, +1, +1, +1, +1	215	-1, -1, +1, -1, +1, -1, -1, +1
44	+1, +1, -1, +1, -1, +1, -1, -1	130	-1, +1, +1, +1, +1, +1, +1, -1	216	-1, -1, +1, -1, +1, -1, -1, -1
45	+1, +1, -1, +1, -1, -1, +1, +1	131	-1, +1, +1, +1, +1, +1, -1, +1	217	-1, -1, +1, -1, -1, +1, +1, +1
46	+1, +1, -1, +1, -1, -1, +1, -1	132	-1, +1, +1, +1, +1, +1, -1, -1	218	-1, -1, +1, -1, -1, +1, +1, -1
47	+1, +1, -1, +1, -1, -1, -1, +1	133	-1, +1, +1, +1, +1, -1, +1, +1	219	-1, -1, +1, -1, -1, +1, -1, +1
48	+1, +1, -1, +1, -1, -1, -1, -1	134	-1, +1, +1, +1, +1, -1, +1, -1	220	-1, -1, +1, -1, -1, +1, -1, -1
49	+1, +1, -1, -1, +1, +1, +1, +1	135	-1, +1, +1, +1, +1, -1, -1, +1	221	-1, -1, +1, -1, -1, -1, +1, +1
50	+1, +1, -1, -1, +1, +1, +1, -1	136	-1, +1, +1, +1, +1, -1, -1, -1	222	-1, -1, +1, -1, -1, -1, +1, -1
51	+1, +1, -1, -1, +1, +1, -1, +1	137	-1, +1, +1, +1, -1, +1, +1, +1	223	-1, -1, +1, -1, -1, -1, -1, +1
52	+1, +1, -1, -1, +1, +1, -1, -1	138	-1, +1, +1, +1, -1, +1, +1, -1	224	-1, -1, +1, -1, -1, -1, -1, -1
53	+1, +1, -1, -1, +1, -1, +1, +1	139	-1, +1, +1, +1, -1, +1, -1, +1	225	-1, -1, -1, +1, +1, +1, +1, +1
54	+1, +1, -1, -1, +1, -1, +1, -1	140	-1, +1, +1, +1, -1, +1, -1, -1	226	-1, -1, -1, +1, +1, +1, +1, -1
55	+1, +1, -1, -1, +1, -1, -1, +1	141	-1, +1, +1, +1, -1, -1, +1, +1	227	-1, -1, -1, +1, +1, +1, -1, +1
56	+1, +1, -1, -1, +1, -1, -1, -1	142	-1, +1, +1, +1, -1, -1, +1, -1	228	-1, -1, -1, +1, +1, +1, -1, -1
57	+1, +1, -1, -1, -1, +1, +1, +1	143	-1, +1, +1, +1, -1, -1, -1, +1	229	-1, -1, -1, +1, +1, -1, +1, +1
58	+1, +1, -1, -1, -1, +1, +1, -1	144	-1, +1, +1, +1, -1, -1, -1, -1	230	-1, -1, -1, +1, +1, -1, +1, -1
59	+1, +1, -1, -1, -1, +1, -1, +1	145	-1, +1, +1, -1, +1, +1, +1, +1	231	-1, -1, -1, +1, +1, -1, -1, +1
60	+1, +1, -1, -1, -1, +1, -1, -1	146	-1, +1, +1, -1, +1, +1, +1, -1	232	-1, -1, -1, +1, +1, -1, -1, -1
61	+1, +1, -1, -1, -1, -1, +1, +1	147	-1, +1, +1, -1, +1, +1, -1, +1	233	-1, -1, -1, +1, -1, +1, +1, +1
62	+1, +1, -1, -1, -1, -1, +1, -1	148	-1, +1, +1, -1, +1, +1, -1, -1	234	-1, -1, -1, +1, -1, +1, +1, -1
63	+1, +1, -1, -1, -1, -1, -1, +1	149	-1, +1, +1, -1, +1, -1, +1, +1	235	-1, -1, -1, +1, -1, +1, -1, +1
64	+1, +1, -1, -1, -1, -1, -1, -1	150	-1, +1, +1, -1, +1, -1, +1, -1	236	-1, -1, -1, +1, -1, +1, -1, -1
65	+1, -1, +1, +1, +1, +1, +1, +1	151	-1, +1, +1, -1, +1, -1, -1, +1	237	-1, -1, -1, +1, -1, -1, +1, +1
66	+1, -1, +1, +1, +1, +1, +1, -1	152	-1, +1, +1, -1, +1, -1, -1, -1	238	-1, -1, -1, +1, -1, -1, +1, -1
67	+1, -1, +1, +1, +1, +1, -1, +1	153	-1, +1, +1, -1, -1, +1, +1, +1	239	-1, -1, -1, +1, -1, -1, -1, +1
68	+1, -1, +1, +1, +1, +1, -1, -1	154	-1, +1, +1, -1, -1, +1, +1, -1	240	-1, -1, -1, +1, -1, -1, -1, -1
69	+1, -1, +1, +1, +1, -1, +1, +1	155	-1, +1, +1, -1, -1, +1, -1, +1	241	-1, -1, -1, -1, +1, +1, +1, +1
70	+1, -1, +1, +1, +1, -1, +1, -1	156	-1, +1, +1, -1, -1, +1, -1, -1	242	-1, -1, -1, -1, +1, +1, +1, -1
71	+1, -1, +1, +1, +1, -1, -1, +1	157	-1, +1, +1, -1, -1, -1, +1, +1	243	-1, -1, -1, -1, +1, +1, -1, +1
72	+1, -1, +1, +1, +1, -1, -1, -1	158	-1, +1, +1, -1, -1, -1, +1, -1	244	-1, -1, -1, -1, +1, +1, -1, -1
73	+1, -1, +1, +1, -1, +1, +1, +1	159	-1, +1, +1, -1, -1, -1, -1, +1	245	-1, -1, -1, -1, +1, -1, +1, +1
74	+1, -1, +1, +1, -1, +1, +1, -1	160	-1, +1, +1, -1, -1, -1, -1, -1	246	-1, -1, -1, -1, +1, -1, +1, -1
75	+1, -1, +1, +1, -1, +1, -1, +1	161	-1, +1, -1, +1, +1, +1, +1, +1	247	-1, -1, -1, -1, +1, -1, -1, +1
76	+1, -1, +1, +1, -1, +1, -1, -1	162	-1, +1, -1, +1, +1, +1, +1, -1	248	-1, -1, -1, -1, +1, -1, -1, -1
77	+1, -1, +1, +1, -1, -1, +1, +1	163	-1, +1, -1, +1, +1, +1, -1, +1	249	-1, -1, -1, -1, -1, +1, +1, +1
78	+1, -1, +1, +1, -1, -1, +1, -1	164	-1, +1, -1, +1, +1, +1, -1, -1	250	-1, -1, -1, -1, -1, +1, +1, -1
79	+1, -1, +1, +1, -1, -1, -1, +1	165	-1, +1, -1, +1, +1, -1, +1, +1	251	-1, -1, -1, -1, -1, +1, -1, +1
80	+1, -1, +1, +1, -1, -1, -1, -1	166	-1, +1, -1, +1, +1, -1, +1, -1	252	-1, -1, -1, -1, -1, +1, -1, -1
81	+1, -1, +1, -1, +1, +1, +1, +1	167	-1, +1, -1, +1, +1, -1, -1, +1	253	-1, -1, -1, -1, -1, -1, +1, +1
82	+1, -1, +1, -1, +1, +1, +1, -1	168	-1, +1, -1, +1, +1, -1, -1, -1	254	-1, -1, -1, -1, -1, -1, +1, -1
83	+1, -1, +1, -1, +1, +1, -1, +1	169	-1, +1, -1, +1, -1, +1, +1, +1	255	-1, -1, -1, -1, -1, -1, -1, +1
84	+1, -1, +1, -1, +1, +1, -1, -1	170	-1, +1, -1, +1, -1, +1, +1, -1	256	-1, -1, -1, -1, -1, -1, -1, -1
85	+1, -1, +1, -1, +1, -1, +1, +1	171	-1, +1, -1, +1, -1, +1, -1, +1		
86	+1, -1, +1, -1, +1, -1, +1, -1	172	-1, +1, -1, +1, -1, +1, -1, -1		

### 5.2.2 No negative sequences

Discarding the antipodal (Ap) sequences (negative combination), the available sequences are reduced to a half, so the new subset size is defined as  $S_{Ap} = 128$ .

Num.	Binary sequence	Num.	Binary sequence	Num.	Binary sequence
1	+1, +1, +1, +1, +1, +1, +1, +1	44	+1, +1, -1, +1, -1, +1, -1, -1	87	+1, -1, +1, -1, +1, -1, -1, +1
2	+1, +1, +1, +1, +1, +1, +1, -1	45	+1, +1, -1, +1, -1, -1, +1, +1	88	+1, -1, +1, -1, +1, -1, -1, -1
3	+1, +1, +1, +1, +1, +1, -1, +1	46	+1, +1, -1, +1, -1, -1, +1, -1	89	+1, -1, +1, -1, -1, +1, +1, +1
4	+1, +1, +1, +1, +1, +1, -1, -1	47	+1, +1, -1, +1, -1, -1, -1, +1	90	+1, -1, +1, -1, -1, +1, +1, -1
5	+1, +1, +1, +1, +1, -1, +1, +1	48	+1, +1, -1, +1, -1, -1, -1, -1	91	+1, -1, +1, -1, -1, +1, -1, +1
6	+1, +1, +1, +1, +1, -1, +1, -1	49	+1, +1, -1, -1, +1, +1, +1, +1	92	+1, -1, +1, -1, -1, +1, -1, -1
7	+1, +1, +1, +1, +1, -1, -1, +1	50	+1, +1, -1, -1, +1, +1, +1, -1	93	+1, -1, +1, -1, -1, -1, +1, +1
8	+1, +1, +1, +1, +1, -1, -1, -1	51	+1, +1, -1, -1, +1, +1, -1, +1	94	+1, -1, +1, -1, -1, -1, +1, -1
9	+1, +1, +1, +1, -1, +1, +1, +1	52	+1, +1, -1, -1, +1, +1, -1, -1	95	+1, -1, +1, -1, -1, -1, -1, +1
10	+1, +1, +1, +1, -1, +1, +1, -1	53	+1, +1, -1, -1, +1, -1, +1, +1	96	+1, -1, +1, -1, -1, -1, -1, -1
11	+1, +1, +1, +1, -1, +1, -1, +1	54	+1, +1, -1, -1, +1, -1, +1, -1	97	+1, -1, -1, +1, +1, +1, +1, +1
12	+1, +1, +1, +1, -1, +1, -1, -1	55	+1, +1, -1, -1, +1, -1, -1, +1	98	+1, -1, -1, +1, +1, +1, +1, -1
13	+1, +1, +1, +1, -1, -1, +1, +1	56	+1, +1, -1, -1, +1, -1, -1, -1	99	+1, -1, -1, +1, +1, +1, -1, +1
14	+1, +1, +1, +1, -1, -1, +1, -1	57	+1, +1, -1, -1, -1, +1, +1, +1	100	+1, -1, -1, +1, +1, +1, -1, -1
15	+1, +1, +1, +1, -1, -1, -1, +1	58	+1, +1, -1, -1, -1, +1, +1, -1	101	+1, -1, -1, +1, +1, -1, +1, +1
16	+1, +1, +1, +1, -1, -1, -1, -1	59	+1, +1, -1, -1, -1, +1, -1, +1	102	+1, -1, -1, +1, +1, -1, +1, -1
17	+1, +1, +1, -1, +1, +1, +1, +1	60	+1, +1, -1, -1, -1, +1, -1, -1	103	+1, -1, -1, +1, +1, -1, -1, +1
18	+1, +1, +1, -1, +1, +1, +1, -1	61	+1, +1, -1, -1, -1, -1, +1, +1	104	+1, -1, -1, +1, +1, -1, -1, -1
19	+1, +1, +1, -1, +1, +1, -1, +1	62	+1, +1, -1, -1, -1, -1, +1, -1	105	+1, -1, -1, +1, -1, +1, +1, +1
20	+1, +1, +1, -1, +1, +1, -1, -1	63	+1, +1, -1, -1, -1, -1, -1, +1	106	+1, -1, -1, +1, -1, +1, +1, -1
21	+1, +1, +1, -1, +1, -1, +1, +1	64	+1, +1, -1, -1, -1, -1, -1, -1	107	+1, -1, -1, +1, -1, +1, -1, +1
22	+1, +1, +1, -1, +1, -1, +1, -1	65	+1, -1, +1, +1, +1, +1, +1, +1	108	+1, -1, -1, +1, -1, +1, -1, -1
23	+1, +1, +1, -1, +1, -1, -1, +1	66	+1, -1, +1, +1, +1, +1, +1, -1	109	+1, -1, -1, +1, -1, -1, +1, +1
24	+1, +1, +1, -1, +1, -1, -1, -1	67	+1, -1, +1, +1, +1, +1, -1, +1	110	+1, -1, -1, +1, -1, -1, +1, -1
25	+1, +1, +1, -1, -1, +1, +1, +1	68	+1, -1, +1, +1, +1, +1, -1, -1	111	+1, -1, -1, +1, -1, -1, -1, +1
26	+1, +1, +1, -1, -1, +1, +1, -1	69	+1, -1, +1, +1, +1, -1, +1, +1	112	+1, -1, -1, +1, -1, -1, -1, -1
27	+1, +1, +1, -1, -1, +1, -1, +1	70	+1, -1, +1, +1, +1, -1, +1, -1	113	+1, -1, -1, -1, +1, +1, +1, +1
28	+1, +1, +1, -1, -1, +1, -1, -1	71	+1, -1, +1, +1, +1, -1, -1, +1	114	+1, -1, -1, -1, +1, +1, +1, -1
29	+1, +1, +1, -1, -1, -1, +1, +1	72	+1, -1, +1, +1, +1, -1, -1, -1	115	+1, -1, -1, -1, +1, +1, -1, +1
30	+1, +1, +1, -1, -1, -1, +1, -1	73	+1, -1, +1, +1, -1, +1, +1, +1	116	+1, -1, -1, -1, +1, +1, -1, -1
31	+1, +1, +1, -1, -1, -1, -1, +1	74	+1, -1, +1, +1, -1, +1, +1, -1	117	+1, -1, -1, -1, +1, -1, +1, +1
32	+1, +1, +1, -1, -1, -1, -1, -1	75	+1, -1, +1, +1, -1, +1, -1, +1	118	+1, -1, -1, -1, +1, -1, +1, -1
33	+1, +1, -1, +1, +1, +1, +1, +1	76	+1, -1, +1, +1, -1, +1, -1, -1	119	+1, -1, -1, -1, +1, -1, -1, +1
34	+1, +1, -1, +1, +1, +1, +1, -1	77	+1, -1, +1, +1, -1, -1, +1, +1	120	+1, -1, -1, -1, +1, -1, -1, -1
35	+1, +1, -1, +1, +1, +1, -1, +1	78	+1, -1, +1, +1, -1, -1, +1, -1	121	+1, -1, -1, -1, -1, +1, +1, +1
36	+1, +1, -1, +1, +1, +1, -1, -1	79	+1, -1, +1, +1, -1, -1, -1, +1	122	+1, -1, -1, -1, -1, +1, +1, -1
37	+1, +1, -1, +1, +1, -1, +1, +1	80	+1, -1, +1, +1, -1, -1, -1, -1	123	+1, -1, -1, -1, -1, +1, -1, +1
38	+1, +1, -1, +1, +1, -1, +1, -1	81	+1, -1, +1, -1, +1, +1, +1, +1	124	+1, -1, -1, -1, -1, +1, -1, -1
39	+1, +1, -1, +1, +1, -1, -1, +1	82	+1, -1, +1, -1, +1, +1, +1, -1	125	+1, -1, -1, -1, -1, -1, +1, +1
40	+1, +1, -1, +1, +1, -1, -1, -1	83	+1, -1, +1, -1, +1, +1, -1, +1	126	+1, -1, -1, -1, -1, -1, +1, -1
41	+1, +1, -1, +1, -1, +1, +1, +1	84	+1, -1, +1, -1, +1, +1, -1, -1	127	+1, -1, -1, -1, -1, -1, -1, +1
42	+1, +1, -1, +1, -1, +1, +1, -1	85	+1, -1, +1, -1, +1, -1, +1, +1	128	+1, -1, -1, -1, -1, -1, -1, -1
43	+1, +1, -1, +1, -1, +1, -1, +1	86	+1, -1, +1, -1, +1, -1, +1, -1		

### 5.2.3 Balance

**Case I: Perfect balance,  $Bal = 0$**

Considering perfect balance, it is possible to define the subset  $M_P$  as follows. Note that the available sequences is 35.

Num.	Binary sequence	Num.	Binary sequence	Num.	Binary sequence
1	+1, +1, +1, +1, -1, -1, -1	13	+1, +1, -1, -1, -1, +1, +1, -1	25	+1, -1, +1, -1, -1, -1, +1, +1
2	+1, +1, +1, -1, +1, -1, -1	14	+1, +1, -1, -1, -1, +1, -1, +1	26	+1, -1, -1, +1, +1, +1, -1, -1
3	+1, +1, +1, -1, -1, +1, -1, -1	15	+1, +1, -1, -1, -1, -1, +1, +1	27	+1, -1, -1, +1, +1, -1, +1, -1
4	+1, +1, +1, -1, -1, -1, +1, -1	16	+1, -1, +1, +1, +1, -1, -1, -1	28	+1, -1, -1, +1, +1, -1, -1, +1
5	+1, +1, +1, -1, -1, -1, -1, +1	17	+1, -1, +1, +1, -1, +1, -1, -1	29	+1, -1, -1, +1, -1, +1, +1, -1
6	+1, +1, -1, +1, +1, -1, -1, -1	18	+1, -1, +1, +1, -1, -1, +1, -1	30	+1, -1, -1, +1, -1, +1, -1, +1
7	+1, +1, -1, +1, -1, +1, -1, -1	19	+1, -1, +1, +1, -1, -1, -1, +1	31	+1, -1, -1, +1, -1, -1, +1, +1
8	+1, +1, -1, +1, -1, -1, +1, -1	20	+1, -1, +1, -1, +1, +1, -1, -1	32	+1, -1, -1, -1, +1, +1, +1, -1
9	+1, +1, -1, +1, -1, -1, -1, +1	21	+1, -1, +1, -1, +1, -1, +1, -1	33	+1, -1, -1, -1, +1, +1, -1, +1
10	+1, +1, -1, -1, +1, +1, -1, -1	22	+1, -1, +1, -1, +1, -1, -1, +1	34	+1, -1, -1, -1, +1, -1, +1, +1
11	+1, +1, -1, -1, +1, -1, +1, -1	23	+1, -1, +1, -1, -1, +1, +1, -1	35	+1, -1, -1, -1, -1, +1, +1, +1
12	+1, +1, -1, -1, +1, -1, -1, +1	24	+1, -1, +1, -1, -1, +1, -1, +1		

**Case II: Soft balance with  $Bal = 0, \pm 2$**

However, considering soft balance, the number of available sequences increases to  $M_{sf1} = 91$ .

