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ma[illegible]al Modelling Based Learning Str

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Abstract
We present the Mathematical Modelling Learning strategy in which students create a model that will predict behaviour of existing phenomena using real data. In our implementation students create models from atmospheric data and solve them to determine which weather conditions favour high levels of pollutants in the atmosphere of Monterrey metropolitan area in Mexico. To carry out the strategy we structure course topics around this single comprehensive and integrative project. Students follow a procedure consisting of 4 stages. In the first stage they do statistical analysis of the data. In the second stage, students interpolate missing data and project component data to a 2D map of the metro area. In the third stage students create the mathematical models by carrying out curve fitting through least squares technique. In the third stage, students solve the models by finding roots, solving systems of equations, solving differential equations or integrating. The final deliverable is to determine under which weather conditions there can be an environmental situation that put people's health in danger. Analysis of the strategy is presented as well as statistical results.

Keywords: Mathematical modelling, Metacognition, Collaborative learning, Kolb's learning cycle

1 Introduction

We understand as a model a simplified description of reality, an abstraction that is used to understand systems and environments and to forecast their future behaviour. Therefore, all humans are born with the ability to model reality. Nevertheless, mathematical modelling takes this ability one step further by using mathematical concepts and language to create abstractions of reality that not only allow a more precise prediction of future behaviour and but also to establish and manage levels of uncertainty. And since mathematical models are the result of positivistic thinking, that is based on proven causal relationships between the components that are been modelled, they also give an accurate description of the inner workings and mechanisms of those systems or environments. The creation of accurate mathematical descriptions of reality requires a deep understanding of complex phenomena.

It is no surprise then, than the ability to do mathematical modelling of reality is a skill sought after by employers and a necessary requirement for undergraduate engineering and science students. And also, it is no surprise that this skill is difficult to acquire and is not easy to transfer from academia to the real world. In order to succeed in transferring this ability to model the observable world, students must learn as much as possible using real data from real life.

Just like any other skill included in labour descriptions and work profiles, mathematical modelling requires training. The Mathematical Modelling Based Learning strategy (MMBL) provides one way to train students on mathematical modelling in such a way that we believe has a better opportunity for transferring the skill into real-world situations.

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In MMBL students are presented with real-world data and are walked through a process similar to data mining. Student analyse data, establish relevant variables, establish relationships between relevant variables, establish characteristic behaviours, deal with missing data, create models that fit the data, and create scenarios of future behaviour of the system that created the data by solving the models created. The most important part of it all, is that the data is real, and the goals that students are asked to achieve have a purpose and are important to their life, and the life of their society and loved ones. The final goal of the project, to put it bluntly, is sexy: "Save the city" or "Save the world".

Mathematical Modelling based learning is itself based in four principles:

1. The Integrated Metacognitive Processes model
2. The Integrated Learning Processes Model
3. Collaborative Learning
4. Extensive use of technology to support learning and collaboration

2 The Integrated Metacognitive Processes Model

The Integrated Metacognitive Processes Model (IMP) establishes certain variables that once an activity model with specific needs instantiates, it produces a unique educational model. The IMP model has the five components shown in Fig. 1 and described below [1].

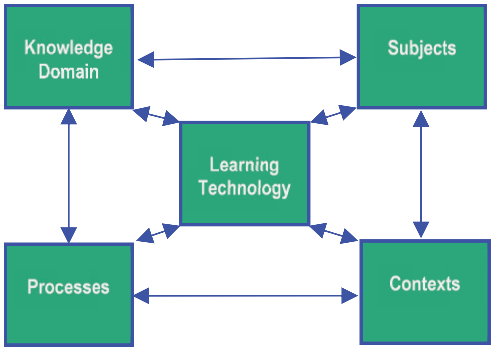

Figure 1. The Integrated Metacognitive Processes Model

1. **Knowledge domain**. This not only describes the knowledge that we desire students to acquire, but also certain different skills, which can be cognitive, affective and motor, or conceptual, procedural and metacognitive. This classification is important, as students must construct knowledge in increasing phases of complexity.
2. **Educational subjects**. It indicates the nature and origins of students and teachers and the relationship between them. Effective learning is achieved when the process is focused on student's interests, culture, and environment, and when new knowledge and skills is constructed upon a well identified academic base (student background).
3. **Contexts and environments**. They establish the variables that make-up the learning space. The same knowledge domain may be taught to different people with different culture, in different locations with different resources, and at different times with different emphasis, by different teachers with different types of communication.

