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Generation of Control by SU(2) Reduction for the Anisotropic Ising Model

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Abstract. Control of entanglement is fundamental in Quantum Information and Quantum Computation towards scalable spin-based quantum devices. For magnetic systems, Ising interaction with driven magnetic fields modifies entanglement properties of matter based quantum systems. This work presents a procedure for dynamics reduction on $SU(2)$ subsystems using a non-local description. Some applications for Quantum Information are discussed.

1. Introduction

Quantum control exploits the fine management of physical variables to improve the capacity and speed of the information processing [1,2]. The magnetic driven Ising interaction [3] generates that control [4] for bipartite qubits, chains and lattices [5–7], involving temperature, field strengths and geometry in the analysis [8–10]. Control for a single or a couple of spin elements is still at the heart of a scalable spin-based quantum computer because the controlled state exchange lets to obtain universal quantum operations [11,12] in terms of DiVincenzo criteria [13] for quantum states reliability. This paper discusses how the bipartite dynamics of Ising model in $SU(4)$ is reduced in two $SU(2)$ subsystems on a non-local basis. Section 2 sets the Hamiltonian, notation and $SU(2)$ reduction. Section 3 discusses some applications: Quantum Error Correction, Evolution Loops, Exchange Operations, $SU(2)$ optimal control and gate design based on Unitary Factorization. Section 4 states the conclusions and future work.

2. $SU(2)$ decomposition for the anisotropic Ising Hamiltonian in a non local basis

Ising interaction has been analyzed for several systems and configurations (XX, XY, XYZ, etc.) [14–16] with structured control effects [17,18] due to the geometry or the physical properties [19–21]. We use a Hamiltonian with a driven magnetic field in $h = 1, 2, 3$ (x, y, z) direction:

$$H_h = \sum_{k=1}^{3} J_k \sigma_{1_k} \sigma_{2_k} - B_{1_h} \sigma_{1_h} - B_{2_h} \sigma_{2_h} \tag{1}$$

including the previous models. Due to the Bell basis is privileged and to comprise the three field cases, we adopt the notation [22]: $|\beta_{\mu\nu}\rangle \equiv |\beta_{AB}\rangle$; $\mu, \nu \in \{-, +\}$; $A, B \in \{0, 1\}$; $A = \frac{1+\mu}{2}$, $B = \frac{1+\nu}{2}$. Capital scripts $A, B, ...$ are $0, 1$ for the computational basis; greek scripts for $\pm$ or ± 1,

because they evolve to ± 1 in algebraic expressions; latin scripts $h, i, j, ...$ for coordinates. Thus

$$E_{h_{\mu\nu}} = \mu J_h + \nu R_{h_{-\mu}} \tag{2}$$

are the eigenvalues for the states $\left| \phi_{\mu\nu}^h \right\rangle$ [22]. $U(t) \in SU(4)$ because they add zero. There

$$R_{h\pm} \equiv \sqrt{B_{h\pm}^2 + J_{\{h\}_\mp}^2}, \quad J_{\{h\}\pm} \equiv J_i \pm J_j, \quad B_{h\pm} \equiv B_{1_h} \pm B_{2_h}, \tag{3}$$

h, i, j is a cyclic permutation of $1, 2, 3$; then $\{h\}$ is equivalent to the pair i, j. The parameters

$$b_{h\pm} = \frac{B_{h\pm}}{R_{h\pm}}, \quad j_{h\pm} = \frac{J_{\{h\}_\mp}}{R_{h\pm}} \in [-1, 1], \quad \Delta_{h\mu}^\nu = \frac{t}{2}(E_{h\mu+} + \nu E_{h\mu-}), \tag{4}$$

$$e_{h\alpha}^\beta = \cos \Delta_{h\alpha}^- + i\beta j_{h-\alpha} \sin \Delta_{h\alpha}^-, \quad d_{h\alpha} = b_{h-\alpha} \sin \Delta_{h\alpha}^-,$$