Num.	Binary sequence	Num.	Binary sequence	Num.	Binary sequence
1	+1, +1, +1, +1, +1, -1, -1, -1	32	+1, +1, -1, -1, +1, -1, +1, -1	63	+1, -1, +1, -1, -1, +1, -1, -1
2	+1, +1, +1, +1, -1, +1, -1, -1	33	+1, +1, -1, -1, +1, -1, -1, +1	64	+1, -1, +1, -1, -1, -1, +1, +1
3	+1, +1, +1, +1, -1, -1, +1, -1	34	+1, +1, -1, -1, +1, -1, -1, -1	65	+1, -1, +1, -1, -1, -1, +1, -1
4	+1, +1, +1, +1, -1, -1, -1, +1	35	+1, +1, -1, -1, -1, +1, +1, +1	66	+1, -1, +1, -1, -1, -1, -1, +1
5	+1, +1, +1, +1, -1, -1, -1, -1	36	+1, +1, -1, -1, -1, +1, +1, -1	67	+1, -1, -1, +1, +1, +1, +1, -1
6	+1, +1, +1, -1, +1, +1, -1, -1	37	+1, +1, -1, -1, -1, +1, -1, +1	68	+1, -1, -1, +1, +1, +1, -1, +1
7	+1, +1, +1, -1, +1, -1, +1, -1	38	+1, +1, -1, -1, -1, +1, -1, -1	69	+1, -1, -1, +1, +1, +1, -1, -1
8	+1, +1, +1, -1, +1, -1, -1, +1	39	+1, +1, -1, -1, -1, -1, +1, +1	70	+1, -1, -1, +1, +1, -1, +1, +1
9	+1, +1, +1, -1, +1, -1, -1, -1	40	+1, +1, -1, -1, -1, -1, +1, -1	71	+1, -1, -1, +1, +1, -1, +1, -1
10	+1, +1, +1, -1, -1, +1, +1, -1	41	+1, +1, -1, -1, -1, -1, -1, +1	72	+1, -1, -1, +1, +1, -1, -1, +1
11	+1, +1, +1, -1, -1, +1, -1, +1	42	+1, -1, +1, +1, +1, +1, -1, -1	73	+1, -1, -1, +1, +1, -1, -1, -1
12	+1, +1, +1, -1, -1, +1, -1, -1	43	+1, -1, +1, +1, +1, -1, +1, -1	74	+1, -1, -1, +1, -1, +1, +1, +1
13	+1, +1, +1, -1, -1, -1, +1, +1	44	+1, -1, +1, +1, +1, -1, -1, +1	75	+1, -1, -1, +1, -1, +1, +1, -1
14	+1, +1, +1, -1, -1, -1, +1, -1	45	+1, -1, +1, +1, +1, -1, -1, -1	76	+1, -1, -1, +1, -1, +1, -1, +1
15	+1, +1, +1, -1, -1, -1, -1, +1	46	+1, -1, +1, +1, -1, +1, +1, -1	77	+1, -1, -1, +1, -1, +1, -1, -1
16	+1, +1, +1, -1, -1, -1, -1, -1	47	+1, -1, +1, +1, -1, +1, -1, +1	78	+1, -1, -1, +1, -1, -1, +1, +1
17	+1, +1, -1, +1, +1, +1, -1, -1	48	+1, -1, +1, +1, -1, +1, -1, -1	79	+1, -1, -1, +1, -1, -1, +1, -1
18	+1, +1, -1, +1, +1, -1, +1, -1	49	+1, -1, +1, +1, -1, -1, +1, +1	80	+1, -1, -1, +1, -1, -1, -1, +1
19	+1, +1, -1, +1, +1, -1, -1, +1	50	+1, -1, +1, +1, -1, -1, +1, -1	81	+1, -1, -1, -1, +1, +1, +1, +1
20	+1, +1, -1, +1, +1, -1, -1, -1	51	+1, -1, +1, +1, -1, -1, -1, +1	82	+1, -1, -1, -1, +1, +1, +1, -1
21	+1, +1, -1, +1, -1, +1, +1, -1	52	+1, -1, +1, +1, -1, -1, -1, -1	83	+1, -1, -1, -1, +1, +1, -1, +1
22	+1, +1, -1, +1, -1, +1, -1, +1	53	+1, -1, +1, -1, +1, +1, +1, -1	84	+1, -1, -1, -1, +1, +1, -1, -1
23	+1, +1, -1, +1, -1, +1, -1, -1	54	+1, -1, +1, -1, +1, +1, -1, +1	85	+1, -1, -1, -1, +1, -1, +1, +1
24	+1, +1, -1, +1, -1, -1, +1, +1	55	+1, -1, +1, -1, +1, +1, -1, -1	86	+1, -1, -1, -1, +1, -1, +1, -1
25	+1, +1, -1, +1, -1, -1, +1, -1	56	+1, -1, +1, -1, +1, -1, +1, +1	87	+1, -1, -1, -1, +1, -1, -1, +1
26	+1, +1, -1, +1, -1, -1, -1, +1	57	+1, -1, +1, -1, +1, -1, +1, -1	88	+1, -1, -1, -1, -1, +1, +1, +1
27	+1, +1, -1, +1, -1, -1, -1, -1	58	+1, -1, +1, -1, +1, -1, -1, +1	89	+1, -1, -1, -1, -1, +1, +1, -1
28	+1, +1, -1, -1, +1, +1, +1, -1	59	+1, -1, +1, -1, +1, -1, -1, -1	90	+1, -1, -1, -1, -1, +1, -1, +1
29	+1, +1, -1, -1, +1, +1, -1, +1	60	+1, -1, +1, -1, -1, +1, +1, +1	91	+1, -1, -1, -1, -1, -1, +1, +1
30	+1, +1, -1, -1, +1, +1, -1, -1	61	+1, -1, +1, -1, -1, +1, +1, -1		
31	+1, +1, -1, -1, +1, -1, +1, +1	62	+1, -1, +1, -1, -1, +1, -1, +1		

**Case III: Soft balance with  $Bal = 0, \pm 2, \pm 4$**

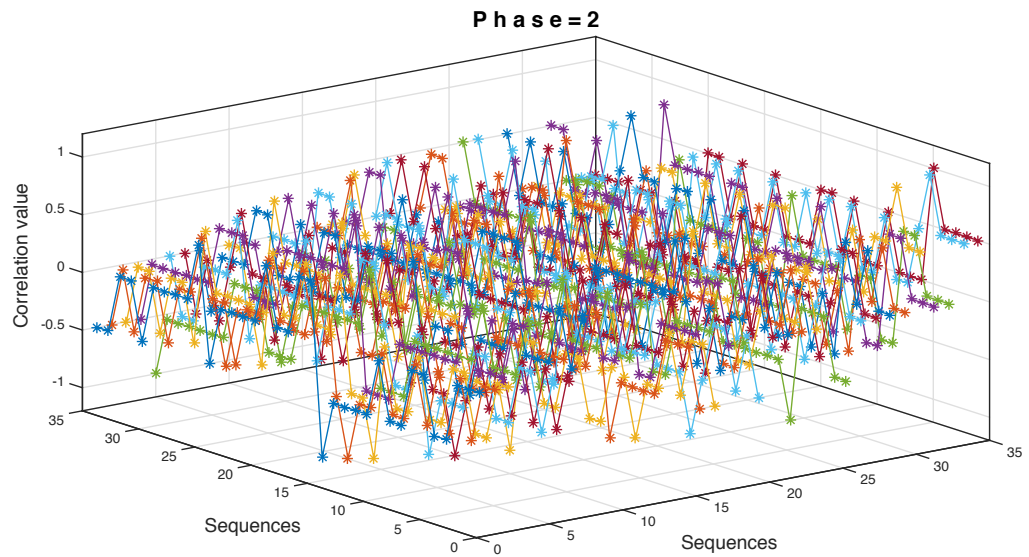
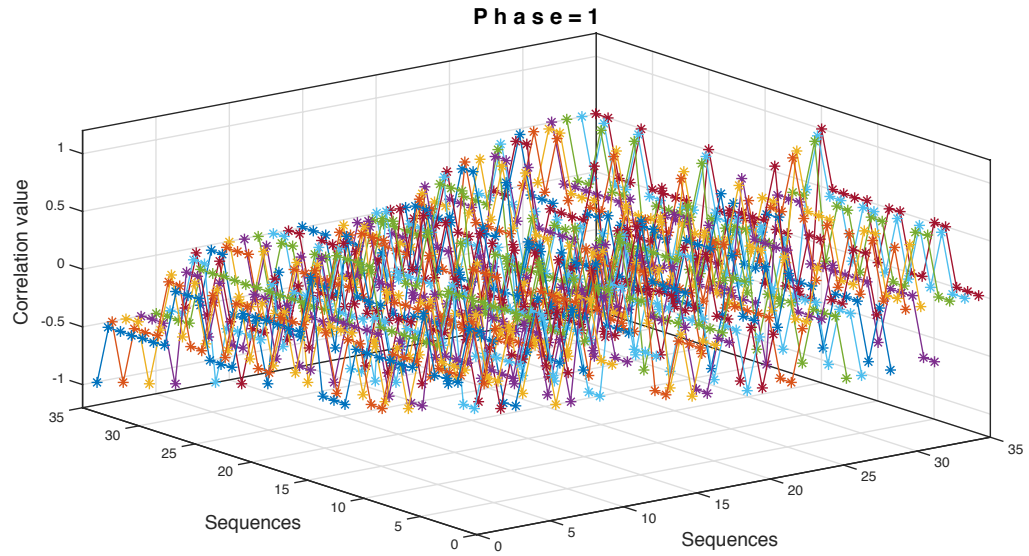
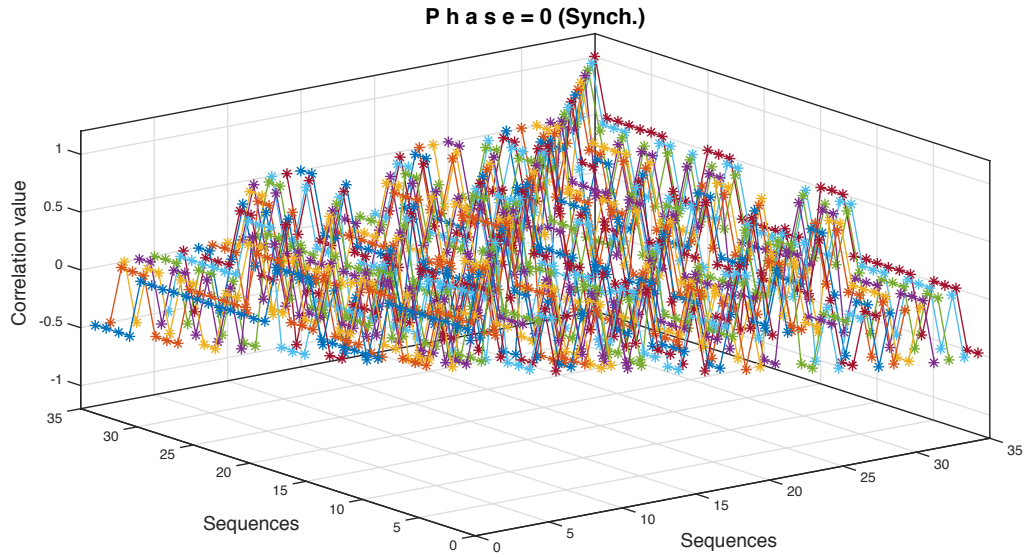
In this case, the number of available sequences increases to  $M_{sf2} = 119$ .

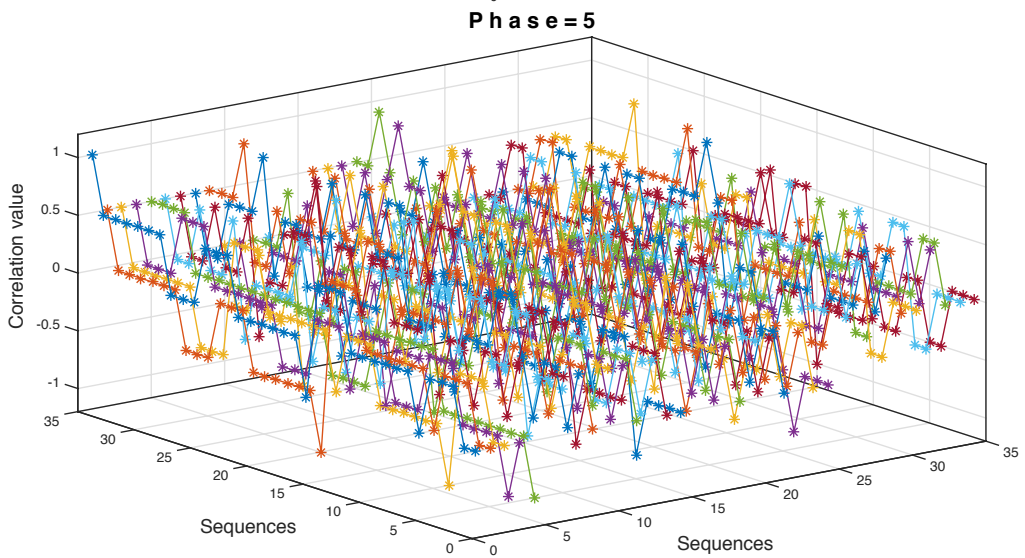
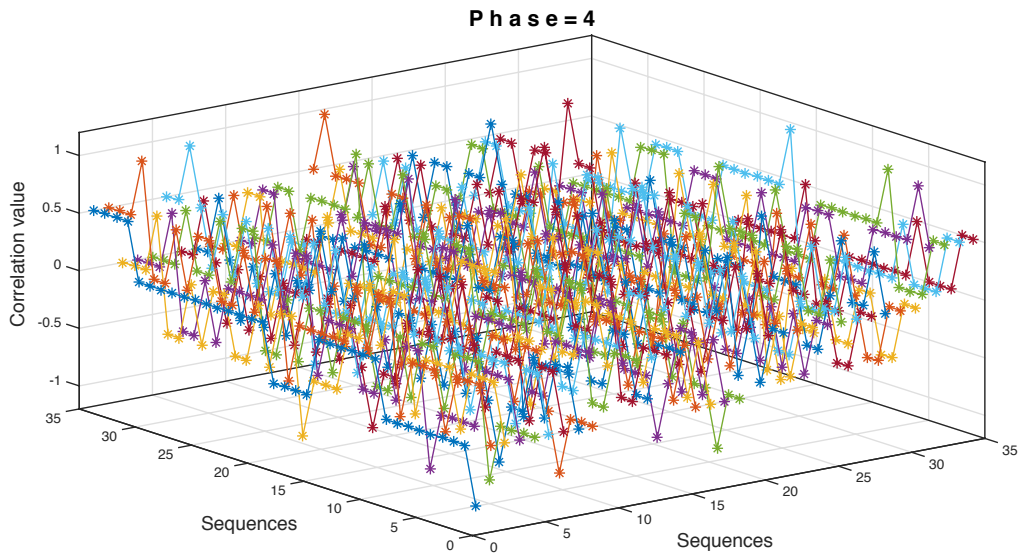
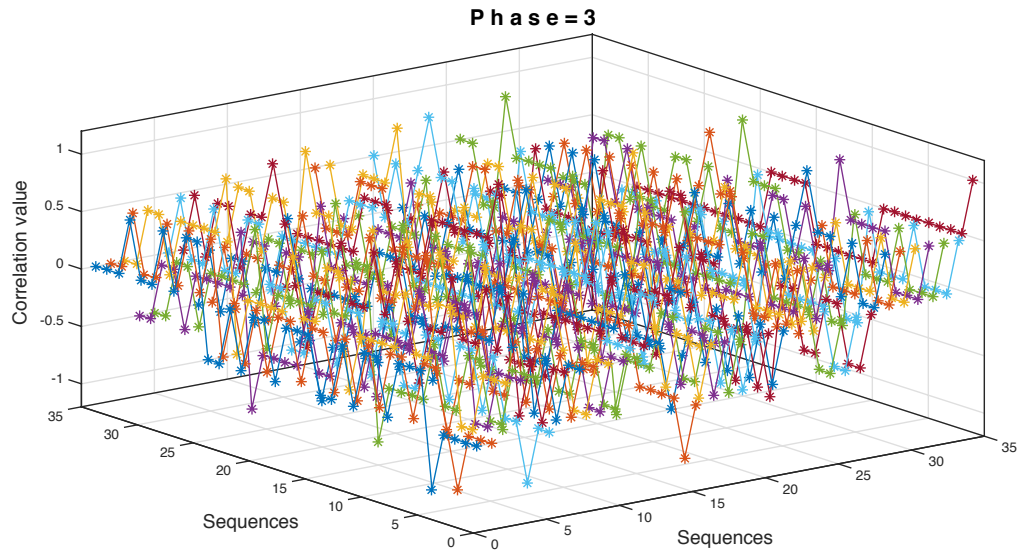
Num.	Binary sequence	Num.	Binary sequence	Num.	Binary sequence
1	+1, +1, +1 +1, +1, +1, -1, -1	41	+1, +1, -1, +1, -1, -1, -1, -1	81	+1, -1, +1, -1, -1, +1, +1, +1
2	+1, +1, +1, +1, +1, -1, +1, -1	42	+1, +1, -1, -1, +1, +1, +1, +1	82	+1, -1, +1, -1, -1, +1, +1, -1
3	+1, +1, +1, +1, +1, -1, -1, +1	43	+1, +1, -1, -1, +1, +1, +1, -1	83	+1, -1, +1, -1, -1, +1, -1, +1
4	+1, +1, +1, +1, +1, -1, -1, -1	44	+1, +1, -1, -1, +1, +1, -1, +1	84	+1, -1, +1, -1, -1, +1, -1, -1
5	+1, +1, +1, +1, -1, +1, +1, -1	45	+1, +1, -1, -1, +1, +1, -1, -1	85	+1, -1, +1, -1, -1, -1, +1, +1
6	+1, +1, +1, +1, -1, +1, -1, +1	46	+1, +1, -1, -1, +1, -1, +1, +1	86	+1, -1, +1, -1, -1, -1, +1, -1
7	+1, +1, +1, +1, -1, +1, -1, -1	47	+1, +1, -1, -1, +1, -1, +1, -1	87	+1, -1, +1, -1, -1, -1, -1, +1
8	+1, +1, +1, +1, -1, -1, +1, +1	48	+1, +1, -1, -1, +1, -1, -1, +1	88	+1, -1, +1, -1, -1, -1, -1, -1
9	+1, +1, +1, +1, -1, -1, +1, -1	49	+1, +1, -1, -1, +1, -1, -1, -1	89	+1, -1, -1, +1, +1, +1, +1, +1
10	+1, +1, +1, +1, -1, -1, -1, +1	50	+1, +1, -1, -1, -1, +1, +1, +1	90	+1, -1, -1, +1, +1, +1, +1, -1
11	+1, +1, +1, +1, -1, -1, -1, -1	51	+1, +1, -1, -1, -1, +1, +1, -1	91	+1, -1, -1, +1, +1, +1, -1, +1
12	+1, +1, +1, -1, +1, +1, +1, -1	52	+1, +1, -1, -1, -1, +1, -1, +1	92	+1, -1, -1, +1, +1, +1, -1, -1
13	+1, +1, +1, -1, +1, +1, -1, +1	53	+1, +1, -1, -1, -1, +1, -1, -1	93	+1, -1, -1, +1, +1, -1, +1, +1
14	+1, +1, +1, -1, +1, +1, -1, -1	54	+1, +1, -1, -1, -1, -1, +1, +1	94	+1, -1, -1, +1, +1, -1, +1, -1
15	+1, +1, +1, -1, +1, -1, +1, +1	55	+1, +1, -1, -1, -1, -1, +1, -1	95	+1, -1, -1, +1, +1, -1, -1, +1
16	+1, +1, +1, -1, +1, -1, +1, -1	56	+1, +1, -1, -1, -1, -1, -1, +1	96	+1, -1, -1, +1, +1, -1, -1, -1
17	+1, +1, +1, -1, +1, -1, -1, +1	57	+1, +1, -1, -1, -1, -1, -1, -1	97	+1, -1, -1, +1, -1, +1, +1, +1
18	+1, +1, +1, -1, +1, -1, -1, -1	58	+1, -1, +1, +1, +1, +1, +1, -1	98	+1, -1, -1, +1, -1, +1, +1, -1
19	+1, +1, +1, -1, -1, +1, +1, +1	59	+1, -1, +1, +1, +1, +1, -1, +1	99	+1, -1, -1, +1, -1, +1, -1, +1
20	+1, +1, +1, -1, -1, +1, +1, -1	60	+1, -1, +1, +1, +1, +1, -1, -1	100	+1, -1, -1, +1, -1, +1, -1, -1
21	+1, +1, +1, -1, -1, +1, -1, +1	61	+1, -1, +1, +1, +1, -1, +1, +1	101	+1, -1, -1, +1, -1, -1, +1, +1
22	+1, +1, +1, -1, -1, +1, -1, -1	62	+1, -1, +1, +1, +1, -1, +1, -1	102	+1, -1, -1, +1, -1, -1, +1, -1
23	+1, +1, +1, -1, -1, -1, +1, +1	63	+1, -1, +1, +1, +1, -1, -1, +1	103	+1, -1, -1, +1, -1, -1, -1, +1
24	+1, +1, +1, -1, -1, -1, +1, -1	64	+1, -1, +1, +1, +1, -1, -1, -1	104	+1, -1, -1, +1, -1, -1, -1, -1
25	+1, +1, +1, -1, -1, -1, -1, +1	65	+1, -1, +1, +1, -1, +1, +1, +1	105	+1, -1, -1, -1, +1, +1, +1, +1
26	+1, +1, +1, -1, -1, -1, -1, -1	66	+1, -1, +1, +1, -1, +1, +1, -1	106	+1, -1, -1, -1, +1, +1, +1, -1
27	+1, +1, -1, +1, +1, +1, +1, -1	67	+1, -1, +1, +1, -1, +1, -1, +1	107	+1, -1, -1, -1, +1, +1, -1, +1
28	+1, +1, -1, +1, +1, +1, -1, +1	68	+1, -1, +1, +1, -1, +1, -1, -1	108	+1, -1, -1, -1, +1, +1, -1, -1
29	+1, +1, -1, +1, +1, +1, -1, -1	69	+1, -1, +1, +1, -1, -1, +1, +1	109	+1, -1, -1, -1, +1, -1, +1, +1
30	+1, +1, -1, +1, +1, -1, +1, +1	70	+1, -1, +1, +1, -1, -1, +1, -1	110	+1, -1, -1, -1, +1, -1, +1, -1
31	+1, +1, -1, +1, +1, -1, +1, -1	71	+1, -1, +1, +1, -1, -1, -1, +1	111	+1, -1, -1, -1, +1, -1, -1, +1
32	+1, +1, -1, +1, +1, -1, -1, +1	72	+1, -1, +1, +1, -1, -1, -1, -1	112	+1, -1, -1, -1, +1, -1, -1, -1
33	+1, +1, -1, +1, +1, -1, -1, -1	73	+1, -1, +1, -1, +1, +1, +1, +1	113	+1, -1, -1, -1, -1, +1, +1, +1
34	+1, +1, -1, +1, -1, +1, +1, +1	74	+1, -1, +1, -1, +1, +1, +1, -1	114	+1, -1, -1, -1, -1, +1, +1, -1
35	+1, +1, -1, +1, -1, +1, +1, -1	75	+1, -1, +1, -1, +1, +1, -1, +1	115	+1, -1, -1, -1, -1, +1, -1, +1
36	+1, +1, -1, +1, -1, +1, -1, +1	76	+1, -1, +1, -1, +1, +1, -1, -1	116	+1, -1, -1, -1, -1, +1, -1, -1
37	+1, +1, -1, +1, -1, +1, -1, -1	77	+1, -1, +1, -1, +1, -1, +1, +1	117	+1, -1, -1, -1, -1, -1, +1, +1
38	+1, +1, -1, +1, -1, -1, +1, +1	78	+1, -1, +1, -1, +1, -1, +1, -1	118	+1, -1, -1, -1, -1, -1, +1, -1
39	+1, +1, -1, +1, -1, -1, +1, -1	79	+1, -1, +1, -1, +1, -1, -1, +1	119	+1, -1, -1, -1, -1, -1, -1, +1
40	+1, +1, -1, +1, -1, -1, -1, +1	80	+1, -1, +1, -1, +1, -1, -1, -1		

### 5.2.4 Correlation

In order to show the correlation values of all the subsets, the 3-D Side-lobes graphs are presented for all cases in all phases. The bigger mutual correlation subsets are presented and, finally, the DC property will be proved.