4. **Educational processes**. In general, this term identifies the activities that must be carried out by students in order to acquire skills and knowledge. Those activities take into account the knowledge domain, student's background and learning contents.
5. **Learning technology**. Learning technology is closely tied to the educational processes for basic learning activities are carried out using them. Learning technology must endeavour to target as many communications channels as possible as well as students' different intelligence capacities such as linguistic, visual, kinaesthetic, musical, logical-mathematic, interpersonal, etc.

To develop new activity models a designer must not only establish a sequence of different activities according to the Integrated Learning Processes, but also according to the Metacognitive Processes model, a range of goals, conditions, restrictions and boundaries must be established and that, will differentiate the activity model from others.

3 The Integrated Learning Processes Model

The Integrated Learning Processes (ILP) model is framework for creating new learning experiences. It's a process that drives students to achieve natural learning by creating scaffoldings in which students acquire ever increasing independence and deeper understanding on the knowledge and skills they will need to perform in society. This is achieved by creating learning experiences in which different parts of the brain that are involved in learning are addressed in a carefully constructed sequence of events. As it is a cycle, the learning experience can start in any part of the Cycle. It is itself, based on three principles:

1. Kolb's [2] learning cycle, which James Zull [3] links learning to specific parts of the brain that are essential for learning as shown in Fig. 2. Based on this, the ILP model is based on the repetition of four stages of learning that activate different areas of the brain as discussed in [4], [1] and [5]. Any new learning strategy must establish how it goes through the learning cycle.

2. The second principle of the ILP model is a progression of increasing knowledge complexity that follows the path: Conceptual and Contextual Knowledge->Procedural and Problem Solving Knowledge->Cognitive complexity knowledge. These three levels of complexity for knowledge do correspond to Boom's taxonomy [6] [7].

Since the ILP model is a cycle, learning activities can start at any point. For example, interactive museums start with action, then let students uses their senses to observe, to infer and finally to understand a theory that was from the beginning intended for them to learn. Socrates' Maieutic started asking questions therefore, he started with reflexion and the construction of knowledge.

3. The third principle is the use of social interaction and the presence clear and present motivators. Motivation is based on the following elements:

   a) An orthodox use of collaborative learning
   b) The presence of Active Learning
   c) Putting the students at risk
   d) Showing that learning is about improving life.

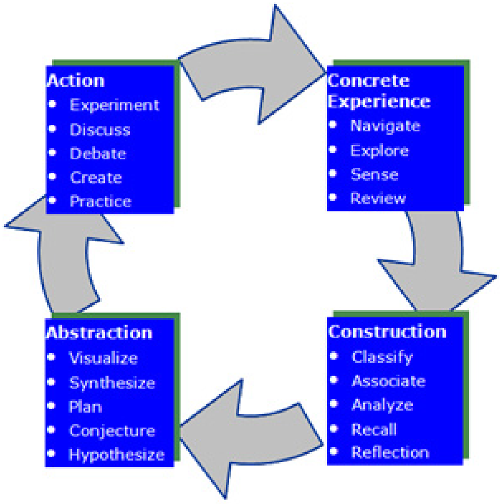

Figure 2. Integrated Learning Processes Educational Model

The Integrated Learning Processes model allows the construction of a learning strategies that drive students to achieve natural learning by creating scaffolding according to Vygotsky's theory in which students acquire ever increasing independence and deeper understanding on the knowledge and skills they will need to perform in society. This is achieved by creating learning experiences in which different parts of the brain that are involved. Also student's social skills are addressed in a carefully constructed sequence of events [5].

4 Collaborative Learning

A common definition of Collaborative Learning (CL) is: "involved joint intellectual effort by students or students and teachers. Groups of students work together in searching for understanding, meaning, solutions or in creating a product" [10].

One of the most important principles in collaborative learning is "Positive Interdependence". Positive interdependence is "the degrees to which participants perceive they are interdependent but that they share a mutual fate and that their success is mutually caused" [11]. Although there are other important aspects of collaborative learning, positive interdependence stands out because it not only encourages knowledge and skills acquisition, but it also encourages the practice of certain attitudes and values such as respect, responsibility to others, personal accountability, self-evaluation, etc. [12] [13].

