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AN APPROACH TO ALGEBRA VOLUME 1

CLAUDIA PATRICIA
CHAPA TAMEZ

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VOLUME 1

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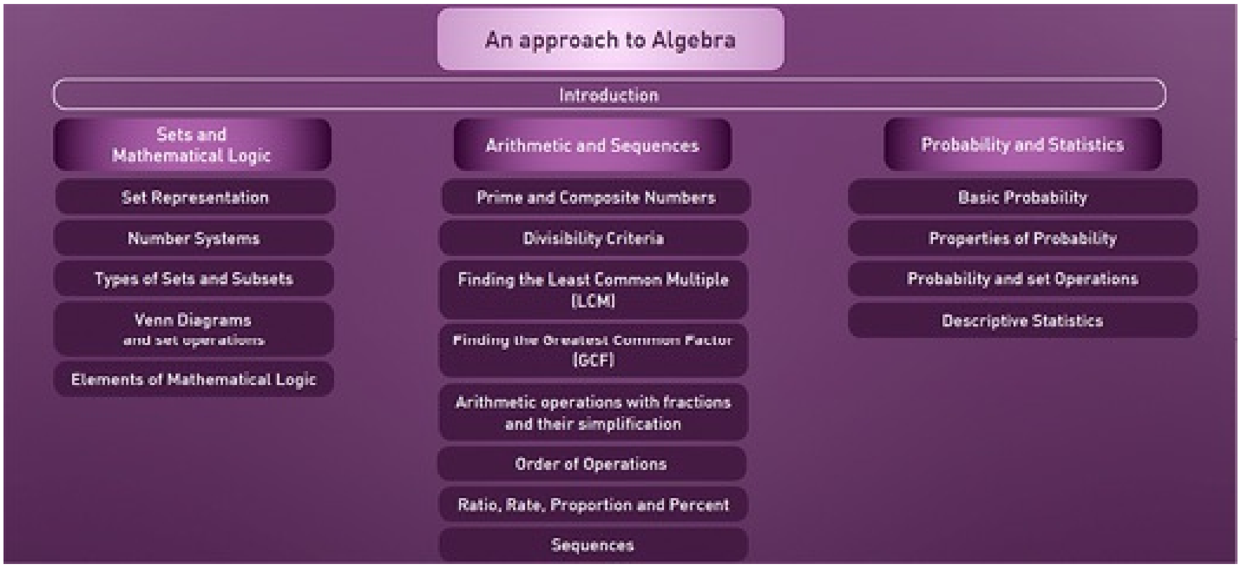
Concept Map


An approach to Algebra
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eBook Introduction

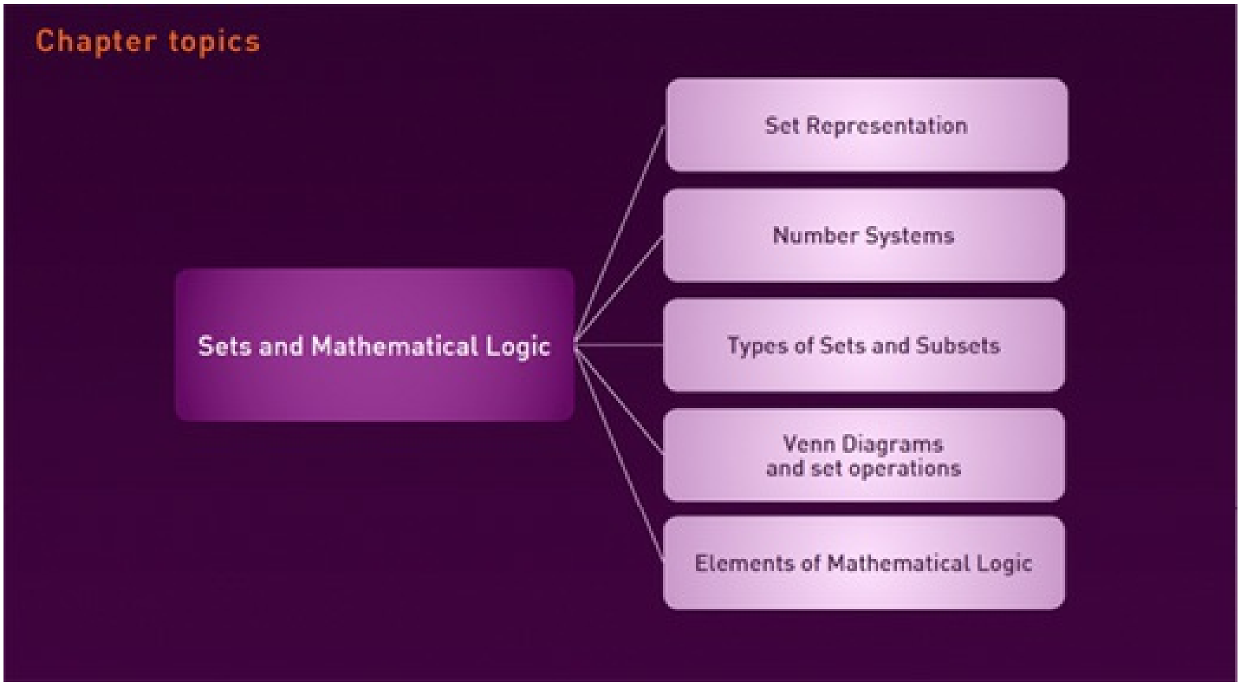
Since mathematical principles have remained the same all throughout the world for centuries, Mathematics has been considered by many the “universal language of numbers”. For some, Mathematics causes anxiety or fear because it seems difficult to understand. One of the objectives of this eBook is to make the material more visually, technologically and multiculturally attractive, with the aid of videos, pictures, games, animations and interactive exercises so that Mathematics can become more interesting and accessible for today’s worldwide students since “evidence is mounting to support technology advocates’ claims that 21st-century information and communication tools, as well as more traditional computer-assisted instructional applications, can positively influence student learning processes and outcomes (Cradler, 2002)”. The role of mathematics in our modern world is crucial for today’s global communication and for a multitude of scientific and technological applications and advances.

