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## Rotating Prism Array for Solar Tracking

Noel León<sup>a</sup>, Carlos Ramírez<sup>a</sup>, Héctor García<sup>a,\*</sup>

<sup>a</sup> *Tecnológico de Monterrey, Eugenio Garza Sada 2501, Monterrey, N.L., México*

### Abstract

Solar energy has become one of the most promising renewable energies being the most widespread used nowadays. In order to achieve an optimum performance, both photovoltaic and solar thermal applications require the use of some kind of solar tracking technique. Every solar concentration system has to constantly reposition itself according to the Sun's position changes throughout the day. This movement must be done in the most effective way as possible to avoid a high negative impact on the system efficiency.

The present paper attempts to describe in detail the design process of a state-of-the-art semi-passive solar tracking concentrator (SPSTC) in which, in order to track the sun, two independent arrays of acrylic prisms are implemented to refract sunlight by rotating said prisms, thus being able to redirect solar radiation as desired. The first set of PMMA prisms is responsible of eliminating one of the directional components of the solar radiation; the task is achieved by rotating the prisms within the array at a specific angle. The second set of "refractive heliostats" deals with another of the sunlight's directional components, transforming its direction into a completely perpendicular pattern to the array. Having downward vertical radiations makes possible to implement a stationary Fresnel lens to concentrate the solar radiation for any application desired.

The system is designed and validated using simulation software to prove the feasibility of the concept.

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### 1. Introduction

Most research efforts on solar energy have focused on photovoltaic (PV) cells development, which generate electricity when a molecular chain reaction that is triggered by a solar energy flux occurs on a thin layer of silicon or germanium based compound. However, commercial PV cells have only achieved an efficiency of 15% to 20% so far [1], while high concentration photovoltaic (HCPV) have efficiency slightly above 40% [2].

\* Corresponding author. Tel.: +52-818-358-2000

E-mail address: [hd.garcia.phd.mty@itesm.mx](mailto:hd.garcia.phd.mty@itesm.mx)

Nomenclature			
$\vec{A}$	Ray direction vector incident to the prim.	$\gamma$	Solar Azimuth ( $^{\circ}$ )
$\vec{B}$	Ray direction vector within the prism.	$\epsilon$	Prism's inner angle ( $^{\circ}$ )
$\vec{C}$	Ray direction vector outside the prim.	$\rho$	Prism's rotation angle ( $^{\circ}$ )
$n$	Refractive index (-)	Subscripts	
PMMA	Poly methyl methacrylate	x	x- vector component
$I_{dir}$	Direct solar irradiance ( $W/m^2$ )	y	y- vector component
Greek symbols		z	z- vector component
$\alpha$	Solar Altitude ( $^{\circ}$ )	I	1st array of prisms
		II	2nd array of prisms

Another research line explores the use of solar thermal energy (STE), in which case two main issues must be overcome. Firstly, solar radiation must be concentrated for high temperature applications; this is due to the low density nature of such energy. Secondly, for most solar concentrators, solar rays must fall perpendicular to the concentrator at all times. Therefore, a solar tracking concentrator (STC) must be used. These requirements also apply for HCPV.

This work was carried on to explore the feasibility of a STC using a semi-passive solar tracking device consisting in two arrays of prisms and a Fresnel lens. A peculiarity of the system is the fixed position of the Fresnel lens therefore, if the prism array can indeed track effectively the sun's path there will be a constant position concentration spot that can be used either in HCPV or CSP applications.

Achieving this constant spot is non-trivial and could represent an important breakthrough since having a non-constant spot means that the receiver which captures the energy must move along with the tracker, elevating in many cases the cost of implementation.

### 1.1. Solar Concentration and Tracking

There are two types of solar concentration, being these, linear and spot; where linear or 2D concentration refers to an area being concentrated to a line and 3D concentration refers to an area being concentrated to a spot. 3D concentration achieves the highest practical concentration level. Solar irradiance intensity varies depending on geographical location, season of the year, weather conditions and time of day. The Liu, B. & Jordan, R. model [3] was used to describe solar irradiance of Monterrey, Mexico (with a latitude of  $25^{\circ}40'$  N). From where it can be seen that the highest irradiance intensities are between 9:00 hours and 15:00 hours, consequently the semi-passive solar tracking concentrator (SPSTC) is designed to work during that time frame.