There are five elements of collaborative learning [11]:

1. **Clearly Perceived Positive Interdependence**. In collaborative learning the success of one person is bound up with the success of others. There are many ways to ensure positive interdependence. Goal sharing is one way. This might include shared subject matter, a particular assessment, joint problem solving, or creating and discovering something of value. Another way is role sharing. This occurs when each group member is given a specific role that gives a person specific responsibilities. The role describes what group activities that person might take and the contribution to the overall task. Also, resource sharing contributes to positive interdependence and exists when each group member has only part of the information, cases, material or other resources necessary for the group to achieve its task. Finally, task

interdependence is structured by creating a division of labour so that the actions of one group member have to be completed before the next member can complete their tasks.

2. **Interaction**. Individual students are encouraged to assist others in the group to complete tasks in order to reach the group's goals. In other words there is an expectation that students will help each other so that common goals can be achieved. Help may be resources, advice, provision of feedback and challenging conclusions.
3. **Individual Accountability and Personal Responsibility**. Everyone is expected to do their fair share of work and it is important for all group members to know that they cannot 'free ride.'
4. **Small Group Skills**. Interpersonal skills such as making decisions and solving conflicts are important. In order to achieve these goals students must:

   i) Get to know and trust each other
   ii) Communicate clearly
   iii) Provide and accept support
   iv) Resolve conflict constructively

5. **Group Processing**. Group work is effective when group participants reflect on how well they function as a group. This reflection assists members to maintain good working relationships. Reflection may focus on such things as relationships between people, facilitation of collaborative skills, rewarding of positive behaviour and the celebration of success.

The correct use of CL is very important to the MMBL strategy. Collaborative Learning has many advantages over traditional learning. Mainly, it gives support to students by creating positive interactions between students, creating shared experiences that derive in comradeship within teams and most importantly, motivating students [5]. And, since MMBL relies so much in teamwork and collaboration, a failure in this part will cause students to simply divide the work among themselves and then getting later together to simply ensemble the whole. This will in turn cause students to learn only a part of the intended learning objective, creating less efficient learning, not better learning.

5 The Mathematical Modelling Based Learning Strategy

The central idea of the MMBL strategy is to orient the learning content of the course around an integration project, one that will require skills learned in different courses and the skills of different people. The pilot MMBL pilot was carried out in a Numerical Methods course, usually taken by third year undergraduate students in engineering careers.

The focus of the project was to create models that would link weather conditions to levels of pollution in the atmosphere of the Monterrey, Mexico, metropolitan area. The objective was to find under which weather conditions there could be environmental emergencies. This would help governments initiate contingency plans. Real data from weather stations was given to students. The data file contains measurements of pollutants such as ozone (O_3), carbon oxides (CO and CO_2), different nitrogen oxides (NOx), Sulphur oxides (SO_2), and particles of less than 10 microns and less than 2.5 microns. The weather information contained in the file is atmospheric pressure, rainfall, humidity, solar radiation, temperature, and wind speed direction. The data files contain reading from 7 weather stations located throughout Monterrey metropolitan area, each hour for an entire year. In total, almost one million records.

Before the project starts, teams of two or three students are formed. Each team is assigned a "mission" that changes slightly the focus of the project. For example, some teams were given the mission to model the data by the hour of the day. Some teams were given the mission of modelling the data by day of year, others by month, and finally others would model by region.

An expert from government is asked to do a presentation of the data so students can be familiarized with the physical and chemical mechanics of their own city's weather system.

The MMBL project is carried out in four phases:

1. Data Analysis
2. Interpolation
3. Model Building
4. Model Solution

In the Data Analysis phase students do a Principal Component Analysis (PCA) according to their team's mission. The first goal of the PCA is to reduce the dimensionality of the data (eliminate variables) and to group them in order to later create mathematical relationships. Students should understand that the eigenvalues of the correlation matrix determine how much variability is captured by each principal component. The second goal was to link environmental variables with pollutant via the square cosines that indicate in which variables there is probably a relationship. The third goal was to find characteristic behaviours by remembering that the PCA of similar time-series will capture the most typical behaviour on the first principal component.

In the Interpolation phase students would interpolate missing data in order to fill holes of missing data. Also they are asked to project a grid over Monterrey's metro area (that is a grid over Monterrey's map), and according to monitoring station location address, place the values in the grid (for example, down town monitoring station data would be approximately in the middle of the grid). Then do 2D interpolation of the first principal component using the component loadings. Then they are asked to create a Contour 2D graph to be projected over the metro area map.