reduce the evolution matrix expressions for the time independent Hamiltonian. Reader should be aware about double scripts to avoid misconceptions. As instance, $R_{h-\mu}$ in 2 is R_{3+} for $\mu = -1$ when driven field is in the z direction. While, B_{1_h}, B_{2_h} are the driven fields in h direction on particles 1,2 respectively. Then, in the Bell basis, evolution operators and bipartite states are

$$U_h(t) = \sum_{\mu,\nu,\gamma,\delta} U_{h\mu\nu,\gamma\delta} |\beta_{\mu\nu}\rangle \langle \beta_{\gamma\delta}| , \quad |\psi\rangle = \sum_{\mu,\nu\in\{-,+\}} \mathcal{B}_{\mu\nu} |\beta_{\mu\nu}\rangle , \tag{5}$$

and $U_h(t)$ becomes split on the direct-sum of subspaces $\mathcal{H}_h^{\otimes 2} = \mathcal{H}_{h,1} \oplus \mathcal{H}_{h,2}$ as function of h [23]:

$$\mathcal{H}_{h,1} \equiv \text{span}(\{|\beta_{--}\rangle , |\beta_{s^0,a s^0,b}\rangle\}), \quad \mathcal{H}_{h,2} \equiv \text{span}(\{|\beta_{s^1,a s^{h,b}}\rangle , |\beta_{+s^1,h}\rangle\}), \tag{6}$$

with $a = \frac{1}{2}(h-2)(h-3), b = \frac{1}{2}(h-1)(h-2), s^{c,d} = (-1)^{(c+d)\text{mod}2}$. Then, $U_h(t)$ becomes the direct sum $U_h(t) = s_{h1} \oplus s_{h2}$ with s_{hj} as a $U(1) \times SU(2) \subset U(2)$ block on each $\mathcal{H}_{h,j}, j = 1, 2$. $|\psi\rangle = \sum_j \alpha_j |\psi_j\rangle$ and $|\psi_j\rangle \in \mathcal{H}_{hj}$ evolving in each subspace. For the time independent case

$$s_{hj} = e^{i\Delta_{h\alpha}^+} \begin{pmatrix} e_{h\alpha}^{\beta*} & -qi^h d_{h\alpha} \\ qi^{*h} d_{h\alpha} & e_{h\alpha}^\beta \end{pmatrix} = e^{i\Delta_{h\alpha}^+} \left(\cos \Delta_{h\alpha}^- \mathbf{I}_{hj} - i \sin \Delta_{h\alpha}^- \mathbf{n} \cdot \mathbf{S}_{hj} \right), \tag{7}$$

where $\mathbf{n} = (qb_{h-\alpha} \sin \frac{h\pi}{2}, qb_{h-\alpha} \cos \frac{h\pi}{2}, \beta j_{h-\alpha}), \alpha = (-1)^{h+j+1}, \beta = (-1)^{j(h+l_j-k_j+1)}, q = \beta(-1)^{h+1}$, h is the direction of magnetic field and $j = 1, 2$ a position label for each block in the whole evolution matrix. k_j, l_j are the labels for its rows. As instance, $k_2 = 3, l_2 = 4$ are the rows of the second block s_{21}, with $j = 2$ in $U_{h=1}(t)$. Pauli matrices $\mathbf{S}_{hj} = (\mathbf{X}_{hj}, \mathbf{Y}_{hj}, \mathbf{Z}_{hj})$ and 2×2 identity $\mathbf{I}_{hj}$ are straight forms in the matrix block to avoid confusion with the traditional operators in the computational basis. $U_h(t)$ forms abelian subgroups for fix $j_{h\pm}, b_{h\pm}$ [22]. Thus, inverses $U_h^\dagger(t)$ are obtained as another $U_h(t')$ for the same $j_{h\pm}, b_{h\pm}$ in the system, so $\mathbf{I}_{hj}$ can be achieved in finite time with the same fields. Properties and applications of the decomposition arise from those known for $SU(2)$ systems. A direct calculation gives the dynamics of the concurrence for the Bell states $|\beta_{\mu\nu}\rangle$ evolution