A wide variety of STC exists which mainly use Fresnel lenses, parabolic mirrors, unconventional geometries and even lenses with variable optical properties [4-11]. In this article, a SPSTC is presented under the premise that reduced but effective sun tracking movement and mechanical effort diminishes the system's energy consumption and complexity, and therefore the costs are reduced. It consists of a couple of PMMA prism arrays that track the sun's rays and a fixed Fresnel lens that concentrates them.

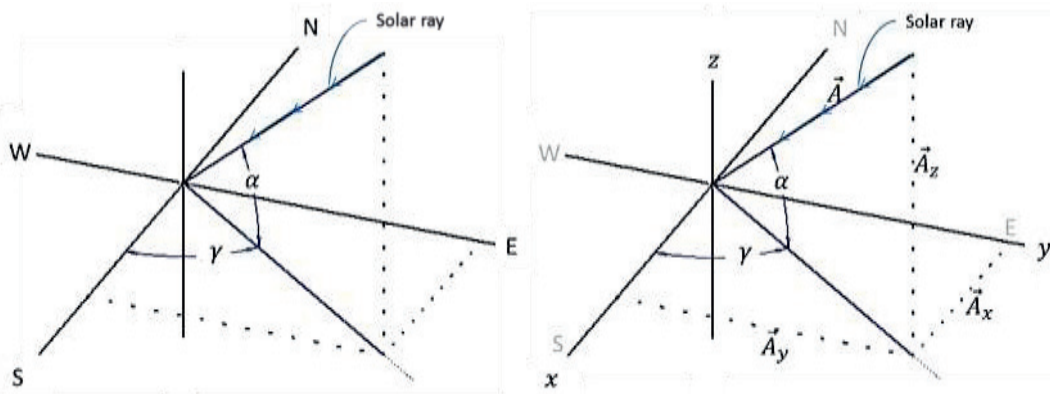


Fig.1. (a) Solar angles; (b) Solar angles and direction vector of the solar ray  $\vec{A}$  with xyz components and coordinate system

### 1.2. Solar angles

The sun’s apparent movement throughout the sky may be described by two angles: altitude and azimuth. The first one is the vertical angle the sun makes with the ground plane, having only the following possible values:  $0^\circ \leq \alpha \leq 90^\circ$ . Conventionally the azimuth is the horizontal angle between the sun and earth’s north, but for practical purposes in this study it has been defined using the south as reference being positive in direction toward East (according to the right hand rule):  $-180^\circ < \gamma < 180^\circ$  as shown in figure 1.

The angles depend not only on the time of the day, but on the time of the year as well (see Fig. 2), analogous to the irradiance variations. The solar angles are a function of the latitude and can be calculated according to established models [12]. The objective of knowing the solar position is to validate our system in actual possible values of solar angles.

## 2. Semi-Passive Solar Tracking Concentrator (SPSTC)

The SPSTC is an optic system that concentrates solar radiation with minimal movement requirements while harvesting the maximum possible energy in a certain area. It has two layers of PMMA prisms that redirect the received solar rays in a constant vertical direction toward a fixed Fresnel lens. The first array