The author brings a variety of expertise to the subject of Algebra, and includes many illustrated material, equations, tables, figures, and other aids that help understanding the text. Unfamiliar terms and concepts are highlighted and defined in a glossary, and at the end of each chapter website links are provided to help students to enrich their knowledge and to help them practice their skills. The author starts the journey of the eBook from the study of sets, numbers and mathematical logic to introduce the student to arithmetic and the study of sequences. Previous knowledge will allow the student to have the most basic fundamentals to understand terms related to probability and statistics. Finally, the student will acquire the essential knowledge of the fundamental concepts of algebra to apply it to the study of functions and their graphs along with the essence of algebra, solving equations.

In the modern world, Algebra is a very important day-to-day tool. It is not only a subject used in a math course but can be applied to many real-life situations. It is not only used by people in daily life, but by many professionals that use it in a wide variety of areas, such as architecture, natural sciences, economy, engineering among others. And the fact is that, as Algebra has advanced in the past, it will continue doing so in the days to come, fulfilling people's worldwide needs in a greater way.

One of the most fundamental concepts in math-ematics is the study of sets, which were developed at the end of the 19th century. The roots of mathematics come from set theory. Venn diagrams were devel-oped to understand it better. Both set theory and Venn diagrams are the basis to comprehend the mathematical logic studied later on in the chapter.

Chapter 1. Sets and Mathematical Logic



Sets and Mathematical Logic

One of the most fundamental concepts in math-ematics is the study of sets, which were developed at the end of the 19th century. The roots of mathematics come from set theory. Venn diagrams were devel-oped to understand it better. Both set theory and Venn diagrams are the basis to comprehend the mathematical logic studied later on in the chapter.

1.1 Set Representation

Aset is a collection of objects, such as a set of numbers in arithmetic, a set of ordered pairs, or a set of letters, which are called elements of the set, or members. Elements in a set are listed within braces { } and usually sets are named with uppercase letters (A, B, C,…).There are different ways to represent a set and they are shown in Table 1.1.

Table 1.1 *Ways to represent a set*

Description	Roster form	Set builder notation
Brief sentence to describe the set in words.	List of elements within braces and are separated by commas. Points of ellipsis (…) are used to denote that the pattern continues or that the list of numbers continues on forever.	Uses specific symbology to represent and describe a set. It is written within braces and uses symbology $\{x\|x\}$ which is read as "x such that x". Generally when working with numbers and letters it is written as an algebraic expression.
Examples: A= The vowels B= Days of the week C= Whole numbers D= Natural numbers	Examples: A={a,e,i,o,u} B={Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} C={0,1,2,3,4,5, …} D={4,5,6,7,…}	Examples: A=$\{x\|x$ is a vowel$\}$ B=$\{x\|x$ is a day of the week$\}$ C=$\{x\|x$ is a whole number$\}$ or C=$\{x\|x=W\}$ D=$\{x \in N\|x>3\}$

REVIEW ACTIVITIES

- Set notation is a way of describing the membership of and relationships between collections of objects and it is used in a wide range of documents and contexts, such as in logic, mathematics and computer science. So, that is why it is important to understand different ways to represent it.