In the Model Building phase students create a model using curve fitting, using the original data to create mathematical relationships of variables grouped together by PCA's cosine statistics. They are asked to propose one or several math models that relate the level of certain pollutant according to weather variables. The models would look like: $O_3 = a_0 + a_1 * WS + a_2 * \frac{\delta O_3}{\delta WS} + a_3 * HR^2$. Students need to do several minimum square error curve fittings to find at least one viable model. Students are asked to use plenty of x-vs-y scatter plots to try to determine if there is a relationship between variables. The use Excel's "Trend" option over x-vs-y plots is also suggested.

In the Model Solution phase students solve the model using root finding (Secant, Newton-Raphson) and differential equations (Runge-Kutta, Euler) or even integration methods (Simpson´s rule, quadrature). Students are asked to determine under which weather conditions there would be an environmental emergencies. In order to determine which levels of which pollutant are considered dangerous students are asked to include in their models the official weather health regulation by using the IMECA pollution measure.

Also, students are asked to prepare a presentation to present their conclusions to the entire classroom.

This is a large project that requires a lot of time to be carried out. Since the amount of data is very large, students have to select the data they need to analyse and therefore, every team's project is different from the project of every other team, even if they have the same mission. This leads students to confusion and concern, since students are used to rely on the opinion of their peers. In order to reduce the level of stress, a rubric was created that helped students understand what exactly is asked of them and how the evaluation by the teacher would be carried out. The rubric is shown in Table 1.

The project is carried out during the course's regular four month period. In order to support the project effort and make sure that all course objectives were covered, the course lectures were restructured around the project. To do the first part, data analysis, the lessons include matrix computations, including matrix algebra, determinants, systems of linear equations and eigenvalues. For the second part, interpolation, several techniques for interpolation are taught to students. For the third part, model creation, several lectures about curve fitting by least squares, linear, non-linear and multivariable are given. For the last part, model solving, students are given lectures on root finding, solving differential equations and integration.

Item	Beginner (50%)	Competent (80%)	Proficient (100%)
PCA (25%)	Is able to do a PCA with XLSTAT or similar tool	Does PCA arranging correctly the info and carrying out correctly mission assigned (without averaging). Selection of data from original file is completely justified	Several PCAs were carried out. Typical behaviour for 03, PM2.5 and PM10 according to mission was found
Interpolation (25%)	Missing values are Interpolated	A grid for Monterrey metro area is created and values are computed by interpolation from data of monitoring stations. Some data is not consistent due to problems in the interpolation process. A contour 2D of the first principal component is created and superimposed over the metro area map (implicit analysis by region)	An advanced interpolation method to fill the city grid is found and used. Interpolation software is used.
Curve Fitting (25%)	A minimum square analysis is carried out	Several plots x-vs-y where y is a contaminant and x is a weather condition are reported. The model form is justified	Pollutant -vs- weather models include nonlinear terms (even a differential equation)
Model Solving (25%)	Scenarios in which weather conditions would create environmental hazards are shown. Mostly lineal models are used	Non lineal models are used and solved using scilab or Excel using numerical methods	Differential equations are solved

Table 1. Final project evaluation rubric.

Also, students are asked to do research in several subjects that are not included as class objectives but that would help on their project. For examples, student teams research and explain to the group 2D interpolation, QR factoring, singular value decomposition, Gram-Schmidt vector orthogonalization, quadrature integration, etc.

6 Analysis of MMBL Approach

As we have discussed already, the Mathematical Based Modelling Learning strategy is based on several theories such as the Integrated Learning Processes model, Collaborative learning and the Integrated Metacognitive Processes model. We now briefly explain how MMBL corresponds to the principles of each.

6.1 Compliance with Collaborative Learning

Collaborative learning is an intrinsic part of MMBL. In Table 2 we describe how the elements of CL are implemented in the MMBL.

Element of CL	MMLP Implementation
Positive Interdependence	Positive interdependence is achieved with goal sharing of course, resource sharing because software licences expire each month so only one student from the team has a licence for the software and task sharing, because the amount information is so large, that the only way to achieve the goal is to try several independent approaches in parallel
Student Interaction	Students need to be in constant communication. Dividing workload will just not work. Students need to put their heads together to analyse data by mission and build successful mathematical models
Individual Accountability	Work cannot be split and then put together. Since no student has a complete picture, all students in the team depend on the work of others. If one fails, all fail. This promotes responsibility
Small Group Skills	Students need to learn to offer and accept help. Since the data can be interpreted in many ways, students seek help from peers and from the teacher. Collective decision making is essential
Group Processing	Successful teams are recognized and problems are sought before they become too big to handle. Students are taught negotiation to resolve conflicts.