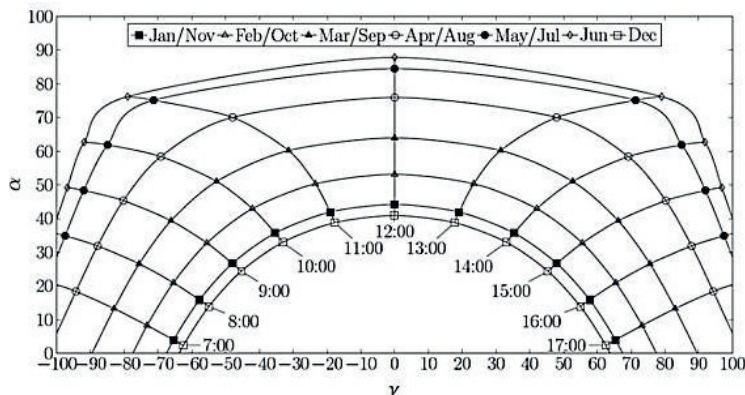


Fig.2. Altitude and Azimuth throughout the year

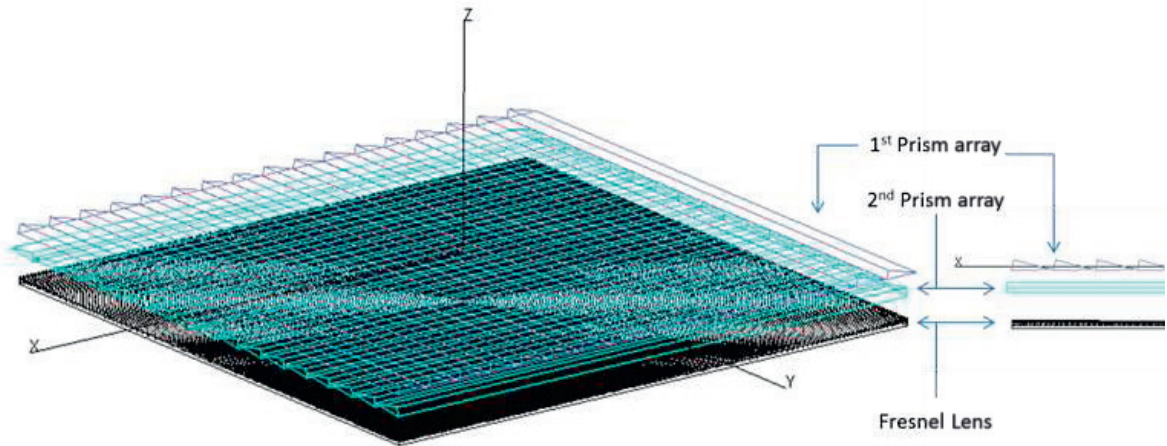


Fig.3. SPSTC formed by two prisms arrays and a Fresnel lens

of prisms rotate all of its components to the same angle  $\rho_I$ , the rotation axis is parallel to the  $\hat{y}$  axis. In a similar manner, the second set rotates all the prisms a value of  $\rho_{II}$  but having as rotation axis the  $\hat{x}$  component.

This configuration allows leaving the Fresnel lens in a fixed horizontal position; which provides several advantages; the wind loads on the system are greatly reduced and more importantly it offers the virtue of having a stationary concentration spot, unlike many other STC which have a robust mechanical tracker that has to support all the moving components.

### 2.1. Prism arrays

The prisms are located in two arrays perpendicular to each other just above the Fresnel lens (see Fig 3). Each prism in the upper array rotate on the y-axis as shown on figure 5b, these prisms are responsible of eliminating one of the solar ray's direction vector components, as its illustrated in figure 5c, ideally leaving  $C_{I,x} = 0$ , afterwards the rays  $C_I$  enter the prisms in the second set which must rotate on the x-axis a corresponding angle  $\rho_{II}$  in order to eliminate the other direction vector component, leaving the resultant beam with just one fixed direction: downwards perpendicular to the plane XY.

A feature worth mentioning is that all the prisms within one array will rotate the same angle  $\rho$ , therefore only one actuator for the mechanical movement is needed. The mechanism is similar to a window's shutter but instead of blocking the sun's light, they redirect those solar rays by the refraction of the PMMA prisms.

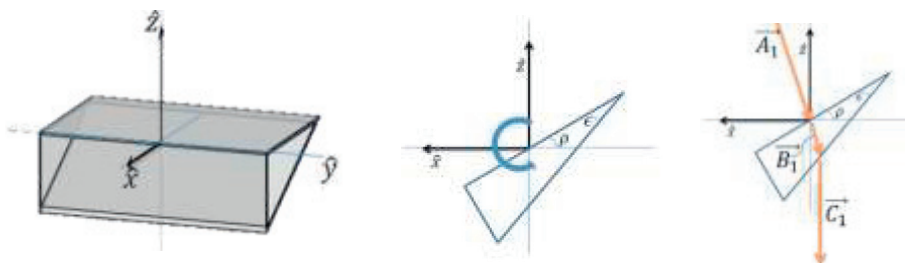


Fig.4. (a) Spatial orientation of a prism from 1<sup>st</sup> array; (b) Prism rotated  $\rho$ ; (c) Ray's path through prism 1

### 2.2. Fresnel lens

It is a type of compact lens that is able to concentrate solar energy in a reduced area; it is essentially a chain of prisms, in which each one represent the slope of the lens surface [13]. The function of this optical device in the system proposed is to receive the rays refracted by the two arrays of prisms and concentrate them in a receiver. This concentration is achieved by two important factors: the change in refractive index of the medium through which the beam travels and the geometry of the lens.