$$\mathcal{C}_{\mu\nu}^h = 1 - 4j_{h-f_{\mu\nu}^h}^2 b_{h-f_{\mu\nu}^h}^2 \sin^4 \Delta_{h\ f_{\mu\nu}^h}^-, \quad f_{\mu\nu}^h = \mu\delta_{1h} + \mu\nu\delta_{2h} + \nu\delta_{3h}, \tag{8}$$

showing a simple behavior on only one Rabi frequency (contrasting with separable states [18]), an inherited feature from the separation of $\mathcal{H}^{\otimes 2}$ in two subspaces. There are not a maximal entangled evolution path with finite constant fields, more than trivial $j_{h f_{\mu\nu}^h} = 0$ or $b_{h f_{\mu\nu}^h} = 0$. Intermediate separable states are possible only for the tuning $4j_{h f_{\mu\nu}^h}^2 b_{h f_{\mu\nu}^h}^2 = 1$.

3. Properties and potential applications of $SU(2)$ block reduction

3.1. Quantum error correction

Expression for a desired s_{hj} in terms of $\mathbf{S}_{hj}$ on each subspace sets a discrete language for quantum errors on it and their related syndromes. As instance, for s_{12}, the subspace is span by $|\beta_{+-}\rangle$, $|\beta_{++}\rangle$, then, $\mathbf{X}_{1_2}$ is a bit flip in the second qubit; $\mathbf{Z}_{1_2}$ is a phase flip on both qubits; and $\mathbf{Y}_{1_2} = -i\mathbf{Z}_{1_2}\mathbf{X}_{1_2}$ is the combination of both errors. Traditional quantum error correction codes can be applied on each sector [24] and addressed on a specific evolution noting if $\mathcal{D} \equiv \delta\mathbf{p} \cdot \nabla_{\mathbf{p}}$, then, a tiny imprecision in the prescription parameters $\delta\mathbf{p}$ $(t, \Delta_{h_\alpha}^{\pm}, j_{h\pm}, b_{h\pm})$ to reproduce s_{hj}^0, induces an error $\delta s_{hj} \approx \mathcal{D}s_{hj}^0 + \frac{1}{2}\mathcal{D}^2 s_{hj}^0$, thus, for a desired final state $\left|\psi_{fj}\right\rangle$ and its fidelity $\mathcal{F}$:

$$\delta\left|\psi_{fj}\right\rangle \approx \delta s_{hj} s_{hj}^{0\dagger}\left|\psi_{fj}\right\rangle, \quad 1 - \mathcal{F}^2 \approx \left\langle s_{hj}^0 \mathcal{D}s_{hj}^{0\dagger} \mathcal{D}s_{hj}^0 s_{hj}^{0\dagger}\right\rangle - \left|\left\langle \mathcal{D}s_{hj}^0 s_{hj}^{0\dagger}\right\rangle\right|^2, \quad (9)$$

where s_{hj} unitary properties are used. The quadratic dependence on $\delta\mathbf{p}$ shows a mild impact.
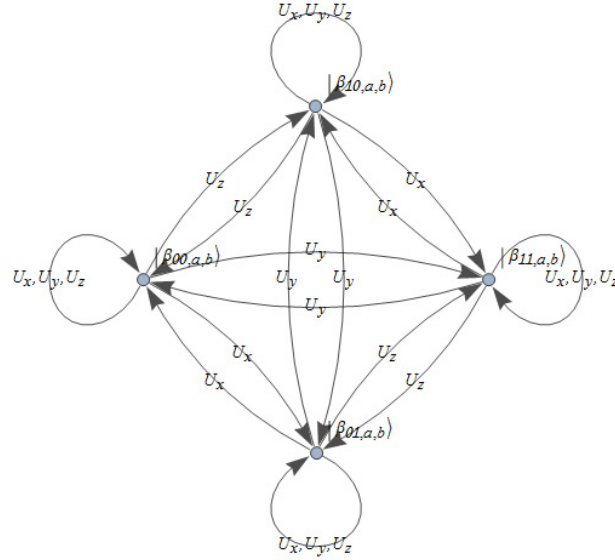
Figure 1. Bell states transformations through Evolution Loops and Exchange Operations.