Table 2. Collaborative Learning and MMBL

6.2 Compliance with the IMP Model

IMP Model Component	MMLP Implementation
Knowledge Domain	Actually knowledge domain was an important issue. One of the groups had students from all branches of engineering and had no problem modelling weather. Whereas in a class with only student of a career focused mainly on programming resented having to model something not related to computing.
Subjects	Although it might be trivial to figure out the main subjects are students, identifying whom this project benefits was a different matter. Since the atmosphere affect us all, this project had a component of citizenship, which is very important to Tecnologico de Monterrey
Contexts	Context was very important also. Although these students are savvy at using technology to communicate, they knew less than what they thought about collaborative learning. Therefore, some training on CL had to be done.
Processes	Designing the learning process was critical since modelling quality of air is something done rather on a graduate level, not by 5th semester students. Therefore, to reduce stress and anxiety students project pace should be slow and that is why the project consists of four stages. Students concerns needed to be carefully taken into consideration.
Educational Technology	The chosen educational technology a composition of learning platforms, social media and mobile devices.

Table 3. The IMP model and MMBL

Understanding the metacognitive environment is very important. Ignoring it can cause the initiative to fail. For example, students from a different numerical methods cohorts were taught using MMBL with the exact same final project. A group cohort consisted of students from IT related careers and resented doing a project not related to IT. The strategy failed with this group as they were very unhappy about it and very much not interested in the weather and the spread of pollutants. Table 3 compares the IMP and MMBL. Also, positive interdependence, interaction, individual accountability and personal responsibility, small group skills and group processing are metacognitive skills that belong to the processes category carried out by subjects, in contexts, with technology.

6.3 Compliance with the ILP Model

We now present the relationship between the ILP and the MMBL in Table 4.

ILP Stage	MMLP Implementation
Concrete Experience	An expert on air quality is presented to students. He then gives a brief talk about the quality of air in the metropolitan area and the principles of atmospheric modelling. Students also review previous projects
Construction	When the goals of the projects are presented, students are asked to discuss about their personal experiences with air quality. Several similes are presented to students trying to establish the mechanics of pollution spread Students work to interpret the result of a few PCAs not related to the spread of contaminants
Abstraction	Students must create a hypothesis in order to do a model fit through minimum square errors. They must evaluate if the models do in fact workout according to their data
Action	Students must constantly make decisions upon each phase of the process, evaluate and take action. They complete the project by phases, and in each phase, past decisions must be reviewed and confirmed or corrected

Table 4. ILP model and MMBL

7 Educational Experiments and Analysis of Results

The results of the MMBL were compared using statistical tests between two cohorts of students. We analysed data of two cohorts belonging to two different classes of the Numerical Methods course. One cohort took the course from August to December 2013 (Aug-13) and the other one from August to December 2014 (Aug-14). Two sets of data are analysed. The first one is the results of the first partial examination. In this exam, since students are learning the basics of matrix algebra, the MMBL is not implemented yet. That is to say, that students do not start working on their project until after the first exam. The second set is the final grade. The final grade integrates students' oral presentations (5%), two partial exams (45%), homework (10%), final project (15%) and final exam (25%). We argue that the final grade represents the results of the MMBL applied to the entire course. Each group consisted of 34 students. The exam scores are graded from 0, all answers wrong, to 100, all answers correct. A Test was carried out to verify that the difference in the mean score observed was statistically significant [14].

For the first data set, first partial exam, we obtained the student score statistical summary shown in Table 5.

	Aug-13	Aug-14
Count	34	34
Mean	69.41	75.32
Variance	235.40	250.15
Std Dev	15.34	15.82

Table 5. Statistical results for first partial exam

	Aug-13	Aug-14
Count	35	35
Mean	73.91	74.89
Variance	140.67	85.99
Std Dev	11.86	9.27

Table 6. Statistical results for the final grade

The hypothesis tested was that there is no statistical difference in the means, and therefore the means are statistically equivalent. From tables we observed that t critical was to be 1.6882 [14]. As the computed t was 6.4501 and since t critical is less than t computed, we cannot reject the hypothesis and thus, the means are statistically equivalent. This seems to be evidence that both groups started out at an equivalent level of performance, even though the mean for Aug-14 is higher than Aug-13.