## 3. Mathematical Model

### 3.1. Light refraction

The angles of the rotating prisms depend on the solar position, which at each given time defines a directional vector as shown in figure 1b. The physical phenomenon involved in this system in both the prisms arrays and the Fresnel lens is known as refraction of the light. Snell’s law only applies when we work in the same plane, however when working with solar rays, we find the problem of double angle, which makes the working plane to vary constantly. Therefore vector analysis must be performed to find an extrapolation of Snell’s law for a 3-D environment.

Start with the plane formed by the incident light beam and the normal vector in the point of incident as seen in Fig 5. In that plane the resulting vector after refraction in terms of  $\theta_2$  is:

$$\hat{s}_2 = (\sin \theta_2) \hat{v} + (\cos \theta_2) \hat{N}_2 \tag{1}$$

With basic trigonometry:

$$\cos \theta_2 = \sqrt{1 - \sin^2 \theta_2} \tag{2}$$

On the other hand Snell’s law states:

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 \tag{3}$$

Substituting (2) and (3) onto (1) the following expression is obtained:

$$\hat{s}_2 = \left(\frac{n_1}{n_2} \sin \theta_1\right) \hat{v} - \hat{N} \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1} \tag{4}$$

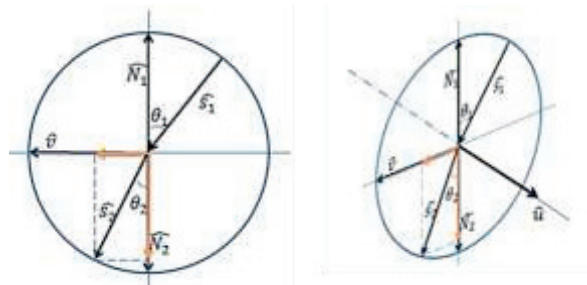


Fig.5. (a) Snell’s law in a plane; (b) Snell’s law in space

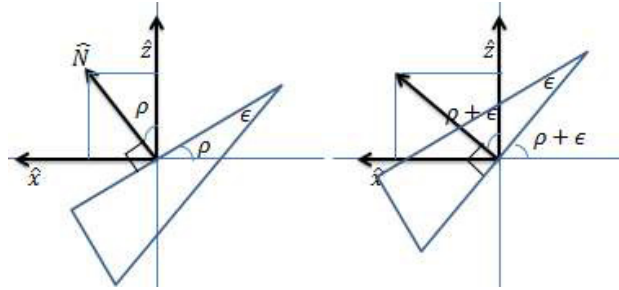


Fig.6. (a) Normal vector for rays entering the prism; (b) Normal vector for rays leaving the prism

This formula is a good approach, notice that in space the cross product of unitary vectors  $\hat{N}_1$  and  $\hat{s}_1$  is:

$$\hat{N}_1 \times \hat{s}_1 = |\hat{N}_1| \cdot |\hat{s}_1| \sin \theta_1 \cdot \hat{u} = \sin \theta_1 \cdot \hat{u} \quad (5)$$

Where  $\hat{u}_1$  is a unitary vector normal to the initial plane, obeying the right hand rule, as shown in Fig 6b. A clever manipulation using dot product in expression (5) yields the following:

$$\sin^2 \theta_1 = (\hat{N} \times \hat{s}_1) \cdot (\hat{N} \times \hat{s}_1) \quad (6)$$

Remember  $\hat{v}$  is normal to the plane defined by  $\hat{N}_1$  and  $\hat{u}$ , therefore:

$$\hat{v} = \hat{N}_1 \times -\hat{u} \quad (7)$$

Merging equation (5) with (7):

$$\hat{v} = \frac{1}{\sin \theta_1} \hat{N} \times (-\hat{N} \times \hat{s}_1) \quad (8)$$

Finally, substituting (6) and (8) in (4) an extrapolation of Snell's Law is obtained, in function of vectors  $\hat{s}_1$  and  $\hat{N}$  only, which are the incident ray, and the normal to the plane, respectively.

$$\hat{s}_2 = \frac{n_1}{n_2} \hat{N} \times (-\hat{N} \times \hat{s}_1) - \hat{N} \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 (\hat{N} \times \hat{s}_1) \cdot (\hat{N} \times \hat{s}_1)} \quad (9)$$

In order to exploit equation (9), the normal vector must be calculated for both cases when the ray is entering the prism, as for when it is going out of it. Both of them depend on the rotation of the prism  $\rho$ , as it would be expected, and is appreciated in Fig. 6. But  $\epsilon$  once it's defined it won't change throughout the operation of the system.