3.2. Diagonal and antidiagonal basic forms

If the dynamics in $t = T$ reduces to the forms (diagonal and antidiagonal cases) [22]:

$$s_{hj} = \pm\mathbf{I}_{hj} \tag{10}$$

$$s_{hj} = \pm\mathbf{X}_{hj} \quad \text{or:} \quad s_{hj} = \pm i\mathbf{Y}_{hj} \tag{11}$$

we could achieve evolution loops [25–27] and exchange operations [18] in $\mathcal{H}^{\otimes 2}$ combining them, recovering or switching Bell states. Last operations let manipulate the Bell states in a planned way by applying a sequence of magnetic field pulses in adequate directions. Figure 1 summarizes the achievable loops and transitions. There are several ways to s_{hj} fulfills (10) or (11). Using only one field pulse, a direct analysis shows $s_{hj} = (-1)^{m_\alpha}\mathbf{I}_{hj}$ at $t = T$ if $n_\alpha, m_\alpha \in \mathbf{Z}$:

$$T = \frac{m_\alpha - n_\alpha}{\alpha J_h}\pi > 0 \quad , \quad B_{h-\alpha}{}^2 = (\frac{J_h n_\alpha}{m_\alpha - n_\alpha})^2 - J_{\{h\}_\alpha}{}^2. \tag{12}$$

To get the evolution loop $U_h(T) = (-1)^{m_\alpha}\mathbf{I_4}$, (12) should be fulfilled in both blocks $\pm\alpha$. It is possible if $m_\alpha - n_\alpha = n_{-\alpha} - m_{-\alpha}$ and $m_{\pm\alpha}$ have the same parity. If (11) is required in one block and (10) in another, $U_h(T)$ becomes an exchange operation transforming two Bell states between them, while the remaining become unchanged. Nevertheless, there are not solutions for block antidiagonalization (11) in one pulse compatible with (12) without restrict J_h and use non finite fields. But a two blocks product (7) for consecutive pulses $s'_{hj}s_{hj}$ fulfill it (α, β, q are the same). Complementary diagonalization prescriptions (10) in two pulses for block α are [23]:

$$\Delta_{h\alpha}^- + \text{sign}(J_{\{h\}_\alpha}J'_{\{h\}_\alpha})\Delta'^{\,-}_{h\alpha} = n_\alpha\pi, \quad \frac{B_{h-\alpha}}{J_{\{h\}_\alpha}} = \frac{B'_{h-\alpha}}{J'_{\{h\}_\alpha}}, \quad \Delta_{h\alpha}^+ + \Delta'^{\,+}_{h\alpha} = (m_\alpha + n_\alpha)\pi, \quad (13)$$

$m_\alpha, n_\alpha \in \mathbf{Z}$, giving: $s'_{hj}s_{hj} = (-1)^{m_\alpha}\mathbf{I}_{hj}$. Antidiagonalization (11) for block $-\alpha$ gives [23]:

$$\Delta_{h-\alpha}^- = \frac{2n_{-\alpha}+1}{2}\pi, \quad \Delta'^{\,-}_{h-\alpha} = \frac{2n'_{-\alpha}+1}{2}\pi, \quad \frac{B_{h\alpha}}{J_{\{h\}-\alpha}} = -\frac{J'_{\{h\}-\alpha}}{B'_{h\alpha}} \tag{14}$$