For the second data set, final grade, we obtained the student score statistical summary shown in Table 6. Again, the hypothesis tested was that there is no statistical difference in the means, and therefore the means are statistically equivalent. From tables we observed that t critical was to be 1.6676. As the computed t was -1.597 and since t critical is greater than t computed, we reject the hypothesis and thus, the means are statistically different. This seems to be evidence that cohort Aug-14 in fact did better than cohort Aug-13.

8 Conclusions

We have designed a learning strategy based on three pedagogical models. One of them is the Integrated Learning Processes which take students through neurology based cycles of ever increasing knowledge complexity in which students observe, reflect/analyse, plan/synthesize and act, and then start again. Also, the ILP model helps establish motivators to drive students using collaborative learning not only as motivator but also as affective support. The second pedagogical model is thus Collaborative Learning. CL success' is supported by the Integrated Metacognitive Processes model, our third model, that helps to establish the ecosystem of the learning strategy determining what are our academic objectives, what are our learning subjects, contexts and processes, and what should be the appropriate learning technology.

We have been able to present evidence that the Mathematical Modelling Based Learning strategy could help students learn new concepts, new procedures, to transform knowledge, and to uses their creativity and imagination by motivating students and taking them through the correct neurological process of learning in a sustainable ecosystem for learning.

References

[1] N. S. Alexandrov and R. V. Ramirez-Velarde, "The Integrated Learning Processes, Metacognition, and Collaborative Learning," in *Technological Advances in Interactive Collaborative Learning*, Boca Raton, USA, CRC Press, 2013, pp. 145-161.

[2] A. Kolb and D. A. Kolb, Experiential Learning Theory Bibliography 1971-2001, Boston, USA: McBer and Co, 2001.

[3] J. Zull, The Art of Changing the Brain, Sterling, USA: Stylus Publishing, 2002.

[4] N. S. Alexandrov, R. V. Ramirez-Velarde and V. N. Alexandrov, "Natural Learning and Collaborative Learning, , 2013.," in *"Technological Advances in Interactive Collaborative Learning*, Boca Raton, USA, CRC Press, 2013, pp. 121-143.

[5] N. S. Alexandrov, R. Ramirez-Velarde and A. V. N., "Education 2.0: Student Generated Learning Materials through Collaborative Work," *Procedia Computer Science,* vol. 29, pp. 1835-1845, 2014.

[6] B. S. Bloom and D. R. Krathwohl, Taxonomy of Educational Objectives Book 1: Cognitive Domain, Addison Wesley Publishing Company, 1984.

[7] R. D. Tennyson, "An Educational Learning Theory for Instructional Design," *Educational Technology,* vol. 32, no. 1, pp. 36-41, 1992.

[8] J. J. G. Van Merriënboer, P. A. Kirschner and L. Kester, "Taking the load of a learners' mind: Instructional design for complex learning," *Educational Psychologist,* vol. 38, no. 1, pp. 5-13, 2002.

[9] N. V. Cherenkova and N. S. Alexandrov, "Transfer of E-Learning Metacognitive Skills Using Games," in *" in "Technological Advances in Interactive Collaborative Learning*, Boca Raton, USA,, CRC Press, 2013, pp. 163-175.

[10] "Collaborative Learning," Wikipedia, [Online]. Available: http://en.wikipedia.org/wiki/Collaborative_learning. [Accessed 2013].

[11] D. W. Johnson and R. T. Johnson, "Positive Interdependence: Key to Effective Cooperation," in *Interaction in Cooperative Groups: The Theoretical Anatomy of Group Learning*, Cambridge, USA, Cambridge University Press, 1991, pp. 174-199.

[12] E. Barkley, P. Cross and C. H. Major, Collaborative Learning Techniques: A Handbook for College Faculty, Jossey-Bass, 2004.

[13] V. H. M. Dale, L. Nasir and M. Sullivan, "Evaluation of Student Attitudes to Cooperative Learning in Undergraduate Veterinary Medicine," *Journal of Veterinary Medical Education,* vol. 32, no. 4, pp. 511-516, 2005.

[14] D. Remenyi, G. Onofrei and J. English, An introduction to statistics using Microsoft Excel, Academic Conferences Limited, 2009.