#### 3-D Side-lobe graphs of Perfect balance subset (C-I)







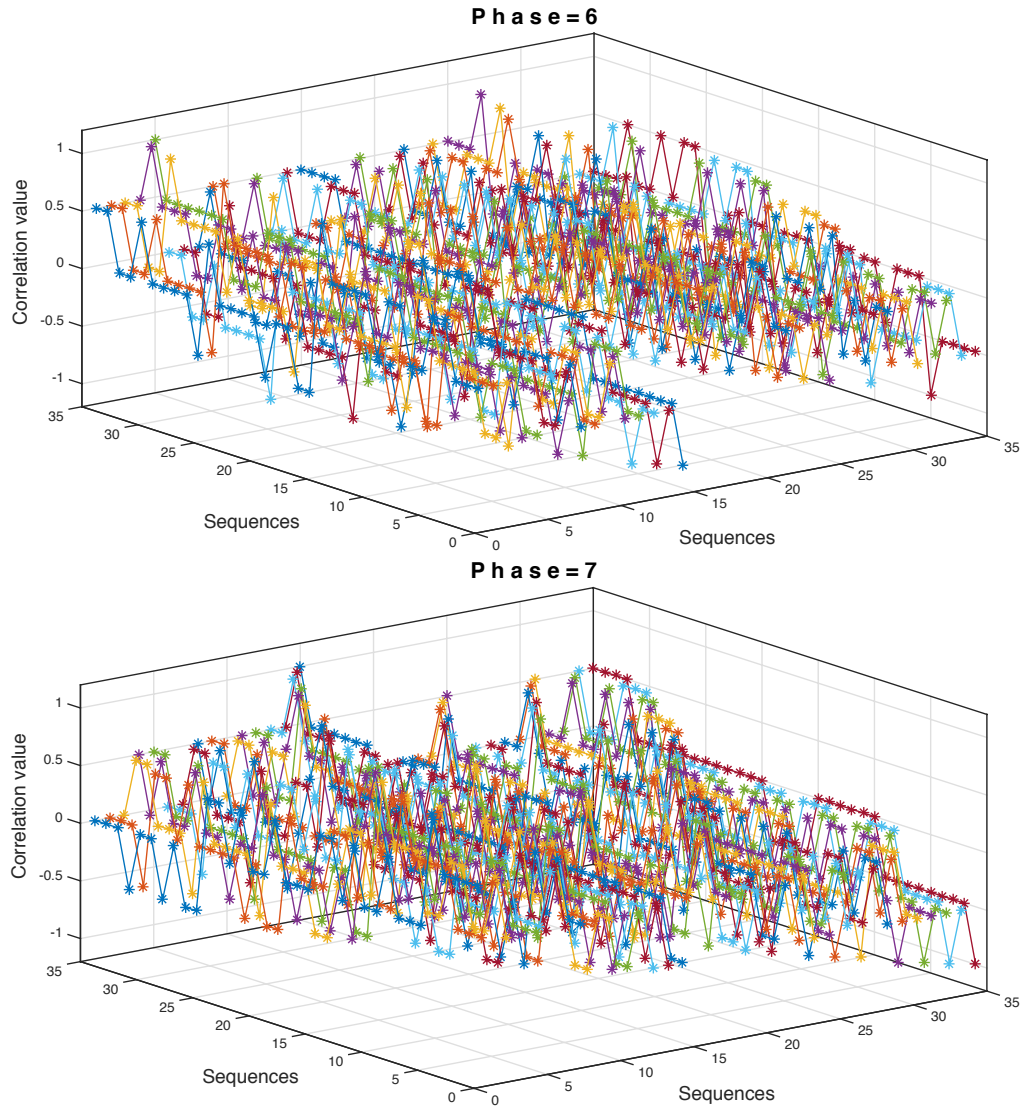


Figure 5.2: Perfect balance subset.

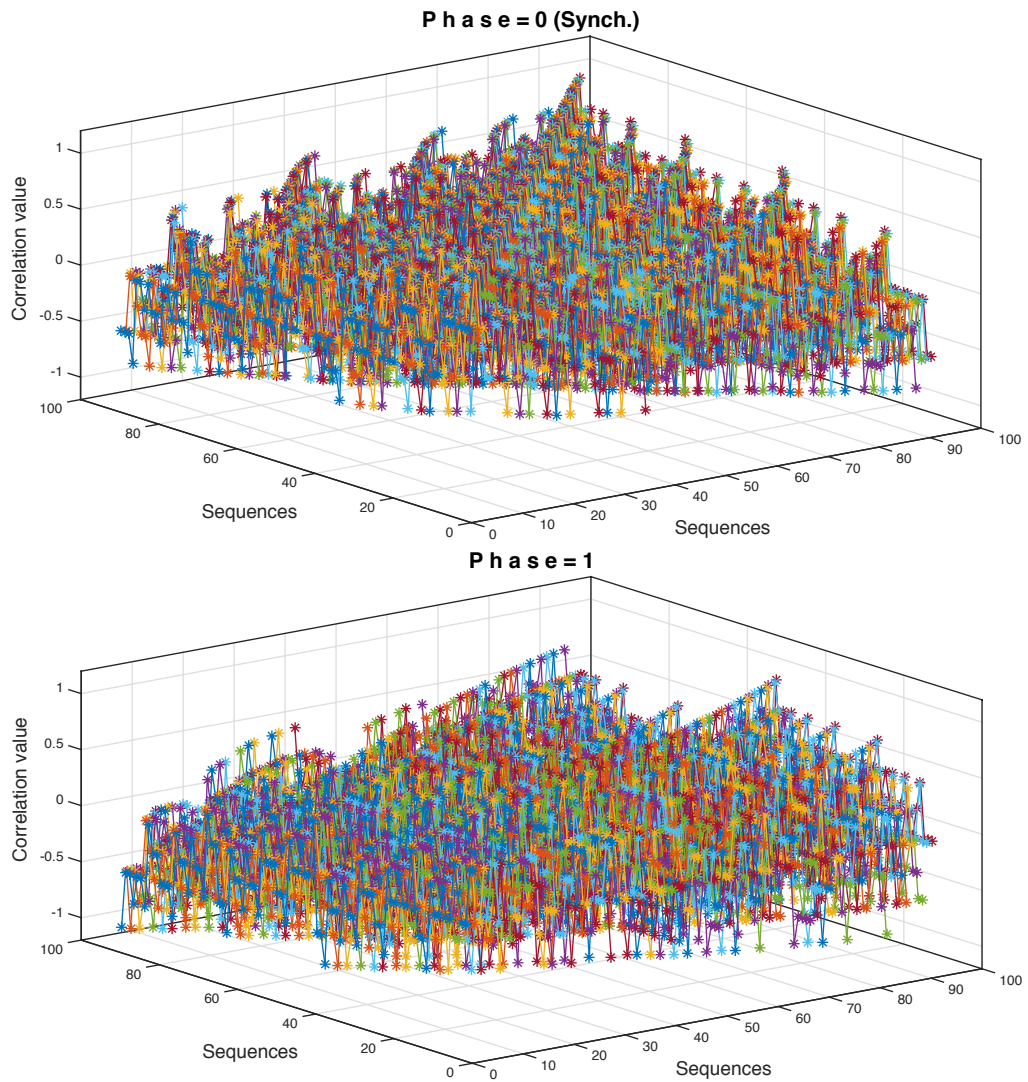
**C-I correlation: Autocorrelation & cross-correlation**

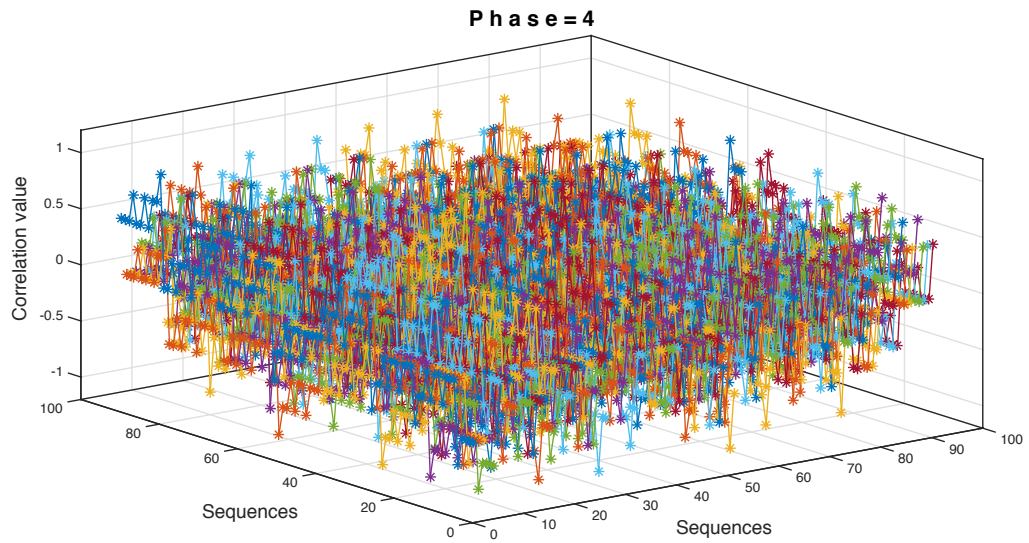
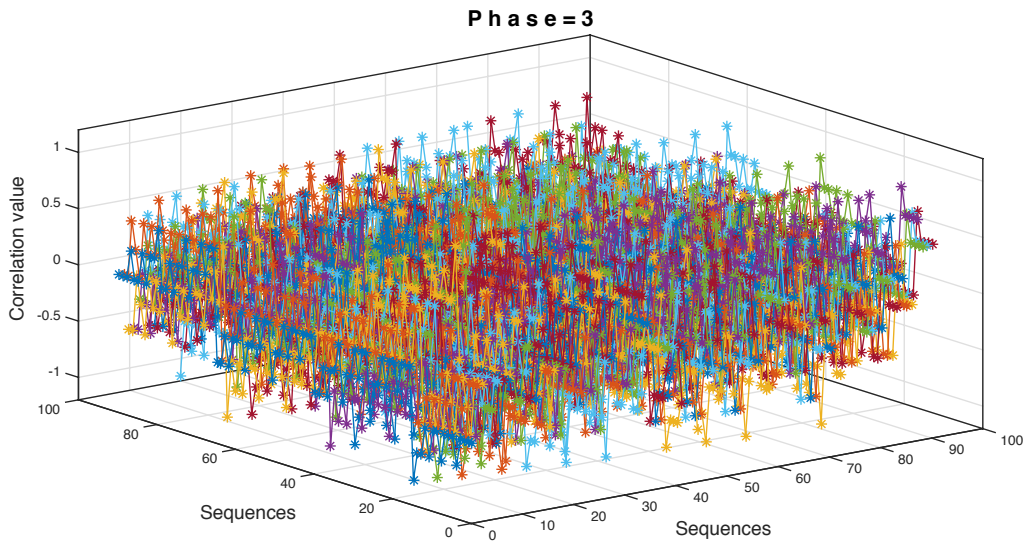
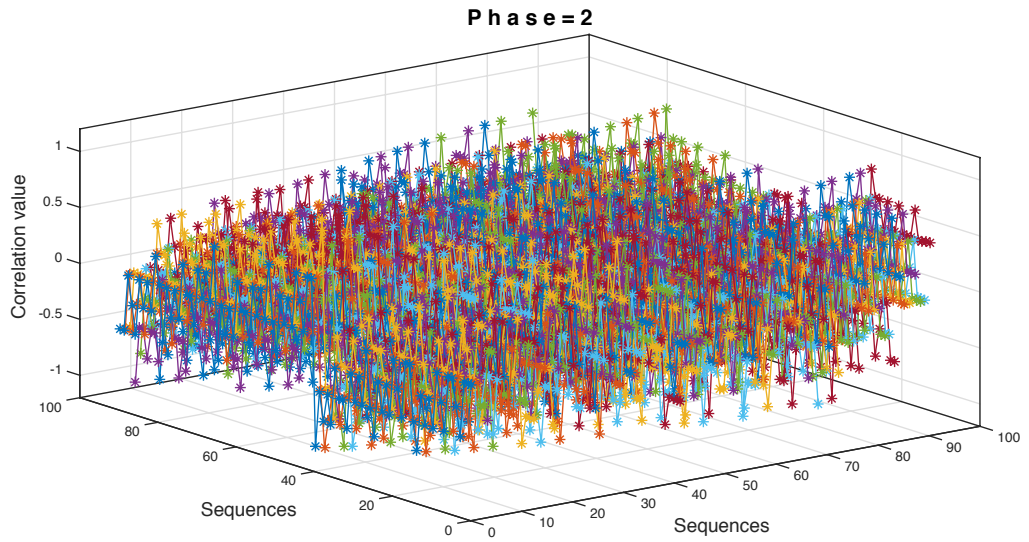
Aforementioned, it is only considered ideal values due to computational restrictions. Recall that perfect values are given by equations 4.2 and 4.3. Thus, the bigger subsets of sequences with mutual correlation from C-I have 7 elements and their indexes are given as  $MS1_{\#}^{(7)} = \{S^i, S^j, S^k, S^l, S^m, S^n, S^o\}$  as are shown in the following list.



#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>
1	1	10	15	21	24	28	29	11	2	9	13	17	25	27	33	21	4	7	12	16	24	31	32
2	1	10	15	22	23	27	30	12	2	9	13	18	24	26	34	22	4	7	12	19	20	27	35
3	1	11	14	20	25	28	29	13	3	6	15	18	22	30	32	23	4	9	10	16	24	29	34
4	1	11	14	22	23	26	31	14	3	6	15	19	21	29	33	24	4	9	10	17	22	27	35
5	1	12	13	20	25	27	30	15	3	8	12	16	25	30	32	25	5	6	13	17	21	31	33
6	1	12	13	21	24	26	31	16	3	8	12	19	21	26	35	26	5	6	13	18	20	30	34
7	2	7	15	18	24	28	32	17	3	9	11	16	25	29	33	27	5	7	11	16	23	31	33
8	2	7	15	19	23	27	33	18	3	9	11	18	22	26	35	28	5	7	11	18	20	28	35
9	2	8	14	17	25	28	32	19	4	6	14	17	22	31	32	29	5	8	10	16	23	30	34
10	2	8	14	19	23	26	34	20	4	6	14	19	20	29	34	30	5	8	10	17	21	28	35

**3-D Side-lobe graphs of Soft balance 1 subset (C-II)**







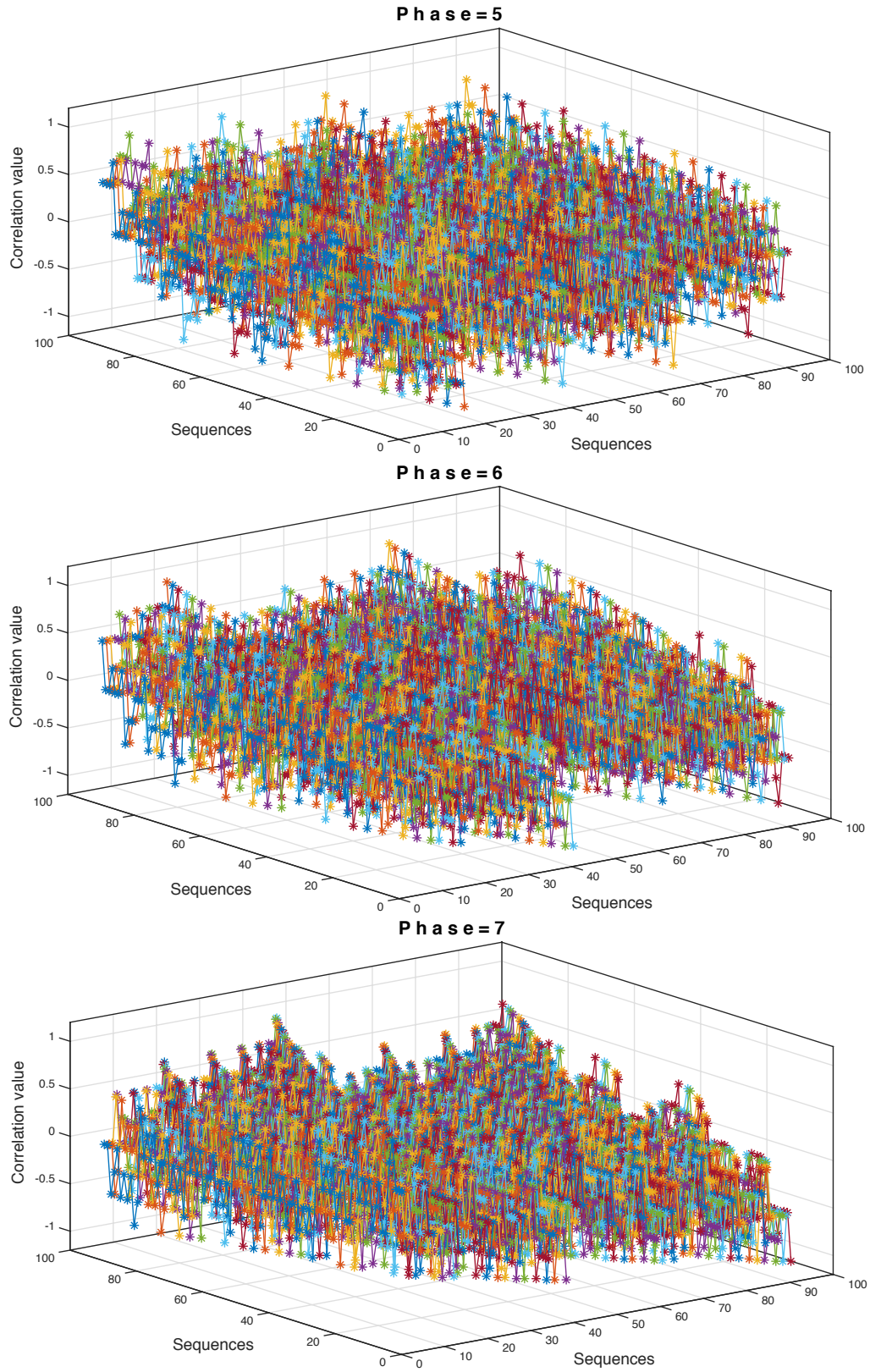


Figure 5.3: Soft balance 1 subset.

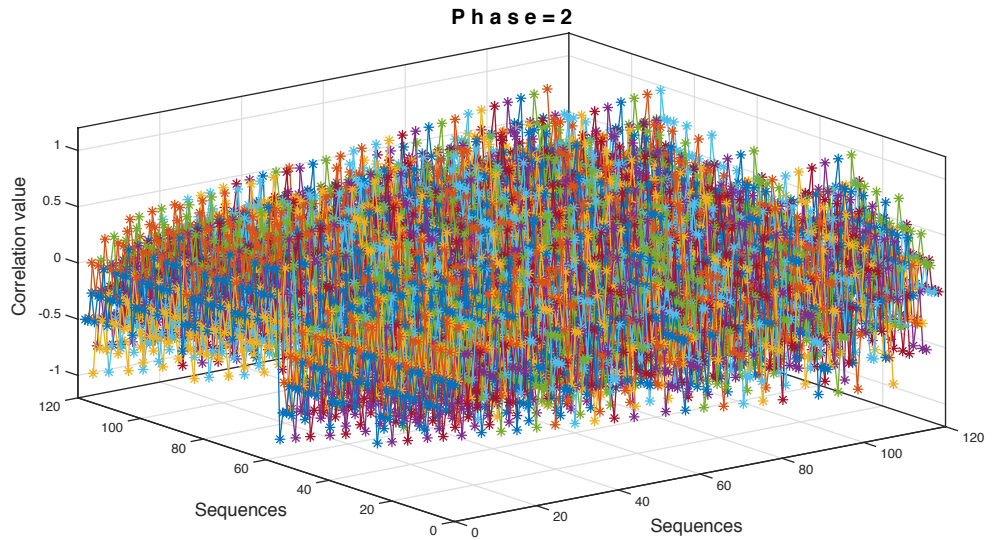
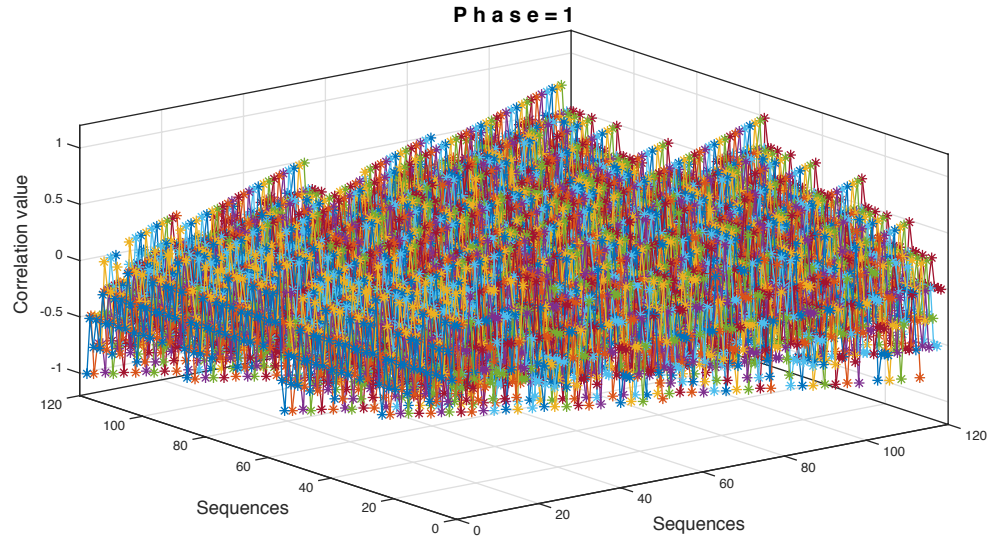
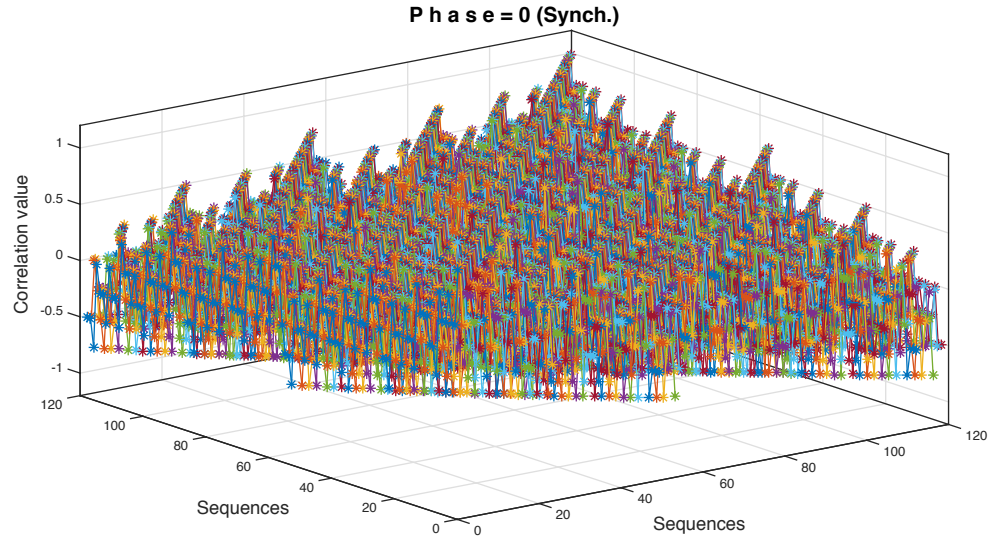
**C-II correlation: Autocorrelation & cross-correlation**

The bigger subsets from C-II also have 7 elements and their indexes are given as  $MS2_{\#}^{(7)} = \{S^i, S^j, S^k, S^l, S^m, S^n, S^o\}$  as following list shows. Recall that the analysis only considers  $Phase = 0$ .