Given values  $\alpha$ ,  $\gamma$ ; the direction vector of the solar ray is defined by:

$$\vec{A}_r = [-\cos \alpha * \cos \gamma, -\cos \alpha * \sin \gamma, -\sin \alpha] \quad (10)$$

For every angle  $\rho$ , everything else is determined, therefore the path that a solar rays traces is unique. First with (9) and  $\vec{A}$  we obtain the ray inside the prism:  $\vec{B}$ , then with  $\vec{B}$  and (9):  $\vec{C}$  can be found. Once the analysis is complete for the first array of prisms, it follows the same way for the second set.

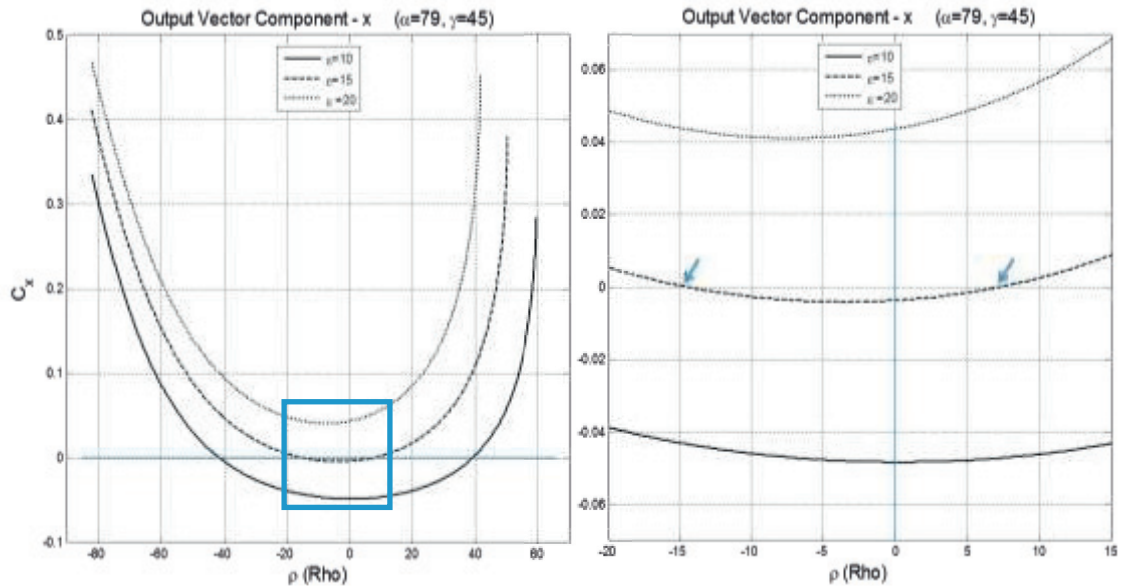


Fig.7. (a) Output response  $C_x$  for prism rotating  $\rho$  for distinct values of  $\epsilon$ ; (b) Zoom view

### 3.2. First layer of prisms

With the help of Matlab, the behavior of a ray through the prism for all possible rotations may be obtained. Being of interest the component  $C_x$ , since the purpose of the first prisms array is to eliminate such component.

It may be observed in figure 7 that for a particular  $\alpha$  and  $\gamma$  if  $\epsilon$  increases the system won't be able to eliminate the desired component  $x$ ; meanwhile when it does, there are two values  $\rho$  that accomplish the desired value. But if  $\epsilon$  decreases too much, even though they are still have two possible solutions, remembering that if the prism has a bigger rotation (positive or negative) then the area available to collect the sun rays will be smaller. Therefore, an adequate value of  $\epsilon$  that ensures  $C_x = 0$  and minimizes the  $|\rho|$  throughout the day and along the months must be chosen.