$$\Delta_{h-\alpha}^+ + \Delta'^{\,+}_{h-\alpha} = \frac{\pi}{2}M_{h,q,-\alpha,\beta,n_{-\alpha},n'_{-\alpha},s_{-\alpha}}$$
$$\equiv -\frac{\pi}{2}(h + \text{sign}(q\beta b'_{h\alpha}j_{h\alpha}) + 2(n_{-\alpha} + n'_{-\alpha} - s_{-\alpha} + 1)),$$

with: $s_{-\alpha}, n_{-\alpha}, n'_{-\alpha} \in \mathbf{Z}$, then $s'_{hj}s_{hj} = (-1)^{s_{-\alpha}}i^{h\bmod 2}\mathbf{S}_{hj}^{1+h\bmod 2}$ ($1 + h\bmod 2$ component of $\mathbf{S}_{hj}$) as (11). $s_{-\alpha}$ and h introduce a phase when the Bell states are exchanged, while $s_{-\alpha}$ is not an integer for all h values due to it depends on the restrictions for $\Delta_{h-\alpha}^+ + \Delta'^{\,+}_{h-\alpha}$ in another sector.

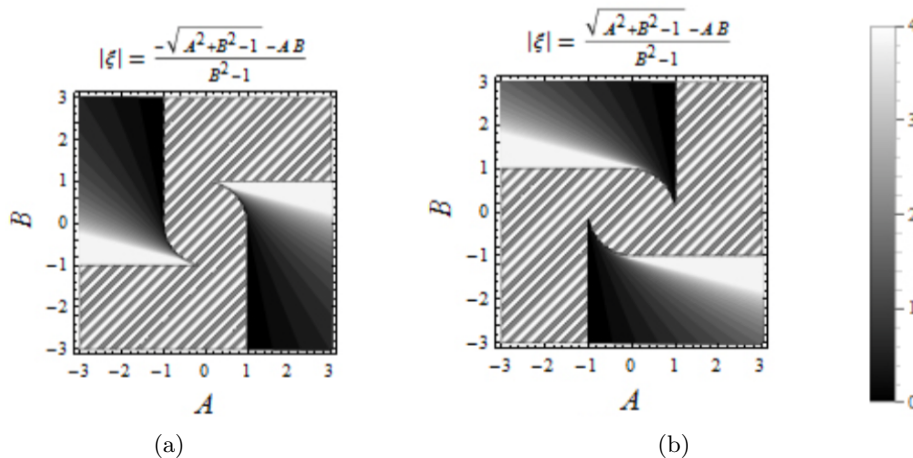### 3.3. Evolution loops and exchange operations

Evolution loops and exchange operations are involved in quantum characterization, teleportation, state discrimination and repreparation [18]. As it was stated, evolution loops are reached in one field pulse using (12) for both blocks $\pm\alpha$: $U_h(T) = (-1)^{m_\alpha}\mathbf{I_4}$. Exchange operations are possible in two pulses depending on: $B_{h\pm\alpha}, B'_{h\pm\alpha}, t, t'$. Due to (14), the antidiagonal-antidiagonal case implies an extra condition that (13), limiting J_i. Thus, we analyze only the diagonal-antidiagonal case where only two Bell states are exchanged while other remain unchanged. Combining (13) and (14), solutions are reached as a program to get $B_{h-\alpha}, B'_{h\pm\alpha}, t, t'$ in terms of $B_{h\alpha}$. We set $\pm\alpha$ for the diagonal/antidiagonal block. Decoupling $B_{h\alpha}$:

$$|\xi| = \frac{-AB \pm \sqrt{A^2 + B^2 - 1}}{B^2 - 1}, \quad A = \frac{(2n_{-\alpha}+1)J_h}{2(m_\alpha+n_\alpha)|J_{\{h\}-\alpha}|}, \quad B = \frac{(2n'_{-\alpha}+1)J'_h}{2(m_\alpha+n_\alpha)|J'_{\{h\}-\alpha}|}, \quad (15)$$