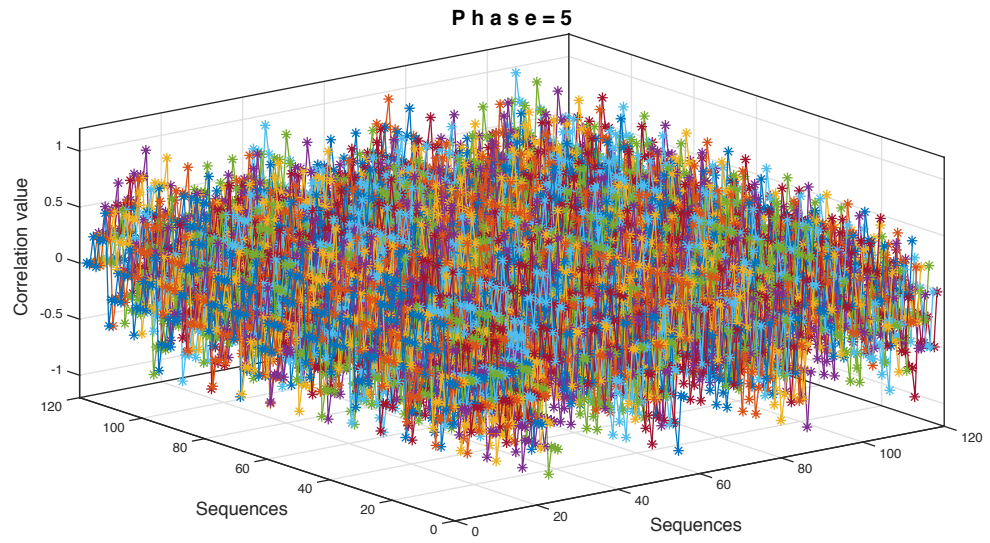
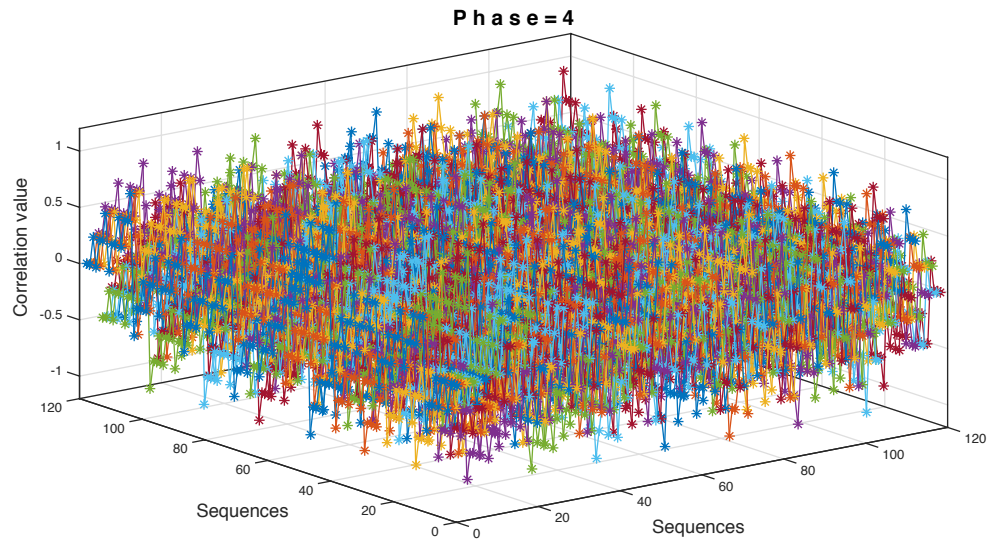
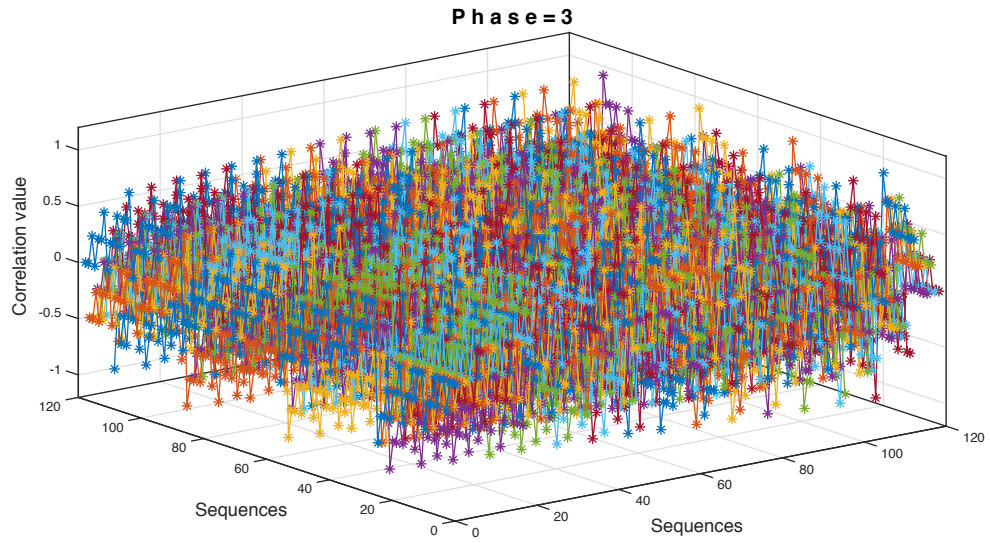
	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>
1	1	10	22	31	49	54	67	91	4	6	18	35	46	56	68	181	9	23	39	50	62	72	82
2	1	10	22	31	66	79	84	92	4	6	18	35	65	77	87	182	9	23	39	51	61	71	83
3	1	10	24	29	47	56	67	93	4	6	21	31	43	60	68	183	9	25	37	48	64	72	82
4	1	10	24	29	66	77	86	94	4	6	21	31	65	73	90	184	9	25	37	51	61	69	85
5	1	10	41	47	56	79	84	95	4	6	40	43	60	77	87	185	9	26	36	48	64	71	83
6	1	10	41	49	54	77	86	96	4	6	40	46	56	73	90	186	9	26	36	50	62	69	85
7	1	11	21	31	49	53	68	97	4	7	17	35	46	54	70	187	10	17	41	47	59	79	81
8	1	11	21	31	65	80	84	98	4	7	17	35	63	79	87	188	10	17	41	52	54	74	86
9	1	11	24	28	46	56	68	99	4	7	21	29	42	60	70	189	10	18	41	49	59	77	81
10	1	11	24	28	65	77	87	100	4	7	21	29	63	73	91	190	10	18	41	52	56	74	84
11	1	11	40	46	56	80	84	101	4	7	38	42	60	79	87	191	10	22	34	42	66	79	81
12	1	11	40	49	53	77	87	102	4	7	38	46	54	73	91	192	10	22	34	52	54	67	91
13	1	13	21	29	47	53	70	103	4	10	17	31	43	54	74	193	10	24	34	43	66	77	81
14	1	13	21	29	63	80	86	104	4	10	17	31	59	79	90	194	10	24	34	52	56	67	90
15	1	13	22	28	46	54	70	105	4	10	18	29	42	56	74	195	10	27	29	42	66	74	86
16	1	13	22	28	63	79	87	106	4	10	18	29	59	77	91	196	10	27	29	47	59	67	91
17	1	13	38	46	54	80	86	107	4	10	34	42	56	79	90	197	10	27	31	43	66	74	84
18	1	13	38	47	53	79	87	108	4	10	34	43	54	77	91	198	10	27	31	49	59	67	90
19	1	21	41	47	65	70	84	109	4	17	40	43	63	74	87	199	11	17	40	46	59	80	81
20	1	21	41	49	63	68	86	110	4	17	40	46	59	70	90	200	11	17	40	52	53	74	87
21	1	22	40	46	66	70	84	111	4	18	38	42	65	74	87	201	11	19	40	49	59	77	81
22	1	22	40	49	63	67	87	112	4	18	38	46	59	68	91	202	11	19	40	52	56	74	84
23	1	24	38	46	66	68	86	113	4	21	34	42	65	70	90	203	11	21	34	42	65	80	81
24	1	24	38	47	65	67	87	114	4	21	34	43	63	68	91	204	11	21	34	52	53	68	91
25	1	28	41	54	65	70	77	115	4	29	40	56	63	73	74	205	11	24	34	44	65	77	81
26	1	28	41	56	63	68	79	116	4	29	40	59	60	70	77	206	11	24	34	52	56	68	89
27	1	29	40	53	66	70	77	117	4	31	38	54	65	73	74	207	11	27	28	42	65	74	87
28	1	29	40	56	63	67	80	118	4	31	38	59	60	68	79	208	11	27	28	46	59	68	91
29	1	31	38	53	66	68	79	119	4	34	35	54	65	70	77	209	11	27	31	44	65	74	84
30	1	31	38	54	65	67	80	120	4	34	35	56	63	68	79	210	11	27	31	49	59	68	89
31	2	7	19	35	49	54	67	121	5	30	39	57	62	72	75	211	12	20	39	50	58	76	82
32	2	7	19	35	66	79	84	122	5	30	39	58	61	71	76	212	12	20	39	51	57	75	83
33	2	7	24	29	44	60	67	123	5	32	37	55	64	72	75	213	12	25	33	45	64	76	82
34	2	7	24	29	66	73	89	124	5	32	37	58	61	69	78	214	12	25	33	51	57	69	88
35	2	7	41	44	60	79	84	125	5	33	36	55	64	71	76	215	12	26	32	45	64	75	83
36	2	7	41	49	54	73	89	126	5	33	36	57	62	69	78	216	12	26	32	50	58	69	88
37	2	8	18	35	49	53	68	127	6	13	18	22	44	46	81	217	13	18	38	46	59	80	81
38	2	8	18	35	65	80	84	128	6	13	18	22	52	87	89	218	13	18	38	52	53	74	87
39	2	8	24	28	43	60	68	129	6	13	19	21	43	47	81	219	13	19	38	47	59	79	81
40	2	8	24	28	65	73	90	130	6	13	19	21	52	86	90	220	13	19	38	52	54	74	86
41	2	8	40	43	60	80	84	131	6	13	27	43	47	87	89	221	13	21	34	43	63	80	81
42	2	8	40	49	53	73	90	132	6	13	27	44	46	86	90	222	13	21	34	52	53	70	90
43	2	13	18	29	44	53	74	133	6	18	41	44	65	77	81	223	13	22	34	44	63	79	81
44	2	13	18	29	59	80	89	134	6	18	41	52	56	68	89	224	13	22	34	52	54	70	89
45	2	13	19	28	43	54	74	135	6	19	40	43	66	77	81	225	13	27	28	43	63	74	87
46	2	13	19	28	59	79	90	136	6	19	40	52	56	67	90	226	13	27	28	46	59	70	90
47	2	13	34	43	54	80	89	137	6	21	41	47	65	73	81	227	13	27	29	44	63	74	86
48	2	13	34	44	53	79	90	138	6	21	41	52	60	68	86	228	13	27	29	47	59	70	89

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>
49	2	18	41	44	65	74	84	139	6	22	40	46	66	73	81	229	14	20	37	48	58	78	82
50	2	18	41	49	59	68	89	140	6	22	40	52	60	67	87	230	14	20	37	51	55	75	85
51	2	19	40	43	66	74	84	141	6	27	31	43	66	68	89	231	14	23	33	45	62	78	82
52	2	19	40	49	59	67	90	142	6	27	31	44	65	67	90	232	14	23	33	51	55	71	88
53	2	24	34	43	66	68	89	143	6	27	35	46	66	68	86	233	14	26	30	45	62	75	85
54	2	24	34	44	65	67	90	144	6	27	35	47	65	67	87	234	14	26	30	48	58	71	88
55	2	28	41	54	65	73	74	145	7	11	17	24	44	46	81	235	15	20	36	48	57	78	83
56	2	28	41	59	60	68	79	146	7	11	17	24	52	87	89	236	15	20	36	50	55	76	85
57	2	29	40	53	66	73	74	147	7	11	19	21	42	49	81	237	15	23	32	45	61	78	83
58	2	29	40	59	60	67	80	148	7	11	19	21	52	84	91	238	15	23	32	50	55	72	88
59	2	34	35	53	66	68	79	149	7	11	27	42	49	87	89	239	15	25	30	45	61	76	85
60	2	34	35	54	65	67	80	150	7	11	27	44	46	84	91	240	15	25	30	48	57	72	88
61	3	6	19	35	47	56	67	151	7	17	41	44	63	79	81	241	16	17	24	43	47	87	89
62	3	6	19	35	66	77	86	152	7	17	41	52	54	70	89	242	16	17	24	44	46	86	90
63	3	6	22	31	44	60	67	153	7	19	38	42	66	79	81	243	16	17	31	43	54	80	89
64	3	6	22	31	66	73	89	154	7	19	38	52	54	67	91	244	16	17	31	44	53	79	90
65	3	6	41	44	60	77	86	155	7	21	41	49	63	73	81	245	16	17	35	46	54	80	86
66	3	6	41	47	56	73	89	156	7	21	41	52	60	70	84	246	16	17	35	47	53	79	87
67	3	8	17	35	47	53	70	157	7	24	38	46	66	73	81	247	16	18	22	42	49	87	89
68	3	8	17	35	63	80	86	158	7	24	38	52	60	67	87	248	16	18	22	44	46	84	91
69	3	8	22	28	42	60	70	159	7	27	29	42	66	70	89	249	16	18	29	42	56	80	89
70	3	8	22	28	63	73	91	160	7	27	29	44	63	67	91	250	16	18	29	44	53	77	91
71	3	8	38	42	60	80	86	161	7	27	35	46	66	70	84	251	16	18	35	46	56	80	84
72	3	8	38	47	53	73	91	162	7	27	35	49	63	67	87	252	16	18	35	49	53	77	87
73	3	11	17	31	44	53	74	163	8	10	17	24	43	47	81	253	16	19	21	42	49	86	90
74	3	11	17	31	59	80	89	164	8	10	17	24	52	86	90	254	16	19	21	43	47	84	91
75	3	11	19	28	42	56	74	165	8	10	18	22	42	49	81	255	16	19	28	42	56	79	90
76	3	11	19	28	59	77	91	166	8	10	18	22	52	84	91	256	16	19	28	43	54	77	91
77	3	11	34	42	56	80	89	167	8	10	27	42	49	86	90	257	16	19	35	47	56	79	84
78	3	11	34	44	53	77	91	168	8	10	27	43	47	84	91	258	16	19	35	49	54	77	86
79	3	17	41	44	63	74	86	169	8	17	40	43	63	80	81	259	16	21	29	42	60	80	86
80	3	17	41	47	59	70	89	170	8	17	40	52	53	70	90	260	16	21	29	47	53	73	91
81	3	19	38	42	66	74	86	171	8	18	38	42	65	80	81	261	16	21	31	43	60	80	84
82	3	19	38	47	59	67	91	172	8	18	38	52	53	68	91	262	16	21	31	49	53	73	90
83	3	22	34	42	66	70	89	173	8	22	40	49	63	73	81	263	16	22	28	42	60	79	87
84	3	22	34	44	63	67	91	174	8	22	40	52	60	70	84	264	16	22	28	46	54	73	91
85	3	28	41	56	63	73	74	175	8	24	38	47	65	73	81	265	16	22	31	44	60	79	84
86	3	28	41	59	60	70	77	176	8	24	38	52	60	68	86	266	16	22	31	49	54	73	89
87	3	31	38	53	66	73	74	177	8	27	28	42	65	70	90	267	16	24	28	43	60	77	87
88	3	31	38	59	60	67	80	178	8	27	28	43	63	68	91	268	16	24	28	46	56	73	90
89	3	34	35	53	66	70	77	179	8	27	35	47	65	70	84	269	16	24	29	44	60	77	86
90	3	34	35	56	63	67	80	180	8	27	35	49	63	68	86	270	16	24	29	47	56	73	89

**3-D Side-lobe graphs of Soft balance 2 subset (C-III)**







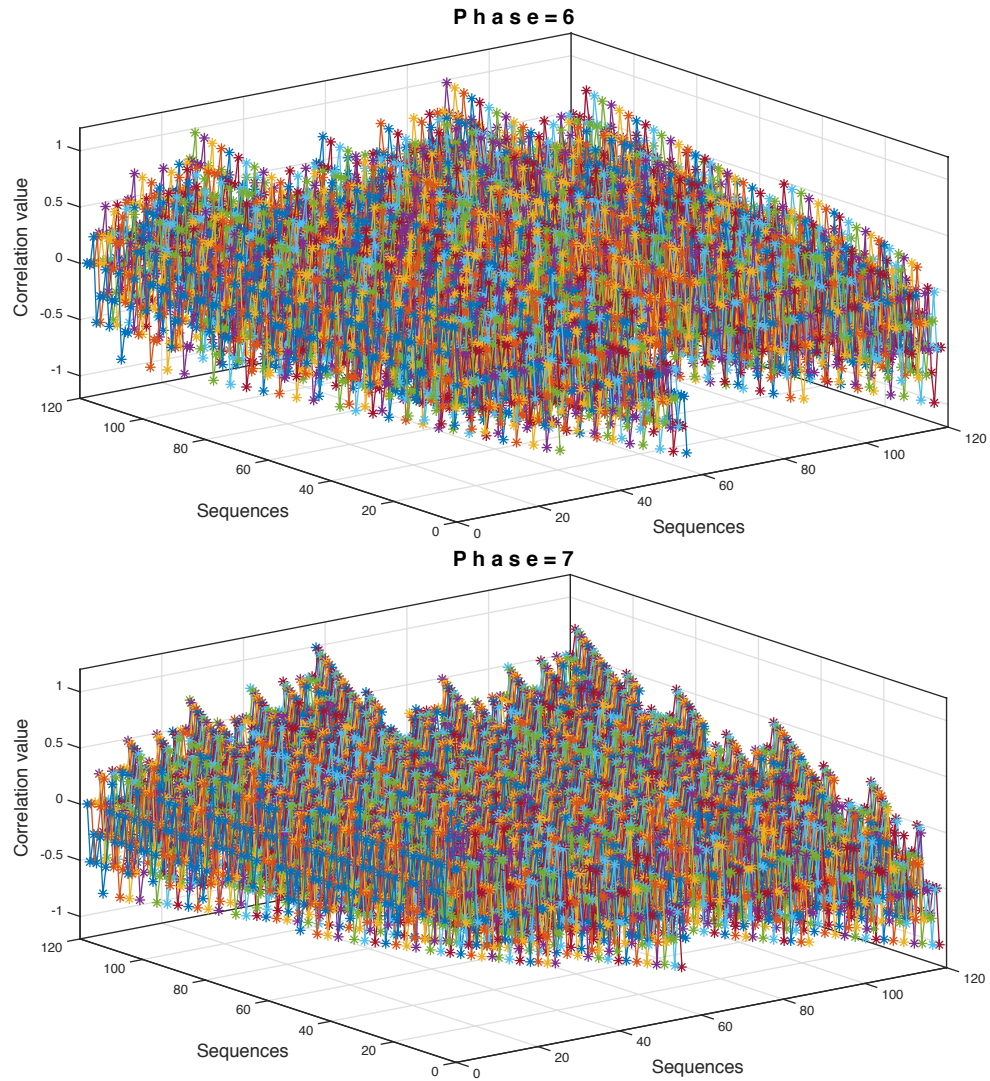


Figure 5.4: Soft balance 2 subset.