### 3.3. Second layer of prisms

Using exactly the same analysis as above, instead of having an incoming solar ray  $A_I$ , the ray comes from the previous prism array, therefore has a known direction  $C_I$ . In other words it holds that incoming vector

$$A_{II} = C_I \tag{11}$$

With the main difference that since  $C_{I,x} = 0$  then the rays  $A_{II}, B_{II}, C_{II}$  which describe the path through the prism in the second array will all be in the same plane  $YZ$  and the goal of the prisms will be to eliminate the other component:  $C_{II,y} = 0$ .

When the ray travels through the same plane  $YZ$ , equation 3 may be used to determine the path without the use of vector, this analysis is similar to the analysis necessary for designing a Fresnel lens and

is well described in [3], however the ideal value  $C_{I,x} = 0$  may not be always possible; therefore the goal of both sets of prisms is to minimize the respective components so the Fresnel lens can operate efficiently.

#### 4. Computer Simulation

After having obtained values with Matlab, the model is evaluated using TracePro in order to verify the trajectory the solar rays will follow corroborating the results predicted in the calculations. It also helps to a better understanding of the behaviour of the rays in a 3D environment. For instance, in Fig. 8b it can be observed how  $C_I$  has a  $y$ -component which will be dealt with by the lower prism.

Additionally to the simulation shown in figure 8 of just one pair of prisms and one solar ray, the whole system can be visualized in figure 9 for a given time of the day, were the concentration spot formed by the Fresnel lens is shown.

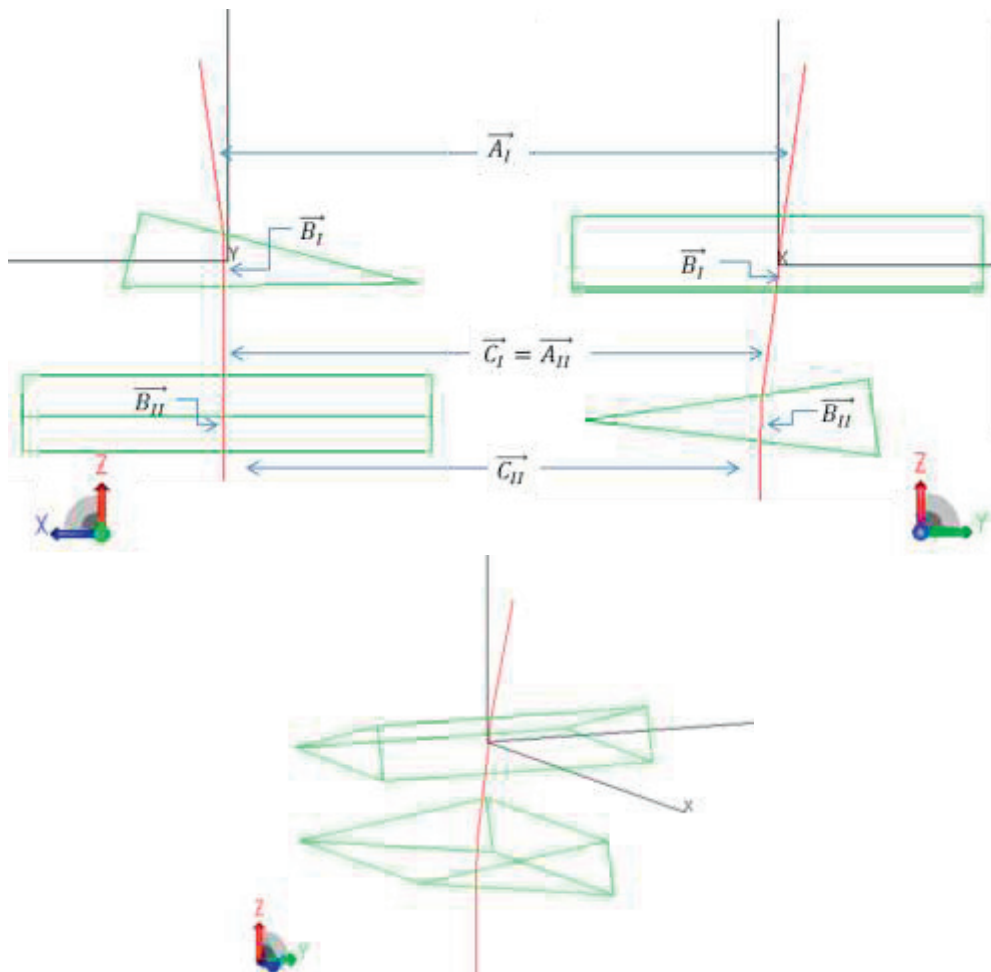


Fig 8. Solar path through prism I and II: (a) Front view; (b) Lateral view; (c) Isometric view