where $\xi \equiv \frac{B_{h\alpha}}{J_{\{h\}-\alpha}}$. It has solutions if $A^2 + B^2 \geq 1$ and positivity is fulfilled. It is possible for finite and anisotropic strengths selecting $n_{-\alpha}, n'_{-\alpha}, n_\alpha, m_\alpha$ properly. Solutions are mainly around of opposite signs for A and B, limiting physical cases. $n_{-\alpha}, n'_{-\alpha} \geq 0$ and $m_\alpha + n_\alpha, J_h, J'_h$ determine the selection of A, B. Figure 2 shows the regions with solutions for $|\xi|$ in the plane AB. Note there are solutions in the four quadrants for finite J_h, J'_h with an adequate selection of $n_{-\alpha}, n'_{-\alpha}, n_\alpha, m_\alpha$. Dashed regions have not solutions for (15). Remaining prescriptions are

$$\frac{B_{h-\alpha}^2}{J_{\{h\}_\alpha}^2} \equiv \chi^2 = \left(\frac{2n_\alpha\sqrt{\xi^2+1}}{S_\alpha(2n_\alpha+1)+P_\alpha S'_\alpha(2n'_{-\alpha}+1)|\xi|}\right)^2 - 1, \quad \frac{B'_{h\alpha}}{J'_{\{h\}-\alpha}} = -\xi^{-1}, \quad \frac{B'_{h-\alpha}}{J'_{\{h\}_\alpha}} = \chi, \quad (16)$$

$$\frac{|J_{\{h\}-\alpha}|t}{(2n_\alpha+1)} = \frac{|J'_{\{h\}-\alpha}|t'}{(2n'_{-\alpha}+1)|\xi|} = \frac{\pi}{2\sqrt{\xi^2+1}}, \quad P_\alpha = \text{sign}(J'_{\{h\}_\alpha}J_{\{h\}_\alpha}), \quad S_\alpha = |\frac{J_{\{h\}_\alpha}}{J_{\{h\}-\alpha}}|, \quad S'_\alpha = |\frac{J'_{\{h\}_\alpha}}{J'_{\{h\}-\alpha}}|,$$

in addition, block phases should be synchronized: $2(m_\alpha + n_\alpha) = -M_{h,q,-\alpha,\beta,n_{-\alpha},n'_{-\alpha},s_{-\alpha}}$. Then, $m_\alpha + n_\alpha$ fixes $s_{-\alpha}$ value in $s'_{hj}s_{hj}$. Note our analysis has preserved the non decisive possibility of the strengths could change during each pulse, a few common, but not impossible situation.

Figure 2. Solution for $|\xi|$ values in gray scale for a) Negative and b) Positive signs.

3.4. Remarkable natural quantum gates

An analysis for exchange operations shows the evolution (for $T = t + t'$) as one of the gates

$$
\begin{aligned}
U_1(T): & \quad \mathcal{A}_{1,1} \equiv i\mathbf{Y}_{1_1} \oplus \mathbf{I}_{1_2} = X_1 C^1(iY_2)X_1, & \mathcal{A}_{1,2} \equiv \mathbf{I}_{1_1} \oplus i\mathbf{Y}_{1_2} = C^1(iY_2) \\
U_2(T): & \quad \mathcal{A}_{2,1} \equiv i\mathbf{X}_{2_1} \oplus \mathbf{I}_{2_2} = X_1 C^{1|2}(iX_1X_2)X_1, & \mathcal{A}_{2,2} \equiv \mathbf{I}_{2_1} \oplus i\mathbf{X}_{2_2} = C^{1|2}(iX_1X_2) \\
U_3(T): & \quad \mathcal{A}_{3,1} \equiv \mathbf{I}_{3_1} \oplus i\mathbf{Y}_{3_2} = X_2 C^2(iY_1)X_2, & \mathcal{A}_{3,2} \equiv i\mathbf{Y}_{3_1} \oplus \mathbf{I}_{3_2} = C^2(iY_1)
\end{aligned}
\tag{17}
$$