**C-III correlation: Autocorrelation & cross-correlation**

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>
1	1	8	42	57	78	83	95	98	106	5	18	30	52	71	73	92	118
2	1	8	42	57	79	82	94	99	107	5	18	40	42	59	85	94	116
3	1	8	47	52	73	88	95	98	108	5	18	40	42	61	83	92	118
4	1	8	47	52	79	82	89	104	109	5	25	28	47	61	76	104	113
5	1	8	48	51	73	88	94	99	110	5	25	28	47	64	73	101	116
6	1	8	48	51	78	83	89	104	111	5	25	30	45	59	78	104	113
7	1	15	34	57	70	83	95	106	112	5	25	30	45	64	73	99	118
8	1	15	34	57	71	82	94	107	113	5	25	33	42	59	78	101	116
9	1	15	39	52	65	88	95	106	114	5	25	33	42	61	76	99	118
10	1	15	39	52	71	82	89	112	115	6	12	30	57	64	85	98	107
11	1	15	40	51	65	88	94	107	116	6	12	30	57	70	79	92	113
12	1	15	40	51	70	83	89	112	117	6	12	33	54	61	88	98	107
13	1	19	30	57	70	79	99	106	118	6	12	33	54	70	79	89	116
14	1	19	30	57	71	78	98	107	119	6	12	39	48	61	88	92	113
15	1	19	39	48	61	88	99	106	120	6	12	39	48	64	85	89	116
16	1	19	39	48	71	78	89	116	121	6	15	27	57	64	82	101	107
17	1	19	40	47	61	88	98	107	122	6	15	27	57	70	76	95	113
18	1	19	40	47	70	79	89	116	123	6	15	33	51	58	88	101	107
19	1	24	30	52	65	79	104	106	124	6	15	33	51	70	76	89	119



	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>
20	1	24	30	52	71	73	98	112	125	6	15	39	45	58	88	95	113
21	1	24	34	48	61	83	104	106	126	6	15	39	45	64	82	89	119
22	1	24	34	48	71	73	94	116	127	6	18	27	54	61	82	104	107
23	1	24	40	42	61	83	98	112	128	6	18	27	54	70	73	95	116
24	1	24	40	42	65	79	94	116	129	6	18	30	51	58	85	104	107
25	1	25	30	51	65	78	104	107	130	6	18	30	51	70	73	92	119
26	1	25	30	51	70	73	99	112	131	6	18	39	42	58	85	95	116
27	1	25	34	47	61	82	104	107	132	6	18	39	42	61	82	92	119
28	1	25	34	47	70	73	95	116	133	6	24	27	48	61	76	104	113
29	1	25	39	42	61	82	99	112	134	6	24	27	48	64	73	101	116
30	1	25	39	42	65	78	95	116	135	6	24	30	45	58	79	104	113
31	2	6	42	57	76	85	95	98	136	6	24	30	45	64	73	98	119
32	2	6	42	57	79	82	92	101	137	6	24	33	42	58	79	101	116
33	2	6	45	54	73	88	95	98	138	6	24	33	42	61	76	98	119
34	2	6	45	54	79	82	89	104	139	8	12	28	57	64	83	98	109
35	2	6	48	51	73	88	92	101	140	8	12	28	57	68	79	94	113
36	2	6	48	51	76	85	89	104	141	8	12	33	52	59	88	98	109
37	2	13	34	57	68	85	95	106	142	8	12	33	52	68	79	89	118
38	2	13	34	57	71	82	92	109	143	8	12	37	48	59	88	94	113
39	2	13	37	54	65	88	95	106	144	8	12	37	48	64	83	89	118
40	2	13	37	54	71	82	89	112	145	8	13	27	57	64	82	99	109
41	2	13	40	51	65	88	92	109	146	8	13	27	57	68	78	95	113
42	2	13	40	51	68	85	89	112	147	8	13	33	51	58	88	99	109
43	2	19	28	57	68	79	101	106	148	8	13	33	51	68	78	89	119
44	2	19	28	57	71	76	98	109	149	8	13	37	47	58	88	95	113
45	2	19	37	48	59	88	101	106	150	8	13	37	47	64	82	89	119
46	2	19	37	48	71	76	89	118	151	8	18	27	52	59	82	104	109
47	2	19	40	45	59	88	98	109	152	8	18	27	52	68	73	95	118
48	2	19	40	45	68	79	89	118	153	8	18	28	51	58	83	104	109
49	2	22	28	54	65	79	104	106	154	8	18	28	51	68	73	94	119
50	2	22	28	54	71	73	98	112	155	8	18	37	42	58	83	95	118
51	2	22	34	48	59	85	104	106	156	8	18	37	42	59	82	94	119
52	2	22	34	48	71	73	92	118	157	8	22	27	48	59	78	104	113
53	2	22	40	42	59	85	98	112	158	8	22	27	48	64	73	99	118
54	2	22	40	42	65	79	92	118	159	8	22	28	47	58	79	104	113
55	2	25	28	51	65	76	104	109	160	8	22	28	47	64	73	98	119
56	2	25	28	51	68	73	101	112	161	8	22	33	42	58	79	99	118
57	2	25	34	45	59	82	104	109	162	8	22	33	42	59	78	98	119
58	2	25	34	45	68	73	95	118	163	11	12	28	54	61	83	98	112
59	2	25	37	42	59	82	101	112	164	11	12	28	54	65	79	94	116
60	2	25	37	42	65	76	95	118	165	11	12	30	52	59	85	98	112
61	3	5	42	57	76	85	94	99	166	11	12	30	52	65	79	92	118
62	3	5	42	57	78	83	92	101	167	11	12	34	48	59	85	94	116
63	3	5	45	54	73	88	94	99	168	11	12	34	48	61	83	92	118
64	3	5	45	54	78	83	89	104	169	11	13	27	54	61	82	99	112
65	3	5	47	52	73	88	92	101	170	11	13	27	54	65	78	95	116
66	3	5	47	52	76	85	89	104	171	11	13	30	51	58	85	99	112
67	3	12	34	57	68	85	94	107	172	11	13	30	51	65	78	92	119
68	3	12	34	57	70	83	92	109	173	11	13	34	47	58	85	95	116
69	3	12	37	54	65	88	94	107	174	11	13	34	47	61	82	92	119
70	3	12	37	54	70	83	89	112	175	11	15	27	52	59	82	101	112
71	3	12	39	52	65	88	92	109	176	11	15	27	52	65	76	95	118
72	3	12	39	52	68	85	89	112	177	11	15	28	51	58	83	101	112
73	3	19	27	57	68	78	101	107	178	11	15	28	51	65	76	94	119
74	3	19	27	57	70	76	99	109	179	11	15	34	45	58	83	95	118
75	3	19	37	47	58	88	101	107	180	11	15	34	45	59	82	94	119
76	3	19	37	47	70	76	89	119	181	11	19	27	48	59	78	101	116
77	3	19	39	45	58	88	99	109	182	11	19	27	48	61	76	99	118

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>		<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>
78	3	19	39	45	68	78	89	119	183	11	19	28	47	58	79	101	116
79	3	22	27	54	65	78	104	107	184	11	19	28	47	61	76	98	119
80	3	22	27	54	70	73	99	112	185	11	19	30	45	58	79	99	118
81	3	22	34	47	58	85	104	107	186	11	19	30	45	59	78	98	119
82	3	22	34	47	70	73	92	119	187	12	25	28	39	61	68	112	113
83	3	22	39	42	58	85	99	112	188	12	25	28	39	64	65	109	116
84	3	22	39	42	65	78	92	119	189	12	25	30	37	59	70	112	113
85	3	24	27	52	65	76	104	109	190	12	25	30	37	64	65	107	118
86	3	24	27	52	68	73	101	112	191	12	25	33	34	59	70	109	116
87	3	24	34	45	58	83	104	109	192	12	25	33	34	61	68	107	118
88	3	24	34	45	68	73	94	119	193	13	24	27	40	61	68	112	113
89	3	24	37	42	58	83	101	112	194	13	24	27	40	64	65	109	116
90	3	24	37	42	65	76	94	119	195	13	24	30	37	58	71	112	113
91	5	13	30	57	64	85	99	106	196	13	24	30	37	64	65	106	119
92	5	13	30	57	71	78	92	113	197	13	24	33	34	58	71	109	116
93	5	13	33	54	61	88	99	106	198	13	24	33	34	61	68	106	119
94	5	13	33	54	71	78	89	116	199	15	22	27	40	59	70	112	113
95	5	13	40	47	61	88	92	113	200	15	22	27	40	64	65	107	118
96	5	13	40	47	64	85	89	116	201	15	22	28	39	58	71	112	113
97	5	15	28	57	64	83	101	106	202	15	22	28	39	64	65	106	119
98	5	15	28	57	71	76	94	113	203	15	22	33	34	58	71	107	118
99	5	15	33	52	59	88	101	106	204	15	22	33	34	59	70	106	119
100	5	15	33	52	71	76	89	118	205	18	19	27	40	59	70	109	116
101	5	15	40	45	59	88	94	113	206	18	19	27	40	61	68	107	118
102	5	15	40	45	64	83	89	118	207	18	19	28	39	58	71	109	116
103	5	18	28	54	61	83	104	106	208	18	19	28	39	61	68	106	119
104	5	18	28	54	71	73	94	116	209	18	19	30	37	58	71	107	118
105	5	18	30	52	59	85	104	106	210	18	19	30	37	59	70	106	119

**Double-Correlation**

DC subsets can be calculated from the method described in previous chapter. In this work, all the subsets were gotten, however the extension of results are too large to include them. For this reason, they are presented the number of DC subsets for each case and provided an example. This way, the results are presents as follows

- **Perfect balance sets (C-I).** Each subset presents 28 DC subsets with 4 elements, i.e. the total number of DC subsets is given as  $DC_{C1} = 30 * 28 = 840$ . For example, the set formed from C1,  $\{S_1^{C1}, S_{10}^{C1}, S_{15}^{C1}, S_{21}^{C1}, S_{24}^{C1}, S_{28}^{C1}, S_{29}^{C1}\}$  have 28 DC subsets.

#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
1	1	10	15	21	8	1	10	28	29	15	1	21	28	29	22	10	21	24	29
2	1	10	15	24	9	1	15	21	24	16	1	24	28	29	23	10	21	28	29
3	1	10	15	28	10	1	15	21	29	17	10	15	21	28	24	10	24	28	29
4	1	10	15	29	11	1	15	24	28	18	10	15	21	29	25	15	21	24	28
5	1	10	21	24	12	1	15	28	29	19	10	15	24	28	26	15	21	24	29
6	1	10	21	28	13	1	21	24	28	20	10	15	24	29	27	15	21	28	29
7	1	10	24	29	14	1	21	24	29	21	10	21	24	28	28	15	24	28	29

- **Soft balance sets 1 (C-II).** In this case, again, each subset presents 28 DC subsets with 4 elements, so the total number of DC subsets is  $DC_{C2} = 270 * 28 = 7,560$ . For example, the set formed from C2,  $\{S_1^{C2}, S_{10}^{C2}, S_{22}^{C2}, S_{31}^{C2}, S_{49}^{C2}, S_{54}^{C2}, S_{67}^{C2}\}$  have 28 the following DC subsets.

#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
1	1	10	22	49	8	1	10	54	67	15	1	31	49	67	22	10	31	49	54
2	1	10	22	54	9	1	22	31	49	16	1	49	54	67	23	10	31	49	67
3	1	10	22	67	10	1	22	31	54	17	10	22	31	49	24	10	31	49	67
4	1	10	31	49	11	1	22	31	67	18	10	22	31	54	25	22	31	49	67
5	1	10	31	54	12	1	22	49	54	19	10	22	31	67	26	22	31	54	67
6	1	10	31	67	13	1	22	54	67	20	10	22	49	54	27	22	49	54	67
7	1	10	49	67	14	1	31	49	54	21	10	22	49	67	28	31	49	54	67

Each subset presents 28 DC subsets with 4 elements,

- **Soft balance sets 2 (C-III).** This case also considers mutual correlation sets with 7 elements to form DC subsets, however, the total number of DC subsets with 4 elements keeps in 28, i.e.  $DC_{C3} = 1,950 * 28 = 54,600$  subsets can be derived from C-III. For example, the set formed from C3,  $\{S_1^{C3}, S_8^{C3}, S_{42}^{C3}, S_{57}^{C3}, S_{78}^{C3}, S_{83}^{C3}, S_{95}^{C3}\}$  have the following DC subsets.

#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>
1	1	8	42	78	8	1	8	83	95	15	1	57	78	95	22	8	57	78	83
2	1	8	42	83	9	1	42	57	78	16	1	78	83	95	23	8	57	83	95
3	1	8	42	95	10	1	42	57	83	17	8	42	57	78	24	8	78	83	95
4	1	8	57	78	11	1	42	57	95	18	8	42	57	83	25	42	57	78	95
5	1	8	57	83	12	1	42	78	83	19	8	42	57	95	26	42	57	83	95
6	1	8	57	95	13	1	42	83	95	20	8	42	78	83	27	43	78	83	95
7	1	8	78	95	14	1	57	78	83	21	8	42	78	95	28	57	78	83	95

### 5.2.5 Shifting

This property is verified searching windows with desired correlation. This way, the balance between of ACF and CCF values is crucial because defines the robustness and weakness of codes. In this work, the interference caused by multipath signals is prioritized, so the ACF values is favored in order to incorporate this feature to selecting method. They are defined four cases with different window size  $W_1, W_2, W_3,$  and  $W_4$  which describe desired autocorrelation, i.e. zero correlation value in phases  $\phi = 1, 2, 3$  and 4 respectively. Figure (5.5) shows their correlation behavior. Note that  $W_{A1}$  case considers sequence sets that show ideal correlation values in phases 0, 1, establishing the window size in 1, while  $W_{A2}, W_{A3},$  and  $W_{A4}$  cases consider sequence sets with ideal correlation with window sizes  $W_{A2} = 2, W_{A3} = 3,$  and  $W_{A4} = 4$  respectively. On the other hand, CCF values are also characterised in four cases  $W_{C1}, W_{C2},$  and  $W_{C3}$  which also define the windows size.

In following subsections, the results of the exploration of combinations of the values of ACF and CCF from  $C - I, C - II,$  and  $C - III$  cases are shown in order to provide subsets for different applications.

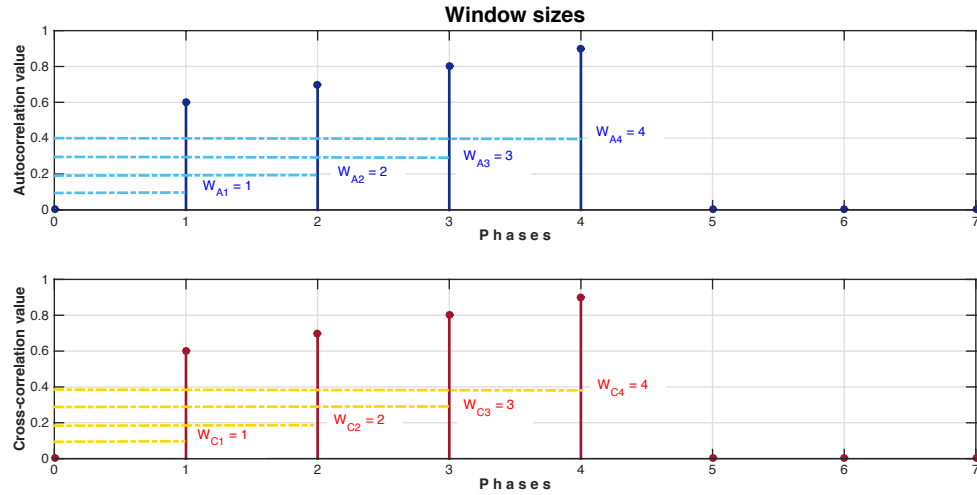


Figure 5.5: Autocorrelation windows:  $W_{A1}$ ,  $W_{A2}$ ,  $W_{A3}$ , and  $W_{A4}$ . Cross-correlation windows:  $W_{C1}$ ,  $W_{C2}$ , and  $W_{C3}$ .

Table 5.2.5 shows the total number of subsets that accomplishes each condition. It considers the mutual correlation codes as  $Mx$  with ACF and CCF window sizes  $W_{Ax}$  and CCF window  $W_{Cx}$  respectively, e.g.  $M2 W_{A1} W_{C1}$  means that they are pairs of codes ( $M = 2$ ) ideal ACF and CCF for phases 0 and 1. Table 5.2.5 shows that from  $C - I$  they are only 20

$Mx$	$W_{Ax}$	$W_{Cx}$	$C - I$	$C - II$	$C - III$
$M2$	$W_{A1}$	$W_{C1}$	$20x2$	$84x2$	$162x2$
$M2$	$W_{A1}$	$W_{C2}$	—	$16x2$	$40x2$
$M2$	$W_{A1}$	$W_{C3}$	—	—	$16x2$
$M2$	$W_{A1}$	$W_{C4}$	—	—	$16x2$
$M2$	$W_{A2}$	$W_{C1}$	—	$16x2$	$42x2$
$M2$	$W_{A2}$	$W_{C2}$	—	—	$8x2$
$M2$	$W_{A2}$	$W_{C3}$	—	—	—
$M2$	$W_{A2}$	$W_{C4}$	—	—	—
$M2$	$W_{A3}$	$W_{C1}$	—	$16x2$	$18x2$
$M2$	$W_{A3}$	$W_{C2}$	—	—	—
$M2$	$W_{A3}$	$W_{C3}$	—	—	—
$M2$	$W_{A3}$	$W_{C4}$	—	—	—
$M2$	$W_{A4}$	$W_{C1}$	—	—	—
$M2$	$W_{A4}$	$W_{C2}$	—	—	—
$M2$	$W_{A4}$	$W_{C3}$	—	—	—
$M2$	$W_{A4}$	$W_{C4}$	—	—	—

pairs that exhibit the relation  $W_{A1} - W_{C1}$  and they are shown in Table 5.1.

**$C - I$  and IFW subsets**

Table 5.1:  $W_{A1} - W_{C1}$  from C-I

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	2	13	5	3	25	9	6	33	13	12	26	17	19	34
2	2	26	6	4	6	10	9	13	14	12	32	18	25	33
3	3	16	7	4	31	11	9	33	15	13	31	19	26	34
4	3	19	8	6	14	12	12	16	16	14	19	20	31	32

**C – II and IFW subsets**

This case increases the number of pairs and include subsets that exhibit  $W_{A2} - W_{C1}$  and  $W_{A3} - W_{C1}$  relationship. It means that the ACF is robustness. Note that both cases have 16 pairs, it means that they are the same as show the Tables 5.2 and 5.3. On the other hand, subsets that exhibit  $W_{A1} - W_{C2}$  are also included.

Table 5.2:  $W_{A1} - W_{C1}$  from C-II

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	2	24	18	8	90	35	19	40	52	31	44	69	52	87
2	2	28	19	9	36	36	19	67	53	31	84	70	60	67
3	2	40	20	9	69	37	19	90	54	33	45	71	60	68
4	2	49	21	10	27	38	20	37	55	33	69	72	60	87
5	2	73	22	10	66	39	20	83	56	33	82	73	64	83
6	3	17	23	10	74	40	24	28	57	34	42	74	66	70
7	3	31	24	11	17	41	24	38	58	34	89	75	66	74
8	3	34	25	11	44	42	24	52	59	36	78	76	66	89
9	3	42	26	11	74	43	24	68	60	37	51	77	67	87
10	3	80	27	11	80	44	26	36	61	38	52	78	67	90
11	6	40	28	11	89	45	26	83	62	38	60	79	69	85
12	6	52	29	12	45	46	27	29	63	40	49	80	70	89
13	6	67	30	12	51	47	27	31	64	40	52	81	73	90
14	8	28	31	12	64	48	27	42	65	42	70	82	74	84
15	8	60	32	14	20	49	27	84	66	44	74	83	78	82
16	8	68	33	14	78	50	29	42	67	49	90	84	80	89
17	8	73	34	17	31	51	29	66	68	51	85			

Table 5.3:  $W_{A1} - W_{C2}$  from C-II

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	2	73	5	10	27	9	29	66	13	42	70
2	3	80	6	11	17	10	31	44	14	49	90
3	6	67	7	19	40	11	34	89	15	52	87
4	8	28	8	24	68	12	38	60	16	74	84

Table 5.4:  $W_{A2} - W_{C1}$  from C-II

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	2	24	5	8	60	9	24	52	13	60	67
2	2	40	6	8	90	10	27	31	14	66	74
3	3	31	7	11	74	11	27	42	15	66	89
4	3	42	8	11	89	12	40	52	16	67	90

Table 5.5:  $W_{A3} - W_{C1}$  from C-II

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	2	24	5	8	60	9	24	52	13	60	67
2	2	40	6	8	90	10	27	31	14	66	74
3	3	31	7	11	74	11	27	42	15	66	89
4	3	42	8	11	89	12	40	52	16	67	90

### *C* – III and IFW subsets

This case also increases the number of pairs and provides new sets with useful properties. There are 162 pairs with  $W_{A1} - W_{C1}$ , see Table 5.6, 40 pairs from  $W_{A1} - W_{C2}$ , see Table 5.7, and subsets with relation  $W_{A1} - W_{C3}$  and  $W_{A1} - W_{C4}$  are incorporated, see Tables 5.8 and 5.9. For  $W_{A2} - W_{C1}$  and  $W_{A2} - W_{C2}$  are 42 and 8 pairs respectively as shown Tables 5.10 and 5.11. Finally,  $W_{A3} - W_{C1}$  condition has 18 pairs, see Table 5.12.