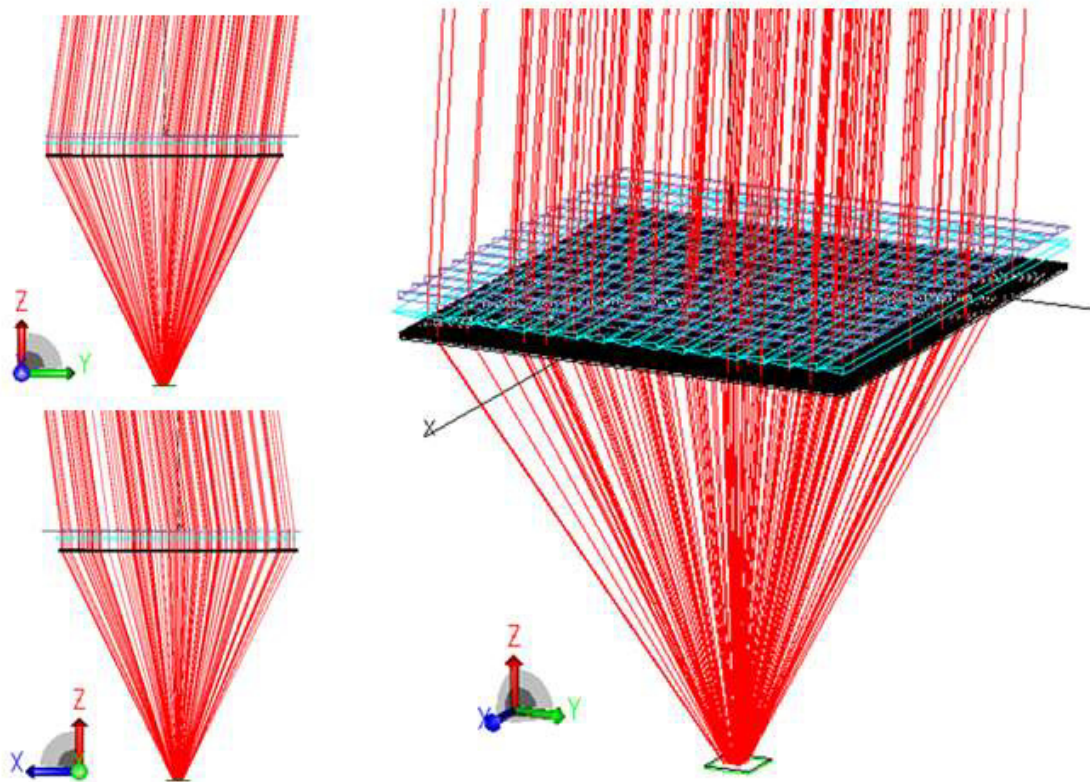


Fig. 9. Simulation for  $\alpha = 79^\circ$ ,  $\gamma = 45^\circ$

## 5. Conclusions and future work

It has been theoretically proved that a SPSTC that uses exclusively PMMA prisms that rotate accordingly to the sun's position is feasible. The current implementation has a fixed Fresnel lens which reduces greatly the mechanical effort of the system, since the tracking is performed by the rotating prisms arrays, which have the virtue of being lightweight individual prisms instead of a robust big heliostat that has to move in a 3D manner.

The tracking is achieved by separating the dual axis movement and implementing two independent rotational movements that work hand by hand to refract efficiently solar rays. Doing so offers a simpler approach in tracking the Sun which ultimately would result in cheaper but reliable solution.

Further analysis must be done to find the optimal values of  $\epsilon_I, \epsilon_{II}$  that manage to operate within the desired time frame. Also it must be considered the prisms material's resistance and stiffness to figure out the feasibility of implementation, since bending and torsion are real problems that may potentially interfere with the correct functionality of the system.

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