remembering $C^a NOT_b$ gates on the $|\beta_{\alpha\beta}\rangle$ scripts (extending our notation under: $0 \leftrightarrow -, 1 \leftrightarrow +$). Realizing $\mathcal{A}_{1,2}$ is $C^1(iY_2)$ gate (understood for the Bell basis), an analysis shows the second expressions in (17) with the traditional X_i, Y_i in the 4×4 matrices. $C^{1|2}(G)$ is a controlled gate G where $1|2$ means $(A+B) \bmod 2$ (using the equivalence with computational scripts $|\beta_{AB}\rangle = |\beta_{\alpha\beta}\rangle$). These operations give alternative gates to those used with computational basis in the circuit model of quantum computation, being an adaptation to the Bell basis and deserving the same treatment (by example, gate $\mathcal{A}_{1,2}$ was used to implement a Teleportation algorithm [28]).

3.5. Optimal cost quantum control and gate design by Unitary Factorization

Compatible trends of the reduction are devised. Optimal control for two level quantum systems is developed for energy cost [29] and minimal time [30] and the Hamiltonian is fully compatible with the current: (1) can be decomposed as $H_h = \sum_j H_{j,h}$, with each $H_{j,h}$ acting on each $\mathcal{H}_{hj}$,

$$
H_{j,h} = -s_0 J_h \mathbf{I}_{2,h_j} + s_1 J_{\{h\}_{s_0}} \mathbf{Z}_{hj} + s_2 B_{h-s_0} \mathbf{S}_{hj}^q,
\tag{18}
$$

where $s_0 = (-1)^{h+j+1}, s_1 = s_0^p, s_2 = (-1)^p s_0^{p+q}, p = 1 + \frac{1}{2}(h-1)(h-2)$ and $q = h \bmod 2 \in \{1, 2\}$.

Another application is the Unitary Factorization gate design [31]. Diagonal-antidiagonal forms on (5) can reproduce unitary matrices $M^{(i,j)\dagger}$ as factors [32]. Then, a unitary gate $U \in SU(4)$ can be written as ($\rightarrow$ states a forward product stacking factors from left to right):

$$
U = \prod_{\substack{1 \le j < n \\ n > i \ge j}}^{\rightarrow} M^{(i,j)\dagger}
\tag{19}
$$

4. Conclusions

Spin-based quantum computing (superconducting integrated systems, superconducting flux qubits, straight nuclear magnetic resonance or quantum dots) exploits Ising interactions [33] where several technological issues have been solved around of stability and decoherence to set stable isolated qubits. There, contemporary control physics reports times and magnetic fields achievable in the experimental setup around of $t \sim 10^{-9} - 10^{-6}s$ and $B \sim 1 - 10T$ operating in regions around of $r \ll 5nm$ [34]. Despite, in nowadays, deep control of physical parameters in quantum magnetic systems is still limited to set a programmable artificial spin network under full control [33]. Current work develops control models including several freedom degrees. Operations constructed under $SU(2)$ decomposition state elementary physical operations to reproduce a planned evolution on a Bell states based grammar, able to be scaled to simulate complex computational problems on them. Moreover, combining these operations (5) for magnetic fields in different directions, we can increase the control possibilities. Creation of universal procedures by factorization as arbitrary gates is an alternative design based completely on $SU(2)$ reduction. Future work should be addressed considering finite temperature effects and deeper error correction procedures for the contemporary experimental limitations. The extension of this formalism to multiqubit case only requires to group arbitrarily the Hamiltonian eigenstates by pairs, but the search of universal basis (as Bell basis was here) is still open.

Acknowledgements

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