Table 5.6:  $W_{A1} - W_{C1}$  from C-III

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	2	45	28	13	71	55	21	97	82	33	52	109	46	108	136	71	109
2	2	48	29	13	85	56	21	103	83	33	58	110	48	59	137	71	118
3	2	73	30	13	92	57	21	114	84	33	107	111	48	64	138	72	111
4	2	92	31	13	106	58	22	27	85	33	116	112	48	73	139	73	92
5	2	95	32	14	55	59	22	34	86	34	45	113	48	92	140	73	95
6	5	33	33	14	72	60	22	64	87	34	71	114	48	106	141	81	90
7	5	40	34	14	90	61	22	71	88	34	95	115	49	60	142	81	91
8	5	64	35	15	33	62	22	85	89	34	118	116	49	114	143	81	111
9	5	71	36	15	45	63	22	104	90	38	43	117	51	65	144	85	107
10	5	85	37	15	58	64	22	118	91	38	53	118	51	88	145	85	116
11	6	45	38	15	95	65	24	30	92	38	72	119	51	101	146	87	93
12	6	51	39	15	107	66	24	33	93	38	91	120	52	65	147	87	97
13	6	88	40	17	43	67	24	65	94	40	51	121	52	71	148	87	114
14	6	95	41	17	81	68	24	101	95	40	59	122	52	104	149	88	95
15	6	101	42	17	91	69	24	104	96	40	107	123	53	72	150	88	101
16	7	38	43	17	96	70	27	48	97	40	116	124	53	81	151	90	111
17	7	43	44	17	115	71	27	52	98	41	44	125	55	69	152	90	115
18	7	55	45	18	27	72	27	109	99	41	46	126	55	72	153	92	109
19	7	69	46	18	51	73	28	112	100	41	60	127	58	95	154	93	114
20	7	96	47	18	59	74	29	46	101	41	108	128	58	107	155	95	118
21	9	29	48	18	92	75	30	51	102	44	60	129	59	101	156	96	115
22	9	46	49	18	116	76	30	64	103	44	87	130	59	109	157	97	108
23	9	49	50	20	41	77	30	92	104	45	58	131	60	93	158	101	106
24	9	60	51	20	87	78	30	106	105	45	73	132	63	97	159	101	116
25	9	103	52	20	97	79	32	55	106	45	88	133	65	106	160	103	114
26	12	61	53	21	29	80	32	90	107	45	118	134	65	107	161	104	107
27	13	64	54	21	63	81	32	115	108	46	63	135	69	115	162	104	109

Table 5.7:  $W_{A1} - W_{C2}$  from C-III

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	2	45	10	15	45	19	32	55	28	45	118	37	73	95
2	2	95	11	15	95	20	34	45	29	46	63	38	88	95
3	5	85	12	17	43	21	34	95	30	49	114	39	95	118
4	6	45	13	18	59	22	38	91	31	52	104	40	97	108
5	6	95	14	20	41	23	40	116	32	53	81			
6	7	96	15	21	29	24	44	87	33	58	95			
7	9	103	16	24	65	25	45	58	34	60	93			
8	13	64	17	27	109	26	45	73	35	69	115			
9	14	90	18	30	106	27	45	88	36	72	111			

### 5.3 Selection of sequences with $N = 16$

In this section, Walsh and Golay Hadamard sets of sequences with length  $N = 16$  are explored in order to overview some results. Thus, the analysis is reduced because they have inherent features of Hadamard sets described previously such as no negative sequences, ideal

Table 5.8:  $W_{A1} - W_{C3}$  from C-III

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	2	45	5	15	45	9	45	58	13	58	95
2	2	95	6	15	95	10	45	73	14	73	95
3	6	45	7	34	45	11	45	88	15	88	95
4	6	95	8	34	95	12	45	118	16	95	118

Table 5.9:  $W_{A1} - W_{C4}$  from C-III

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	2	45	5	15	45	9	45	58	13	58	95
2	2	95	6	15	95	10	45	73	14	73	95
3	6	45	7	34	45	11	45	88	15	88	95
4	6	95	8	34	95	12	45	118	16	95	118

Table 5.10:  $W_{A2} - W_{C1}$  from C-III

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	5	40	10	13	85	19	24	30	28	40	59	37	81	90
2	5	64	11	13	106	20	24	65	29	40	116	38	85	116
3	5	85	12	17	81	21	24	104	30	41	46	39	87	97
4	7	38	13	17	115	22	27	52	31	41	60	40	87	114
5	7	55	14	18	27	23	27	109	32	52	65	41	90	115
6	9	46	15	18	59	24	28	112	33	52	104	42	104	109
7	9	60	16	18	116	25	30	64	34	55	72			
8	12	61	17	21	97	26	30	106	35	59	109			
9	13	64	18	21	114	27	38	72	36	65	106			

Table 5.11:  $W_{A2} - W_{C2}$  from C-III

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	5	85	3	18	59	5	27	109	7	40	116
2	13	64	4	24	65	6	30	106	8	52	104

Table 5.12:  $W_{A3} - W_{C1}$  from C-III

#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$	#	$I_{\#}^{(1)}$	$I_{\#}^{(2)}$
1	7	38	6	17	81	11	38	72	16	87	97
2	7	55	7	17	115	12	41	46	17	87	114
3	9	46	8	21	97	13	41	60	18	90	115
4	9	60	9	21	114	14	55	72			
5	12	61	10	28	112	15	81	90			



correlation in phase, and their balance property is known i.e. Walsh set has perfect balance and Golay set exhibits soft balance. This way, they are only presented the DC subsets.

### 5.3.1 Walsh-Hadamard set $W^{(15)}$

Similar to WH set with  $N = 8$ , the first sequence only composed by elements +1 is discarded, so the set size is reduced to  $M = 15$ . Thus, DC subsets are given in following list. Note that DC subset size reach to six elements as C. Barrera described in [4].

#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>
1	1	2	3	4	8	12	95	1	5	10	12	13	15	189	3	4	5	7	8	13
2	1	2	3	4	9	13	96	1	5	11	12	13	14	190	3	4	5	7	9	12
3	1	2	3	4	10	14	97	1	6	8	9	10	12	191	3	4	5	7	10	15
4	1	2	3	4	11	15	98	1	6	8	9	11	13	192	3	4	5	7	11	14
5	1	2	3	5	8	13	99	1	6	8	10	11	14	193	3	4	6	7	8	14
6	1	2	3	5	9	12	100	1	6	8	12	13	14	194	3	4	6	7	9	15
7	1	2	3	5	10	15	101	1	6	9	10	11	15	195	3	4	6	7	10	12
8	1	2	3	5	11	14	102	1	6	9	12	13	15	196	3	4	6	7	11	13
9	1	2	3	6	8	14	103	1	6	10	12	14	15	197	3	4	8	9	10	12
10	1	2	3	6	9	15	104	1	6	11	13	14	15	198	3	4	8	9	11	13
11	1	2	3	6	10	12	105	1	7	8	9	10	13	199	3	4	8	10	11	14
12	1	2	3	6	11	13	106	1	7	8	9	11	12	200	3	4	8	12	13	14
13	1	2	3	7	8	15	107	1	7	8	10	11	15	201	3	4	9	10	11	15
14	1	2	3	7	9	14	108	1	7	8	12	13	15	202	3	4	9	12	13	15
15	1	2	3	7	10	13	109	1	7	9	10	11	14	203	3	4	10	12	14	15
16	1	2	3	7	11	12	110	1	7	9	12	13	14	204	3	4	11	13	14	15
17	1	2	4	5	8	10	111	1	7	10	13	14	15	205	3	5	6	7	8	15
18	1	2	4	5	9	11	112	1	7	11	12	14	15	206	3	5	6	7	9	14
19	1	2	4	5	12	14	113	2	3	4	6	8	11	207	3	5	6	7	10	13
20	1	2	4	5	13	15	114	2	3	4	6	9	10	208	3	5	6	7	11	12
21	1	2	4	6	8	9	115	2	3	4	6	12	15	209	3	5	8	9	10	13
22	1	2	4	6	10	11	116	2	3	4	6	13	14	210	3	5	8	9	11	12
23	1	2	4	6	12	13	117	2	3	4	7	8	10	211	3	5	8	10	11	15
24	1	2	4	6	14	15	118	2	3	4	7	9	11	212	3	5	8	12	13	15
25	1	2	5	7	8	9	119	2	3	4	7	12	14	213	3	5	9	10	11	14
26	1	2	5	7	10	11	120	2	3	4	7	13	15	214	3	5	9	12	13	14
27	1	2	5	7	12	13	121	2	3	5	6	8	10	215	3	5	10	13	14	15
28	1	2	5	7	14	15	122	2	3	5	6	9	11	216	3	5	11	12	14	15
29	1	2	6	7	8	10	123	2	3	5	6	12	14	217	3	6	8	9	10	14
30	1	2	6	7	9	11	124	2	3	5	6	13	15	218	3	6	8	9	11	15
31	1	2	6	7	12	14	125	2	3	5	7	8	11	219	3	6	8	10	11	12
32	1	2	6	7	13	15	126	2	3	5	7	9	10	220	3	6	8	12	14	15
33	1	2	8	9	12	14	127	2	3	5	7	12	15	221	3	6	9	10	11	13
34	1	2	8	9	13	15	128	2	3	5	7	13	14	222	3	6	9	13	14	15
35	1	2	8	10	12	13	129	2	3	8	10	12	15	223	3	6	10	12	13	14
36	1	2	8	10	14	15	130	2	3	8	10	13	14	224	3	6	11	12	13	15
37	1	2	9	11	12	13	131	2	3	8	11	12	14	225	3	7	8	9	10	15
38	1	2	9	11	14	15	132	2	3	8	11	13	15	226	3	7	8	9	11	14
39	1	2	10	11	12	14	133	2	3	9	10	12	14	227	3	7	8	10	11	13
40	1	2	10	11	13	15	134	2	3	9	10	13	15	228	3	7	8	13	14	15
41	1	3	4	5	8	11	135	2	3	9	11	12	15	229	3	7	9	10	11	12
42	1	3	4	5	9	10	136	2	3	9	11	13	14	230	3	7	9	12	14	15
43	1	3	4	5	12	15	137	2	4	5	6	8	13	231	3	7	10	12	13	15
44	1	3	4	5	13	14	138	2	4	5	6	9	12	232	3	7	11	12	13	14
45	1	3	4	7	8	9	139	2	4	5	6	10	15	233	4	5	8	10	12	15

#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>
46	1	3	4	7	10	11	140	2	4	5	6	11	14	234	4	5	8	10	13	14
47	1	3	4	7	12	13	141	2	4	5	7	8	12	235	4	5	8	11	12	14
48	1	3	4	7	14	15	142	2	4	5	7	9	13	236	4	5	8	11	13	15
49	1	3	5	6	8	9	143	2	4	5	7	10	14	237	4	5	9	10	12	14
50	1	3	5	6	10	11	144	2	4	5	7	11	15	238	4	5	9	10	13	15
51	1	3	5	6	12	13	145	2	4	6	7	8	15	239	4	5	9	11	12	15
52	1	3	5	6	14	15	146	2	4	6	7	9	14	240	4	5	9	11	13	14
53	1	3	6	7	8	11	147	2	4	6	7	10	13	241	4	6	8	9	12	15
54	1	3	6	7	9	10	148	2	4	6	7	11	12	242	4	6	8	9	13	14
55	1	3	6	7	12	15	149	2	4	8	9	10	13	243	4	6	8	11	12	13
56	1	3	6	7	13	14	150	2	4	8	9	11	12	244	4	6	8	11	14	15
57	1	3	8	9	12	15	151	2	4	8	10	11	15	245	4	6	9	10	12	13
58	1	3	8	9	13	14	152	2	4	8	12	13	15	246	4	6	9	10	14	15
59	1	3	8	11	12	13	153	2	4	9	10	11	14	247	4	6	10	11	12	15
60	1	3	8	11	14	15	154	2	4	9	12	13	14	248	4	6	10	11	13	14
61	1	3	9	10	12	13	155	2	4	10	13	14	15	249	4	7	8	9	12	14
62	1	3	9	10	14	15	156	2	4	11	12	14	15	250	4	7	8	9	13	15
63	1	3	10	11	12	15	157	2	5	6	7	8	14	251	4	7	8	10	12	13
64	1	3	10	11	13	14	158	2	5	6	7	9	15	252	4	7	8	10	14	15
65	1	4	5	6	8	14	159	2	5	6	7	10	12	253	4	7	9	11	12	13
66	1	4	5	6	9	15	160	2	5	6	7	11	13	254	4	7	9	11	14	15
67	1	4	5	6	10	12	161	2	5	8	9	10	12	255	4	7	10	11	12	14
68	1	4	5	6	11	13	162	2	5	8	9	11	13	256	4	7	10	11	13	15
69	1	4	5	7	8	15	163	2	5	8	10	11	14	257	5	6	8	9	12	14
70	1	4	5	7	9	14	164	2	5	8	12	13	14	258	5	6	8	9	13	15
71	1	4	5	7	10	13	165	2	5	9	10	11	15	259	5	6	8	10	12	13
72	1	4	5	7	11	12	166	2	5	9	12	13	15	260	5	6	8	10	14	15
73	1	4	6	7	8	12	167	2	5	10	12	14	15	261	5	6	9	11	12	13
74	1	4	6	7	9	13	168	2	5	11	13	14	15	262	5	6	9	11	14	15
75	1	4	6	7	10	14	169	2	6	8	9	10	15	263	5	6	10	11	12	14
76	1	4	6	7	11	15	170	2	6	8	9	11	14	264	5	6	10	11	13	15
77	1	4	8	9	10	14	171	2	6	8	10	11	13	265	5	7	8	9	12	15
78	1	4	8	9	11	15	172	2	6	8	13	14	15	266	5	7	8	9	13	14
79	1	4	8	10	11	12	173	2	6	9	10	11	12	267	5	7	8	11	12	13
80	1	4	8	12	14	15	174	2	6	9	12	14	15	268	5	7	8	11	14	15
81	1	4	9	10	11	13	175	2	6	10	12	13	15	269	5	7	9	10	12	13
82	1	4	9	13	14	15	176	2	6	11	12	13	14	270	5	7	9	10	14	15
83	1	4	10	12	13	14	177	2	7	8	9	10	14	271	5	7	10	11	12	15
84	1	4	11	12	13	15	178	2	7	8	9	11	15	272	5	7	10	11	13	14
85	1	5	6	7	8	13	179	2	7	8	10	11	12	273	6	7	8	10	12	15
86	1	5	6	7	9	12	180	2	7	8	12	14	15	274	6	7	8	10	13	14
87	1	5	6	7	10	15	181	2	7	9	10	11	13	275	6	7	8	11	12	14
88	1	5	6	7	11	14	182	2	7	9	13	14	15	276	6	7	8	11	13	15
89	1	5	8	9	10	15	183	2	7	10	12	13	14	277	6	7	9	10	12	14
90	1	5	8	9	11	14	184	2	7	11	12	13	15	278	6	7	9	10	13	15
91	1	5	8	10	11	13	185	3	4	5	6	8	12	279	6	7	9	11	12	15
92	1	5	8	13	14	15	186	3	4	5	6	9	13	280	6	7	9	11	13	14
93	1	5	9	10	11	12	187	3	4	5	6	10	14							
94	1	5	9	12	14	15	188	3	4	5	6	11	15							

### 5.3.2 Golay-Hadamard set $G^{(16)}$

Aforementioned, this Hadamard set has soft balance and perfect correlation values in phase. Besides, it does not have negative version of sequences, i.e. it has no negative property. Then DC subsets are gotten and presented in following list.

#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	#	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>
1	1	2	3	5	9	16	151	1	6	11	13	14	15	301	3	4	10	11	13	15
2	1	2	3	5	10	15	152	1	6	12	13	14	16	302	3	4	10	11	14	16
3	1	2	3	5	11	14	153	1	7	9	10	11	14	303	3	4	10	12	13	16
4	1	2	3	5	12	13	154	1	7	9	10	12	13	304	3	4	10	12	14	15
5	1	2	3	6	9	15	155	1	7	9	11	12	16	305	3	5	6	7	9	14
6	1	2	3	6	10	16	156	1	7	9	13	14	16	306	3	5	6	7	10	13
7	1	2	3	6	11	13	157	1	7	10	11	12	15	307	3	5	6	7	11	16
8	1	2	3	6	12	14	158	1	7	10	13	14	15	308	3	5	6	7	12	15
9	1	2	3	7	9	14	159	1	7	11	14	15	16	309	3	5	6	8	9	13
10	1	2	3	7	10	13	160	1	7	12	13	15	16	310	3	5	6	8	10	14
11	1	2	3	7	11	16	161	1	8	9	10	11	13	311	3	5	6	8	11	15
12	1	2	3	7	12	15	162	1	8	9	10	12	14	312	3	5	6	8	12	16
13	1	2	3	8	9	13	163	1	8	9	11	12	15	313	3	5	7	8	9	16
14	1	2	3	8	10	14	164	1	8	9	13	14	15	314	3	5	7	8	10	15
15	1	2	3	8	11	15	165	1	8	10	11	12	16	315	3	5	7	8	11	14
16	1	2	3	8	12	16	166	1	8	10	13	14	16	316	3	5	7	8	12	13
17	1	2	4	5	9	15	167	1	8	11	13	15	16	317	3	5	9	10	11	14
18	1	2	4	5	10	16	168	1	8	12	14	15	16	318	3	5	9	10	12	13
19	1	2	4	5	11	13	169	2	3	4	5	9	13	319	3	5	9	11	12	16
20	1	2	4	5	12	14	170	2	3	4	5	10	14	320	3	5	9	13	14	16
21	1	2	4	6	9	16	171	2	3	4	5	11	15	321	3	5	10	11	12	15
22	1	2	4	6	10	15	172	2	3	4	5	12	16	322	3	5	10	13	14	15
23	1	2	4	6	11	14	173	2	3	4	6	9	14	323	3	5	11	14	15	16
24	1	2	4	6	12	13	174	2	3	4	6	10	13	324	3	5	12	13	15	16
25	1	2	4	7	9	13	175	2	3	4	6	11	16	325	3	6	7	8	9	15
26	1	2	4	7	10	14	176	2	3	4	6	12	15	326	3	6	7	8	10	16
27	1	2	4	7	11	15	177	2	3	4	7	9	15	327	3	6	7	8	11	13
28	1	2	4	7	12	16	178	2	3	4	7	10	16	328	3	6	7	8	12	14
29	1	2	4	8	9	14	179	2	3	4	7	11	13	329	3	6	9	10	11	13
30	1	2	4	8	10	13	180	2	3	4	7	12	14	330	3	6	9	10	12	14
31	1	2	4	8	11	16	181	2	3	4	8	9	16	331	3	6	9	11	12	15
32	1	2	4	8	12	15	182	2	3	4	8	10	15	332	3	6	9	13	14	15
33	1	2	5	7	9	12	183	2	3	4	8	11	14	333	3	6	10	11	12	16
34	1	2	5	7	10	11	184	2	3	4	8	12	13	334	3	6	10	13	14	16
35	1	2	5	7	13	16	185	2	3	5	6	9	11	335	3	6	11	13	15	16
36	1	2	5	7	14	15	186	2	3	5	6	10	12	336	3	6	12	14	15	16
37	1	2	5	8	9	11	187	2	3	5	6	13	15	337	3	7	9	10	11	16
38	1	2	5	8	10	12	188	2	3	5	6	14	16	338	3	7	9	10	12	15
39	1	2	5	8	13	15	189	2	3	5	7	9	10	339	3	7	9	11	12	14
40	1	2	5	8	14	16	190	2	3	5	7	11	12	340	3	7	9	14	15	16
41	1	2	6	7	9	11	191	2	3	5	7	13	14	341	3	7	10	11	12	13
42	1	2	6	7	10	12	192	2	3	5	7	15	16	342	3	7	10	13	15	16
43	1	2	6	7	13	15	193	2	3	6	8	9	10	343	3	7	11	13	14	16
44	1	2	6	7	14	16	194	2	3	6	8	11	12	344	3	7	12	13	14	15
45	1	2	6	8	9	12	195	2	3	6	8	13	14	345	3	8	9	10	11	15
46	1	2	6	8	10	11	196	2	3	6	8	15	16	346	3	8	9	10	12	16
47	1	2	6	8	13	16	197	2	3	7	8	9	11	347	3	8	9	11	12	13
48	1	2	6	8	14	15	198	2	3	7	8	10	12	348	3	8	9	13	15	16
49	1	2	9	11	13	16	199	2	3	7	8	13	15	349	3	8	10	11	12	14
50	1	2	9	11	14	15	200	2	3	7	8	14	16	350	3	8	10	14	15	16
51	1	2	9	12	13	15	201	2	3	9	10	13	15	351	3	8	11	13	14	15
52	1	2	9	12	14	16	202	2	3	9	10	14	16	352	3	8	12	13	14	16
53	1	2	10	11	13	15	203	2	3	9	11	13	14	353	4	5	6	7	9	13
54	1	2	10	11	14	16	204	2	3	9	11	15	16	354	4	5	6	7	10	14
55	1	2	10	12	13	16	205	2	3	10	12	13	14	355	4	5	6	7	11	15

56	1	2	10	12	14	15	206	2	3	10	12	15	16	356	4	5	6	7	12	16
57	1	3	4	5	9	14	207	2	3	11	12	13	15	357	4	5	6	8	9	14
58	1	3	4	5	10	13	208	2	3	11	12	14	16	358	4	5	6	8	10	13
59	1	3	4	5	11	16	209	2	4	5	6	9	12	359	4	5	6	8	11	16
60	1	3	4	5	12	15	210	2	4	5	6	10	11	360	4	5	6	8	12	15
61	1	3	4	6	9	13	211	2	4	5	6	13	16	361	4	5	7	8	9	15
62	1	3	4	6	10	14	212	2	4	5	6	14	15	362	4	5	7	8	10	16
63	1	3	4	6	11	15	213	2	4	5	8	9	10	363	4	5	7	8	11	13
64	1	3	4	6	12	16	214	2	4	5	8	11	12	364	4	5	7	8	12	14
65	1	3	4	7	9	16	215	2	4	5	8	13	14	365	4	5	9	10	11	13
66	1	3	4	7	10	15	216	2	4	5	8	15	16	366	4	5	9	10	12	14
67	1	3	4	7	11	14	217	2	4	6	7	9	10	367	4	5	9	11	12	15
68	1	3	4	7	12	13	218	2	4	6	7	11	12	368	4	5	9	13	14	15
69	1	3	4	8	9	15	219	2	4	6	7	13	14	369	4	5	10	11	12	16
70	1	3	4	8	10	16	220	2	4	6	7	15	16	370	4	5	10	13	14	16
71	1	3	4	8	11	13	221	2	4	7	8	9	12	371	4	5	11	13	15	16
72	1	3	4	8	12	14	222	2	4	7	8	10	11	372	4	5	12	14	15	16
73	1	3	5	6	9	12	223	2	4	7	8	13	16	373	4	6	7	8	9	16
74	1	3	5	6	10	11	224	2	4	7	8	14	15	374	4	6	7	8	10	15
75	1	3	5	6	13	16	225	2	4	9	10	13	16	375	4	6	7	8	11	14
76	1	3	5	6	14	15	226	2	4	9	10	14	15	376	4	6	7	8	12	13
77	1	3	5	8	9	10	227	2	4	9	12	13	14	377	4	6	9	10	11	14
78	1	3	5	8	11	12	228	2	4	9	12	15	16	378	4	6	9	10	12	13
79	1	3	5	8	13	14	229	2	4	10	11	13	14	379	4	6	9	11	12	16
80	1	3	5	8	15	16	230	2	4	10	11	15	16	380	4	6	9	13	14	16
81	1	3	6	7	9	10	231	2	4	11	12	13	16	381	4	6	10	11	12	15
82	1	3	6	7	11	12	232	2	4	11	12	14	15	382	4	6	10	13	14	15
83	1	3	6	7	13	14	233	2	5	6	7	9	15	383	4	6	11	14	15	16
84	1	3	6	7	15	16	234	2	5	6	7	10	16	384	4	6	12	13	15	16
85	1	3	7	8	9	12	235	2	5	6	7	11	13	385	4	7	9	10	11	15
86	1	3	7	8	10	11	236	2	5	6	7	12	14	386	4	7	9	10	12	16
87	1	3	7	8	13	16	237	2	5	6	8	9	16	387	4	7	9	11	12	13
88	1	3	7	8	14	15	238	2	5	6	8	10	15	388	4	7	9	13	15	16
89	1	3	9	10	13	16	239	2	5	6	8	11	14	389	4	7	10	11	12	14
90	1	3	9	10	14	15	240	2	5	6	8	12	13	390	4	7	10	14	15	16
91	1	3	9	12	13	14	241	2	5	7	8	9	13	391	4	7	11	13	14	15
92	1	3	9	12	15	16	242	2	5	7	8	10	14	392	4	7	12	13	14	16
93	1	3	10	11	13	14	243	2	5	7	8	11	15	393	4	8	9	10	11	16
94	1	3	10	11	15	16	244	2	5	7	8	12	16	394	4	8	9	10	12	15
95	1	3	11	12	13	16	245	2	5	9	10	11	15	395	4	8	9	11	12	14
96	1	3	11	12	14	15	246	2	5	9	10	12	16	396	4	8	9	14	15	16
97	1	4	5	6	9	11	247	2	5	9	11	12	13	397	4	8	10	11	12	13
98	1	4	5	6	10	12	248	2	5	9	13	15	16	398	4	8	10	13	15	16
99	1	4	5	6	13	15	249	2	5	10	11	12	14	399	4	8	11	13	14	16
100	1	4	5	6	14	16	250	2	5	10	14	15	16	400	4	8	12	13	14	15
101	1	4	5	7	9	10	251	2	5	11	13	14	15	401	5	6	9	11	13	16
102	1	4	5	7	11	12	252	2	5	12	13	14	16	402	5	6	9	11	14	15
103	1	4	5	7	13	14	253	2	6	7	8	9	14	403	5	6	9	12	13	15
104	1	4	5	7	15	16	254	2	6	7	8	10	13	404	5	6	9	12	14	16
105	1	4	6	8	9	10	255	2	6	7	8	11	16	405	5	6	10	11	13	15
106	1	4	6	8	11	12	256	2	6	7	8	12	15	406	5	6	10	11	14	16
107	1	4	6	8	13	14	257	2	6	9	10	11	16	407	5	6	10	12	13	16
108	1	4	6	8	15	16	258	2	6	9	10	12	15	408	5	6	10	12	14	15
109	1	4	7	8	9	11	259	2	6	9	11	12	14	409	5	7	9	10	13	16
110	1	4	7	8	10	12	260	2	6	9	14	15	16	410	5	7	9	10	14	15
111	1	4	7	8	13	15	261	2	6	10	11	12	13	411	5	7	9	12	13	14
112	1	4	7	8	14	16	262	2	6	10	13	15	16	412	5	7	9	12	15	16
113	1	4	9	10	13	15	263	2	6	11	13	14	16	413	5	7	10	11	13	14
114	1	4	9	10	14	16	264	2	6	12	13	14	15	414	5	7	10	11	15	16

115	1	4	9	11	13	14	265	2	7	9	10	11	13	415	5	7	11	12	13	16
116	1	4	9	11	15	16	266	2	7	9	10	12	14	416	5	7	11	12	14	15
117	1	4	10	12	13	14	267	2	7	9	11	12	15	417	5	8	9	10	13	15
118	1	4	10	12	15	16	268	2	7	9	13	14	15	418	5	8	9	10	14	16
119	1	4	11	12	13	15	269	2	7	10	11	12	16	419	5	8	9	11	13	14
120	1	4	11	12	14	16	270	2	7	10	13	14	16	420	5	8	9	11	15	16
121	1	5	6	7	9	16	271	2	7	11	13	15	16	421	5	8	10	12	13	14
122	1	5	6	7	10	15	272	2	7	12	14	15	16	422	5	8	10	12	15	16
123	1	5	6	7	11	14	273	2	8	9	10	11	14	423	5	8	11	12	13	15
124	1	5	6	7	12	13	274	2	8	9	10	12	13	424	5	8	11	12	14	16
125	1	5	6	8	9	15	275	2	8	9	11	12	16	425	6	7	9	10	13	15
126	1	5	6	8	10	16	276	2	8	9	13	14	16	426	6	7	9	10	14	16
127	1	5	6	8	11	13	277	2	8	10	11	12	15	427	6	7	9	11	13	14
128	1	5	6	8	12	14	278	2	8	10	13	14	15	428	6	7	9	11	15	16
129	1	5	7	8	9	14	279	2	8	11	14	15	16	429	6	7	10	12	13	14
130	1	5	7	8	10	13	280	2	8	12	13	15	16	430	6	7	10	12	15	16
131	1	5	7	8	11	16	281	3	4	5	7	9	12	431	6	7	11	12	13	15
132	1	5	7	8	12	15	282	3	4	5	7	10	11	432	6	7	11	12	14	16
133	1	5	9	10	11	16	283	3	4	5	7	13	16	433	6	8	9	10	13	16
134	1	5	9	10	12	15	284	3	4	5	7	14	15	434	6	8	9	10	14	15
135	1	5	9	11	12	14	285	3	4	5	8	9	11	435	6	8	9	12	13	14
136	1	5	9	14	15	16	286	3	4	5	8	10	12	436	6	8	9	12	15	16
137	1	5	10	11	12	13	287	3	4	5	8	13	15	437	6	8	10	11	13	14
138	1	5	10	13	15	16	288	3	4	5	8	14	16	438	6	8	10	11	15	16
139	1	5	11	13	14	16	289	3	4	6	7	9	11	439	6	8	11	12	13	16
140	1	5	12	13	14	15	290	3	4	6	7	10	12	440	6	8	11	12	14	15
141	1	6	7	8	9	13	291	3	4	6	7	13	15	441	7	8	9	11	13	16
142	1	6	7	8	10	14	292	3	4	6	7	14	16	442	7	8	9	11	14	15
143	1	6	7	8	11	15	293	3	4	6	8	9	12	443	7	8	9	12	13	15
144	1	6	7	8	12	16	294	3	4	6	8	10	11	444	7	8	9	12	14	16
145	1	6	9	10	11	15	295	3	4	6	8	13	16	445	7	8	10	11	13	15
146	1	6	9	10	12	16	296	3	4	6	8	14	15	446	7	8	10	11	14	16
147	1	6	9	11	12	13	297	3	4	9	11	13	16	447	7	8	10	12	13	16
148	1	6	9	13	15	16	298	3	4	9	11	14	15	448	7	8	10	12	14	15
149	1	6	10	11	12	14	299	3	4	9	12	13	15							
150	1	6	10	14	15	16	300	3	4	9	12	14	16							

## 5.4 Observations

In this section, all the binary sequences with length  $N = 8$  are explored in order to select sequences with desired properties such as No negative, Balance, correlation, and shifting. This is an exhaustive search that allows to provide interesting comments

- No negative property is usually not considered in conventional code sets, however this work considers it a crucial property in order to avoid the information loss. For example, CC and IFW are vulnerable to this phenomena due to their sets of codes contemplates negative versions.
- Balance property is a hard restriction to define the universe of sequences. For example, perfect balance provides only 35 sequences while soft balance sets increase the useful sequences to 56 and 84 respectively, i.e. the total sequences to C-II is  $M_{sf1} = M_P + 56 = 35 + 56 = 91$  and C-III incorporates 28 sequences, this is  $M_{sf2} = M_{sf1} + 28 = 119$ . Note that C-III includes all the previous cases.
- Considering ideal correlation values, it is possible to identify 210 subsets with eight elements, i.e. set size  $M = 8$ , which increase the traditional idea to use only the Hadamard family.
- Note that considering C-I subsets, Walsh and Paley Hadamard sets can be distinguished, so Walsh-Hadamard set is represented in  $MS_1^{(7)}$ , i.e.  $MS_1^{(7)} = W_n^{(7)}$  and the subset  $MS_{11}^{(7)}$  represents the Paley-Hadamard set ( $MS_{11}^{(7)} = P_n^{(7)}$ ) while Golay-Hadamard is in C-III subsets as subset number 194.
- Shifting property can form subsets from C-I and C-II, and C-III, which follow the relation between ACF and CCF. Then, it is possible to form pairs with  $W_{A1} - W_{C1}$ ,  $W_{A1} - W_{C2}$ ,  $W_{A1} - W_{C3}$ ,  $W_{A1} - W_{C4}$ ,  $W_{A2} - W_{C1}$ ,  $W_{A2} - W_{C2}$ , and  $W_{A3} - W_{C1}$ . The number of pairs depends on the balance property.

On the other hand, Hadamard sets with  $N = 16$  are explored in order to obtain some results. It reduces the selection method due to known properties of these sets such as No negative, Balance, Correlation (AC and CC). DC subsets reach up to six elements just C. Barrera defined. This way, from  $W^{(15)}$  set was obtained 280 available subsets and 448 from  $G^{(16)}$  set which increase the interest of study of C2DMA system and their sequences used as codes.

# Chapter 6

## Simulations and Results

In this chapter, the performance of new code sets are studied for traditional DS-CDMA and C2DMA models. Both models are proved in single user and single path scenario considering different families of sequences. The new IFW subsets are proved considering multiple signal paths, which represent multipath interference (MPI), so IFW sets must perform according to their features.

On the other hand, C2DMA model is proved in usual configurations in RWN which can be extended to explore its performance in bigger networks. These configurations seek to show the robustness of C2DMA model and its capability to confront these usual challenges in RWNs.

### 6.1 CDMA systems and new codes sets

In order to study the new code sets, they are simulated traditional DS-CDMA and C2DMA models which were described in subsections 3.3.1 and 3.3.2 respectively considering single user scenario. They consider codes with length  $N = 8$ , additive white Gaussian noise (AWGN) channel with zero mean and standard deviation  $\sigma = \sqrt{N0/2}$ , BPSK modulation, and synchronous reception. The bit energy is normalized, i.e.  $\mathcal{E}_b = 1$  and bit error rate (BER) is the parameter used to evaluate the models, so the results are compared versus theoretical BER of the BPSK modulation (calculated using 8 chips-sequences), see Appendix A, and the simulation time was set so that 100,000 bits are transmitted for each origin-destination pair.

#### 6.1.1 Single user scenario and single path

This configuration allows to compare the performance of the new sequences and known sets in similar conditions. Firstly, sequences from perfect, soft 1 and soft 2 cases, which are described in previous chapter, and Hadamard sets as Walsh, Paley, and Golay are compared using traditional CDMA model and DC subsets from perfect, soft 1 and soft 2 cases are considered for C2DMA model. Their performance are shown in Figure 6.1. This way, the list of sequences used in DS-CDMA and C2DMA models are in following Table 6.1.

Table 6.1: List of sequences: Single user and single path

		DS-CDMA model				DS-CDMA model	
Family		Binary sequence		Family		Binary sequence	
Perfect Balance set	$S_{24}^P$	+1, -1, +1, -1, -1, +1, -1, +1		Walsh-Hadamard set	$W_1^{(7)}$	+1, -1, +1, -1, +1, -1, +1, -1	
Soft 1 Balance set	$S_{22}^{sf1}$	+1, +1, -1, +1, -1, +1, -1, +1		Paley-Hadamard set	$P_1^{(7)}$	+1, -1, +1, +1, -1, +1, -1, -1	
Soft 2 Balance set	$S_8^{sf2}$	+1, +1, +1, +1, -1, -1, +1, +1		Golay-Hadamard set	$G_1^{(8)}$	+1, +1, +1, -1, +1, +1, -1, +1	
		C2DMA model				C2DMA model	
Family		Tx DC-sequence				Rx DC-sequence	
Perfect Balance set	$S_{24}^P$	+1, -1, +1, -1, -1, +1, -1, +1			$S_{28}^P$	+1, -1, -1, +1, +1, -1, -1, +1	
Soft 1 Balance set	$S_{22}^{sf1}$	+1, +1, -1, +1, -1, +1, -1, +1			$S_{31}^{sf1}$	+1, +1, -1, -1, +1, -1, +1, +1	
Soft 2 Balance set	$S_8^{sf2}$	+1, +1, +1, +1, -1, -1, +1, +1			$S_{42}^{sf2}$	+1, +1, -1, -1, +1, +1, +1, +1	

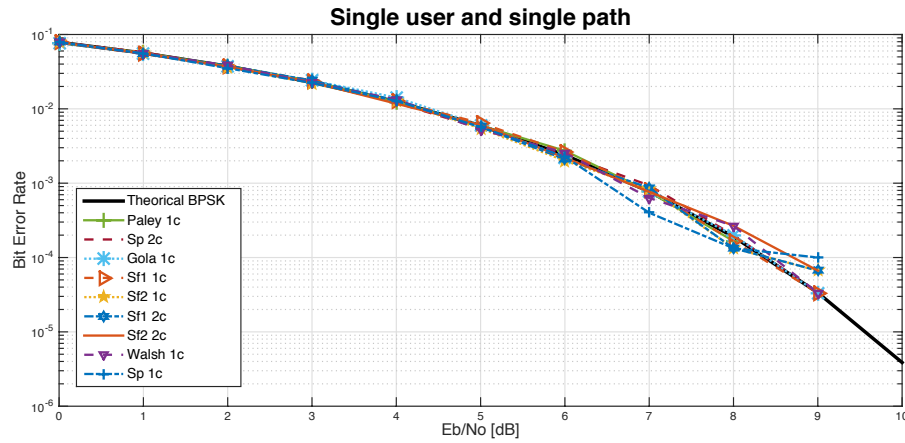


Figure 6.1: CDMA vs C2DMA models: All sequences

### 6.1.2 IFW subsets and DS-CDMA model

In this subsection, they are only considered IFW subsets from case  $C - III$  that promise good correlation within windows size  $W_1 = 1$  and  $W_2 = 2$ , i.e. the pair from  $W_{A1,C1}$  promises resistant to signal out of phase  $\phi = 1$  (delay of one chip), while the pair from  $W_{A2,C2}$  promises resistant to delays  $\phi = 1, 2$ . These sequences are evaluated considering three scenarios, the first one considers a direct path, which is synchronized with the receiver, while a second path has a delay of one chip  $D_1 = 1$ . The second case also considers a synchronized direct path but the second path has a delay of two chips out of phase  $D_2 = 2$  and, finally, the third case considers the synchronized path and the second path has a delay of three chips. Note that the second path represents interference (MPI) to the receiver.

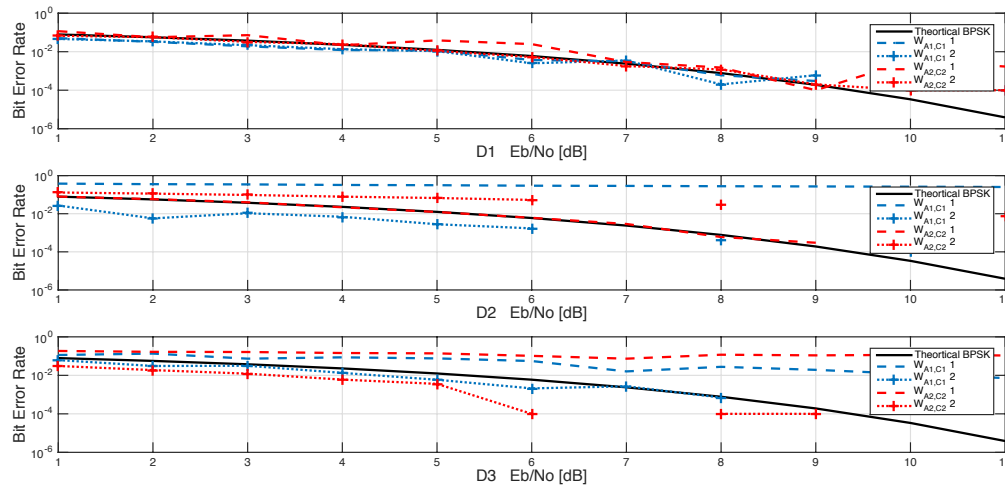
Recall that the new IFW subsets are resistant to signals into the window, i.e. the synchronous signal is accepted while the received signals out of phase and within the window are mitigated. This way, the IFW sequences used are shown in Table 6.2 and their performance are shown in Figure 6.2.

Note that blue lines correspond to  $W_{A1,C1}$  pair and red lines to  $W_{A2,C2}$ . Figure 6.2a



Table 6.2: List of IFW sequences

	Family	Sequence
Case - III, $W_{A1,C1}$	$I^{(1)}$	+1, +1, +1, +1, +1, -1, +1, -1
Case - III, $W_{A1,C1}$	$I^{(2)}$	+1, +1, -1, -1, +1, +1, -1, -1
Case - III, $W_{A2,C2}$	$I^{(1)}$	+1, +1, +1, +1, -1, +1, +1, -1
Case - III, $W_{A2,C2}$	$I^{(2)}$	+1, -1, +1, -1, -1, -1, +1, +1

Figure 6.2: Performance of new IFW subsets considering (a) Delay  $D=1$ , (b) Delay  $D=2$ , and (c) Delay  $D=3$ 

shows the performance of these pairs of codes considering an interfering path with  $D_1$ . Given that the interference signal is within the window, both pairs exhibit good performance. Figure 6.2b considers the interfering path with  $D_2$ , it means that the  $W_{A1,C1}$  pair is vulnerable as shows the blue line of  $W_{A1,C1}$  1. Despite  $W_{A1,C1}$  2 exhibits good performance, the mutual correlation is broken. However, the red lines from  $W_{A2,C2}$  pair are resistant to this signal and mitigate their effects. On the other hand, Figure 6.2c considers the interfering signal with  $D_3$ , i.e. this signal is out of the windows and, as consequence, both subsets are vulnerable. Again, the mutual correlation is broken. These results show the advantages and disadvantages of IFW codes.

## 6.2 RWN and C2DMA model

This section recovered some important features of C2DMA model that makes it attractive to WN. Therefore, considering a RWN with six nodes, they are presented useful configurations that represent traditional issues for these networks with simultaneous users such as *Near-far* problem, multiuser reception, Hidden and Exposed terminal interference. These configurations allow to examine and compare the traditional DS-CDMA and C2DMA models under equal conditions in order to show their performance.

The dynamic topology and connectivity of RWNs, which involve changes in wireless channel conditions and define the scenarios study. The WCS is based on the previous knowledge of network topology, neighbors and the sequences assigned to each user as codes. All the transmissions take place simultaneously in the same channel with synchronous reception and the effects of propagation losses are considered through attenuation factors which are defined by separation distances between pairs of nodes. Note that are considered Golay and CC codes for traditional DS-CDMA model and DC subsets for C2DMA model which play an important role. All the sequences have  $N = 16$ . Finally, bit error rate (BER) is the parameter used to evaluate the models in each scenario and all the results are compared to the theoretical BER of the BPSK modulation, see Appendix A. The simulation time was set so that 100,000 bits were transmitted for each origin destination pair.

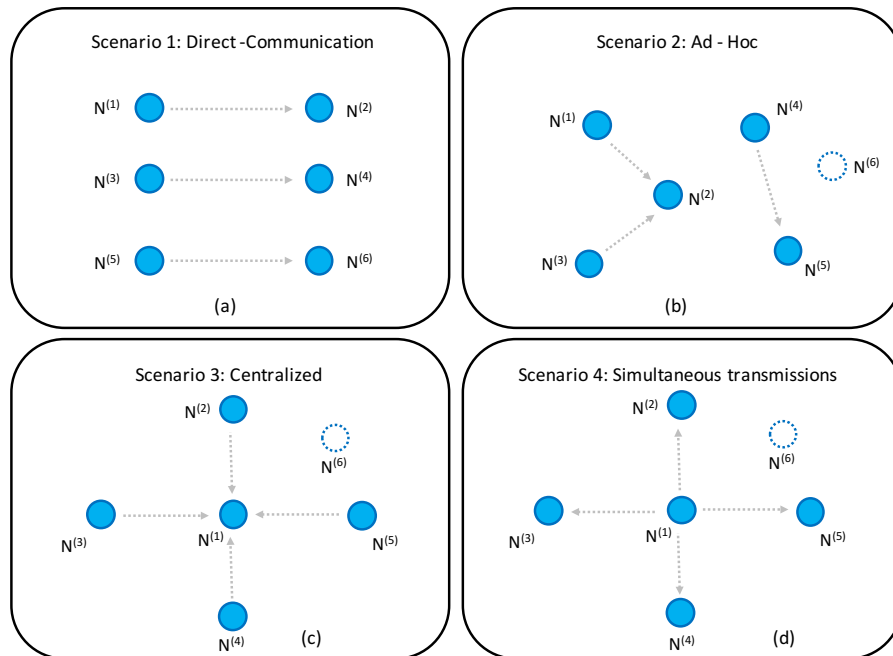


Figure 6.3: Scenarios study.

### 6.2.1 Scenario 1: Direct-Communication configuration

It considers three independent communication links with single path configuration which connect directly each transmitter-receiver pair in the same channel, i.e.  $N^{(1)}$  sends information to  $N^{(2)}$ , similarly  $N^{(3)}$  and  $N^{(5)}$  send information to  $N^{(4)}$  and  $N^{(6)}$  respectively as Figure 6.3a shows. This scenario studies the interference caused only by exposed terminals, so the receivers performance are shown in Figure 6.4.

### 6.2.2 Scenario 2: Ad-Hoc configuration

Figure 6.3b shows other possible configuration of RWN. The communication links are defined as  $N^{(1)}$  and  $N^{(3)}$  send information to  $N^{(2)}$  and  $N^{(4)}$  sends to  $N^{(5)}$ . Note that  $N^{(6)}$  is no active.

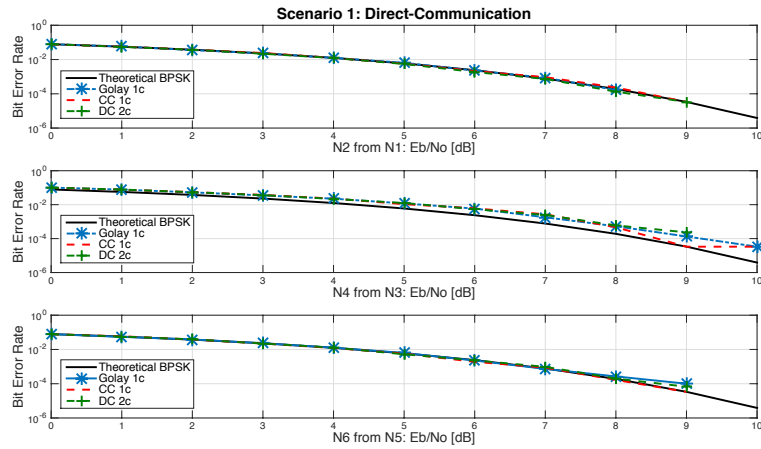


Figure 6.4: Bit error rate: Direct-Communication configuration.

Thus, this scenario considers the reception of multiple users simultaneously, and interference from hidden and exposed terminals. The receivers performance are shown in following Figure.

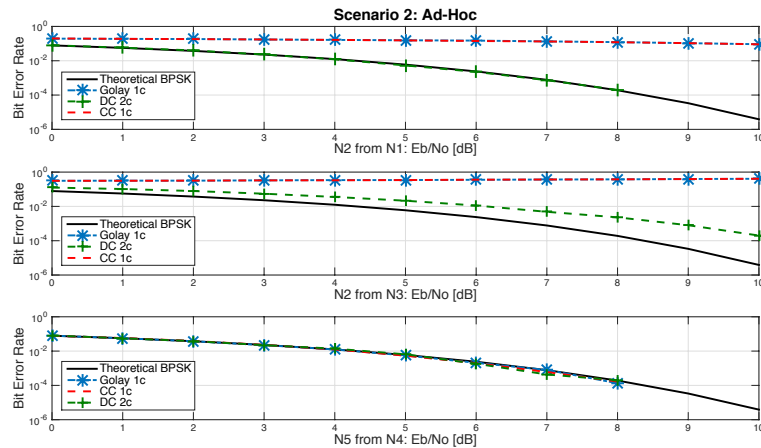


Figure 6.5: Bit error rate: Ad-Hoc scenario

### 6.2.3 Scenario 3: Centralized configuration

Considering a centralized configuration, see Figure 6.3c, the interference caused only by hidden terminals is studied. Then,  $N^{(1)}$  can receive information from  $N^{(2)}$ ,  $N^{(3)}$ , and  $N^{(4)}$  and the received signals in each branch are showed in Figure 6.6. This scenario proves the powerful features of C2DMA as a multi-user reception model.

### 6.2.4 Scenario 4: Simultaneous transmissions (*Broadcast*)

This scenario considers that  $N^{(1)}$  sends information simultaneously to  $N^{(2)}$  and  $N^{(3)}$ . This issue is widely studied in physical layer network coding (PLNC) and an attractive feature to

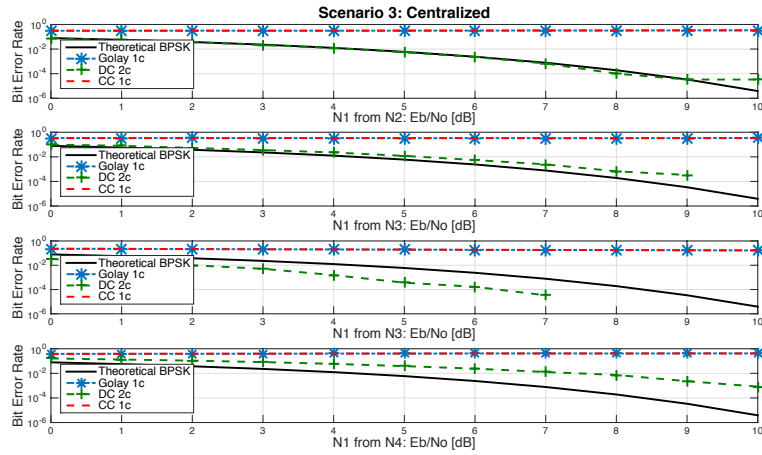


Figure 6.6: Bit error rate: Centralized scenario

RWNs. Thus, the receivers performance in Figure 6.7. It is another powerful advantage of C2DMA model in systems with simultaneous transmission.

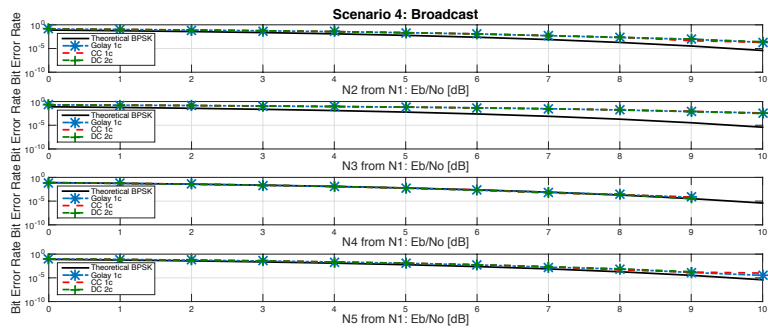


Figure 6.7: Bit error rate: Simultaneous transmissions scenario

The previous scenarios show that C2DMA model is a promising system to RWNs. For example, Figure 6.8 shows a bigger network where some previous scenarios are presented simultaneously, e.g.  $N^{(1)}$  works in *simultaneous transmissions*,  $N^{(8)}$  requires *multiple reception capability*,  $N^{(3)}$  has three exposed terminals as interference, and  $N^{(10)}$  four hidden terminals, i.e. some traditional wireless issues are presented due to dynamic behavior of system, so large sets of mutual DC codes are desirable.

### 6.3 Applications and Architecture networks

In this section, some useful architectures networks are proposed where C2DMA system can be exploited to establish the coexistence scheme and multiple user capability, besides, the number of wireless channels is reduced and, as consequence, the utilization channel can be improved. Therefore, they are considered the following architecture networks.

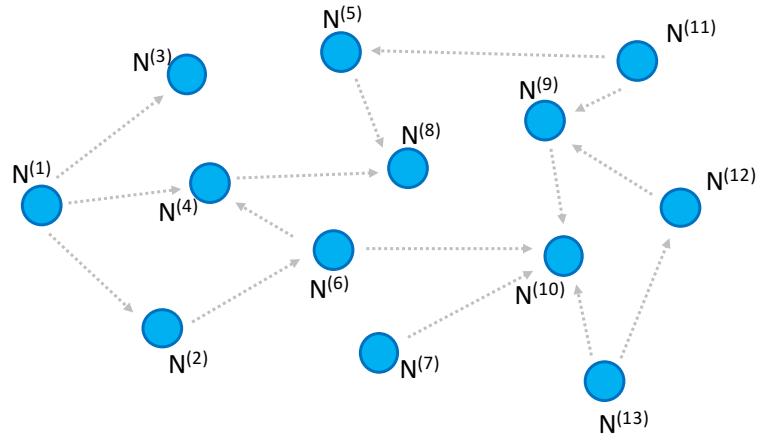


Figure 6.8: RWN and simultaneous communication.

- (a) Infrastructure. It considers to provide the access to WN through access points, i.e. hierarchical structures can form the core network with high capability to manage the information and resources. For example, Figure 6.9a shows an infrastructure where  $T - F1$  defines a network that provides the core using  $F1$  frequency or channel where a subset of codes allows the communication among this *backbone* network while subnetworks  $SecX - F2$  which sectors the functions and/or coverage area using other channel  $F2$  and mutual DC correlation subsets can provided the access to network. Thus, each hierarchical layer defines the rules and scope. Cognitive radio can be an attractive paradigm for this architecture.
- (b) Inter-WNs. This architecture allows interconnect small networks through a *virtual network* formed by head-elements that can interchange the code used to select the network. This head-elements are key to interconnect the elements from different subnets, i.e association of WNs. These structures are useful to mobile and private networks, see Figure 6.9b.
- (c) Cellular. It considers of exploiting cellular strategies as reuse of codes, spatial separation of networks, and hierarchical structures in order to increase the users, see Figure 6.9c.
- (d) Hybrid systems. It represents *hybrid* systems that consider wire and wireless technologies. For example Figure 6.9d shows a system with *Ring* structure and wireless access, this scheme is widely used in telecommunication services such as Wifi and cable TV (CATV). This architecture can also be used to provide access to private services through public network, e.g. Smart cities paradigm and HetNets or IoT. They also allow to combine fix and mobile networks as *Infrastructure-Ad hoc* networks.

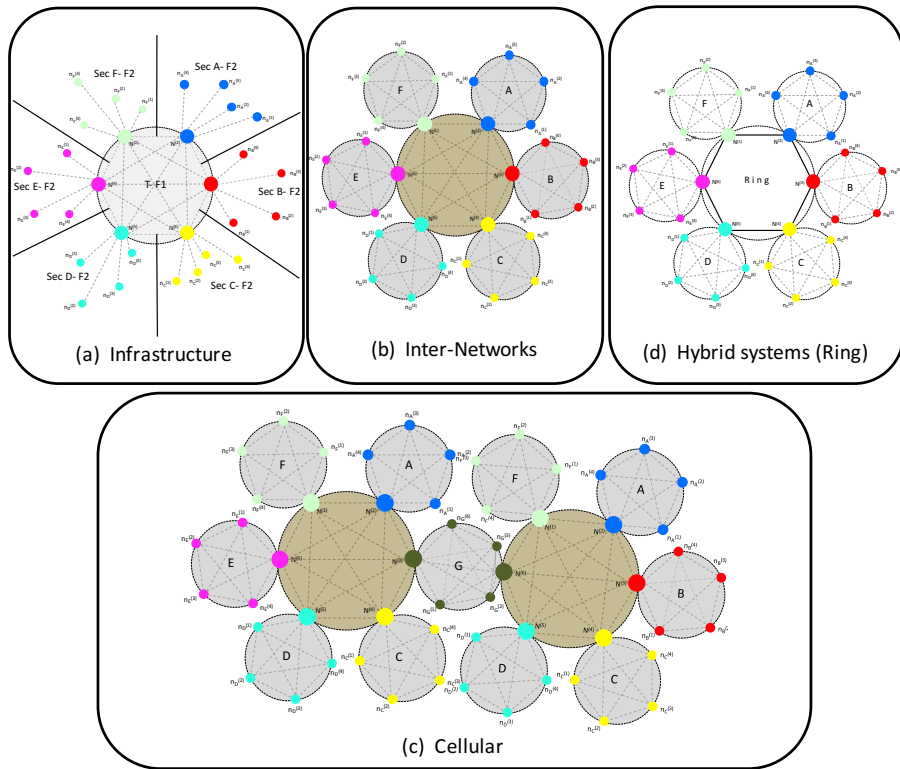


Figure 6.9: Architecture networks

# Chapter 7

## Conclusions and Future work

High density of WEs and their dynamic and intermittent behavior represent important challenges to future WNs. This work considers the attractive features of code division multiple access (CDMA) systems to accost these challenges and recognizes the important role of spreading code which provides their attributes. Thus, a deep study of sequences used as codes is considered in order to explore and prove their properties. Families of known sequences such as Traditional, complementary, and interference free windows are considered which provide the useful properties for these systems. This work also retakes the C2DMA model in order to suggest its implementation for next generation WCS because the double-codification strategy provides powerful features such as simultaneous reception, robustness to hidden and exposed terminal interference, and multiple access. These features are incorporated from physical layer and simplify the employment of upper layers, i.e. it is a helpful cross-layer strategy that decreases the complexity of the communication system.

Due to their crucial role in spreading codes for CDMA systems, this work explores new code sets from their properties. Thus, sequences with *No negative* property that helps to avoid the information loss caused by antipodal signals are considered. The *balance* property is introduced by considering three cases that allow to define the universe of sequences and to consider the direct current component in the receiver. Relaxing this property, the number of codes and their set size can be increased. Besides, the *correlation* properties ensure the decoding of signals and multiple access capability, so autocorrelation and cross-correlation are key to CDMA systems. However, double-correlation property is also incorporated to code sets used in C2DMA model. After that, the *shifting* property is explored in order to consider the windows of correlation that helps to attenuate the interfering signals from multiple paths of propagation.

An exhaustive search of sequences with length  $N = 8$  and some with  $N = 16$  were done in order to provide new code sets for both models, the traditional DS-CDMA and the C2DMA, and to increase their use and to visualise some applications. These models are proved in challenging scenarios in order to examine their performance and persuade about their potential of C2DMA model in WNs. Future works will be focused in the implementation of C2DMA systems and the exploration of new code sets with larger lengths. The results shown in Chapters

5 and 6 suggest that the search of new codes in larger sequences could incorporate new properties such as orthogonality (balance of product of codes) and zero correlation zones (shifting property) that strengthen the C2DMA system. Finally, it is considered that the size of code sets increases as the length of sequences also increases meaning that its implementation can be exploited in more applications areas. This new search will require powerful computing and processing equipment.



# Appendix A

## Bit Error Rate

Bit error rate (BER) is a useful parameter in assessing system that transmit digital data because considers the full end to end performance of a system, i.e. the transmitter, receiver and the medium between them. Thus, BER is the percentage of bits in the transmission that have errors as results of noise, interference or other issues, so BER can be determine as

$$BER = \frac{\text{Number of errors}}{\text{Total number of bits sent}} \quad (\text{A.1})$$

The medium between the transmitter and receiver plays an important role because it establishes the conditions of wireless channel, i.e. the environment and its effects in propagation of signals and, as consequence, the degradation of data. This way, signal-to-noise ratios such as  $E_S/N_0$  (based on energy per symbol) and  $E_b/N_0$  (based on energy per bit) are parameters used to describe this degradation.

Usually, they are used theoretical models to characterize the effects of many random process that occur in wireless channel, for example Additive White Gaussian Noise (AWGN,  $\eta$ ) is a thermal noise model widely used that considers the impairments to communication as a linear addition of wideband or white noise with a constant spectral density and a Gaussian distribution of amplitude with mean  $\mu$  and standard deviation  $\sigma^2$ .

In this work, it is considered the theoretical BER model for binary phase shift keying (BPSK) modulation as reference to evaluate the performance of systems and the different codes. For this reason, in following sections are introduced the mathematical and simulation models used in Chapter 6.

### A.1 BER for Binary Phase Shift Keying (BPSK) modulation

#### A.1.1 Mathematical model

Considering binary digits  $\{\pm 1\}$  that represent the analog levels of bit, i.e.  $+\sqrt{E_b}$  and  $-\sqrt{E_b}$  respectively, so the transmitted waveform gets corrupted by noise  $\eta$  that follows the Gaussian

probability distribution function, i.e.

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (\text{A.2})$$

with  $\mu = 0$  and  $\sigma^2 = \frac{N_0}{2}$ .

Defining the received signal  $y$  as  $y = S_1 + \eta$  when the bit +1 is transmitted and  $y = S_0 + \eta$  when bit -1 is transmitted, so following the analysis in [38], the probability of error is computed by the conditional probability distribution function (PDF) of received signal for both cases

$$\begin{aligned} P_{y/S_0} &= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y+\sqrt{E_b})^2}{N_0}}, \quad \text{and} \\ P_{y/S_1} &= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(y-\sqrt{E_b})^2}{N_0}}, \end{aligned} \quad (\text{A.3})$$

Assuming that  $S_1$  and  $S_0$  are equally probable ( $P(S_1) = P(S_0) = 0.5$ ), the threshold 0 forms the optimal decision boundary, i.e. it is assumed that  $S_1$  was transmitted when the received signal  $y$  is  $y > 0 - S_1$ , while it is assumed  $S_0$  was transmitted when  $y \leq 0$ . This way, the probability of error for both cases are given as

- The probability of error given  $S_1$  is transmitted.

$$\begin{aligned} P(e|S_1) &= \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^0 e^{-\frac{(y-\sqrt{E_b})^2}{N_0}} dy \\ &= \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz \\ &= \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right) \end{aligned}$$

- The probability of error given  $S_0$  is transmitted

$$\begin{aligned} P(e|S_0) &= \frac{1}{\sqrt{\pi N_0}} \int_0^{\infty} e^{-\frac{(y+\sqrt{E_b})^2}{N_0}} dy \\ &= \frac{1}{\sqrt{\pi N_0}} \int_{\sqrt{\frac{E_b}{N_0}}}^{\infty} e^{-z^2} dz \\ &= \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right) \end{aligned}$$

Note that  $\text{erfc}(x)$  is the complementary error function defined as

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-x^2} dx$$

Thus, the total probability of bit error is  $P_b = P(S_1)P(e|S_1) + P(S_0)P(e|S_0)$ . Given that  $S_1$  and  $S_0$  are assumed equally probable, the bit error probability is given as

$$P_b = \frac{1}{2} \text{erfc} \left( \sqrt{\frac{E_b}{N_0}} \right) \quad (\text{A.4})$$

### A.1.2 Simulation models

Aforementioned, Communication System Toolbox of Simulink-Matlab software is used in order to simulate the performance of CDMA systems, so this subsection describes the algorithms that yield the theoretical figures which are used in Chapter 6 as reference. These algorithms are derived from total probability of bit error provides in previous section, see equation A.4. and different values of the ratio of the received signal power per bit and the noise spectral density  $(E_b/N_0)_{dB}$

This way, the theoretical values of the total probability of bit error are calculated from an analysis of discrete signal of the spread signal of  $s(t)$  using codes with  $N$  chips (sequences with length  $N$ ) and simple rate  $f_s$  per chip, so the energy per bit is  $E_b = N \cdot f_s$ . Considering  $dB$  unit, the energy per bit is  $(E_b)_{dB} = 10 \log_{10}(E_b)$  and the the ratio of the received signal power per bit and the noise spectral density is given as

$$\left(\frac{E_b}{N_0}\right)_{dB} = (E_b)_{dB} - (N_0)_{dB} \quad (\text{A.5})$$

Evaluating  $N_0$  for different values of  $(E_b/N_0)_{dB}$ , it is possible to obtain the standard deviation  $\sigma^2 = N_0/2$  of noise from Equation A.5 which is used as parameter in AWGN channel block.

$$(N_0)_{dB} = (E_b)_{dB} - \left(\frac{E_b}{N_0}\right)_{dB} N_0 = 10^{(N_0)_{dB}/10} \quad (\text{A.6})$$

This way, the theoretical values of the total probability of bit error for  $N = 8$  and  $N = 16$  are given in chart A.1 and their curves are showed in Figures A.1.2a and A.1.2b respectively.

Parameter	$N = 8$ and $f_s = 10$	$N = 16$ and $f_s = 5$
★ <i>Energy per bit</i> $E_b = N \cdot F_s$	$E_{b-8} = 10 \cdot 8 = 80$	$E_{b-16} = 16 \cdot 5 = 80$
★ <i>(Energy per bit)<sub>dB</sub></i> $(E_b)_{dB} = 10 \cdot \log_{10}(E_b)$	19.03 dB	19.03 dB

Table A.1: Theoretical parameters

$(E_b/N_0)_{dB}$	$(N_0)_{dB}$	$N_0$	Standard deviation $\sigma^2$	Total probability of error $P_e$	$(N_0)_{dB}$	$N_0$	Standard deviation $\sigma^2$	Total probability of error $P_e$
0	19.03	79.98	39.99	<b>0.0786</b>	19.03	79.98	39.99	<b>0.0786</b>
1	18.03	63.53	31.76	<b>0.0575</b>	18.03	63.53	31.76	<b>0.0575</b>
2	17.03	50.46	25.23	<b>0.0375</b>	17.03	50.46	25.23	<b>0.0375</b>
3	16.03	40.08	20.04	<b>0.0229</b>	16.03	40.08	20.04	<b>0.0229</b>
4	15.03	31.84	15.92	<b>0.0125</b>	15.03	31.84	15.92	<b>0.0125</b>
5	14.03	25.29	12.64	<b>0.0060</b>	14.03	25.29	12.64	<b>0.0060</b>
6	13.03	20.09	10.04	<b>0.0024</b>	13.03	20.09	10.04	<b>0.0024</b>
7	12.03	15.95	7.97	<b>0.0008</b>	12.03	15.95	7.97	<b>0.0008</b>
8	11.03	12.67	6.33	<b>0.0002</b>	11.03	12.67	6.33	<b>0.0002</b>
9	10.03	10.06	5.03	<b>0.0000*</b>	10.03	10.06	5.03	<b>0.0000*</b>
10	9.03	7.99	3.99	<b>0.0000*</b>	9.03	7.99	3.99	<b>0.0000*</b>

\* approximately

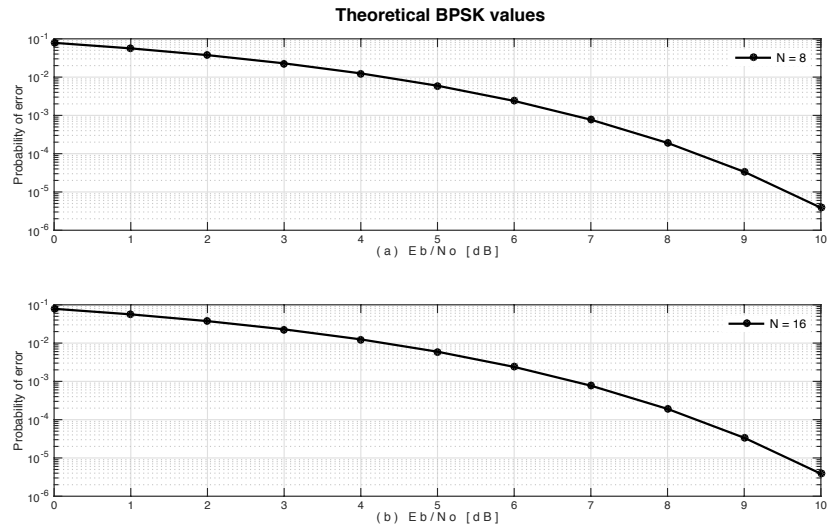


Figure A.1: Theoretical Bit Error Rate figures (BER)

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# Vita

Juan Manuel Velazquez-Gutierrez was born in Puebla, Mexico in 1982. He received the B.S. degree in Electronic engineering from Puebla Institute of Technology, Puebla, Mexico in 2007. He received MS. degree in Electronic Engineering (Telecommunications) from ITESM, Monterrey, N.L., Mexico in 2010 and he is currently working toward PhD. degree at ITESM. He is member of IEEE-HKN society since 2016. He worked in implementation of cellular networks by Huawei Tech. Ltd. and Distribute Antennas Systems design for in-building by Interexport Telecomunicaciones, Mexico. His research interests are personal communication networks, cognitive radio design and applications, software defined radio (SDR) and network architectures. He was accepted in the graduate programs in Information Technologies and Communications in August 2012.

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