Exercise 1.1 Set representation

1.2 Number Systems

The number systems have evolved over time since counting has been very important for human civilization all throughout the world. Today we use numbers for almost everything in our daily life. To comprehend and understand the basics of algebra, there is a group of numbers that is important to know, and it is called real numbers.

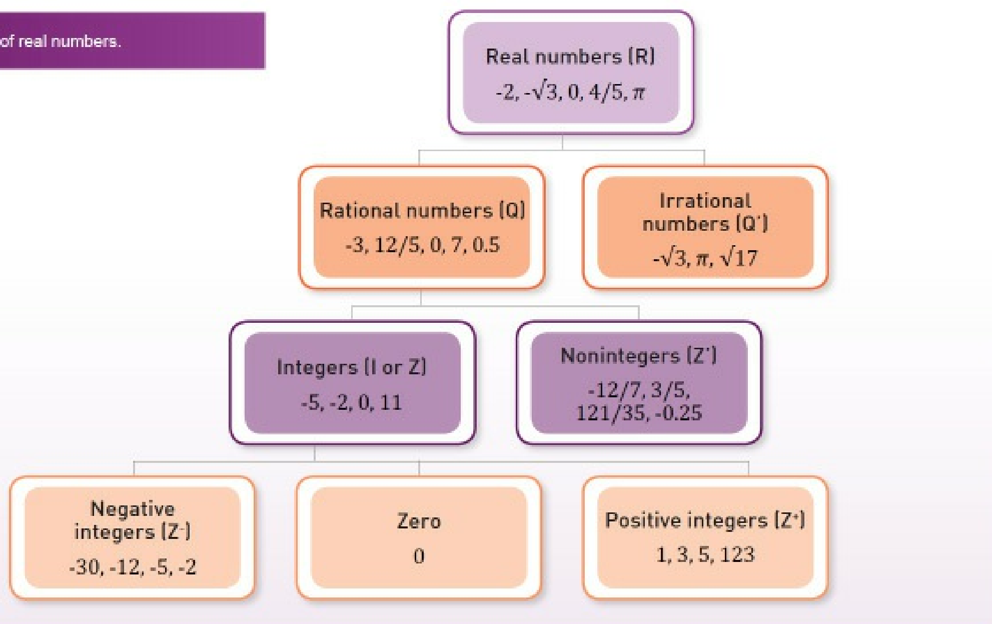
1.2.1 Real Numbers and Field Properties

In Table 1.2 some different sets of numbers are described along with their corresponding letter used to represent each set of numbers.

Table 1.2 *Some important sets of numbers*		
Real numbers	R={x|x is a point on the real number line}	All numbers, including decimals on a number line
Rational numbers	Q={$\frac{p}{q}$ |p and q are integers, $q \neq 0$}	Real numbers that can be expressed as a ratio of integers. Letter "Q" comes from the word quotient.
Irrational numbers	H=Q'={x|x is a real number that is not rational}	Real numbers that cannot be expressed as a ratio of integers
Natural numbers (counting numbers)	N={1,2,3,4,5,...}	All positive integers. Natural numbers are the ones that are used to count
Whole numbers	W={0,1,2,3,4,5,...}	All positive integers and zero. Are also known as non-negative integers
Integers	I=Z={..., -3,-2,-1,0,1,2,3,...}	Whole numbers, including their negatives. Letter "Z" comes from the word *zahe* which in German means "integer".
Negative integers	Z-={-1,-2,-3,-4,-5,...}	Negatives integers
Digits	D={0,1,2,3,4,5,6,7,8,9}	All non-negative integers composed of a single numeral
Prime numbers	P={2,3,5,7,11,13,...}	Natural numbers greater than 1 that can be divisible only by 1 and itself

The diagram shown in Figure 1.1 classifies real numbers and gives some examples for each kind of set of numbers.

Figure 1.1 Classification of real numbers.



In Figure 1.2, the relation between some important subsets of real numbers can be seen clearly.

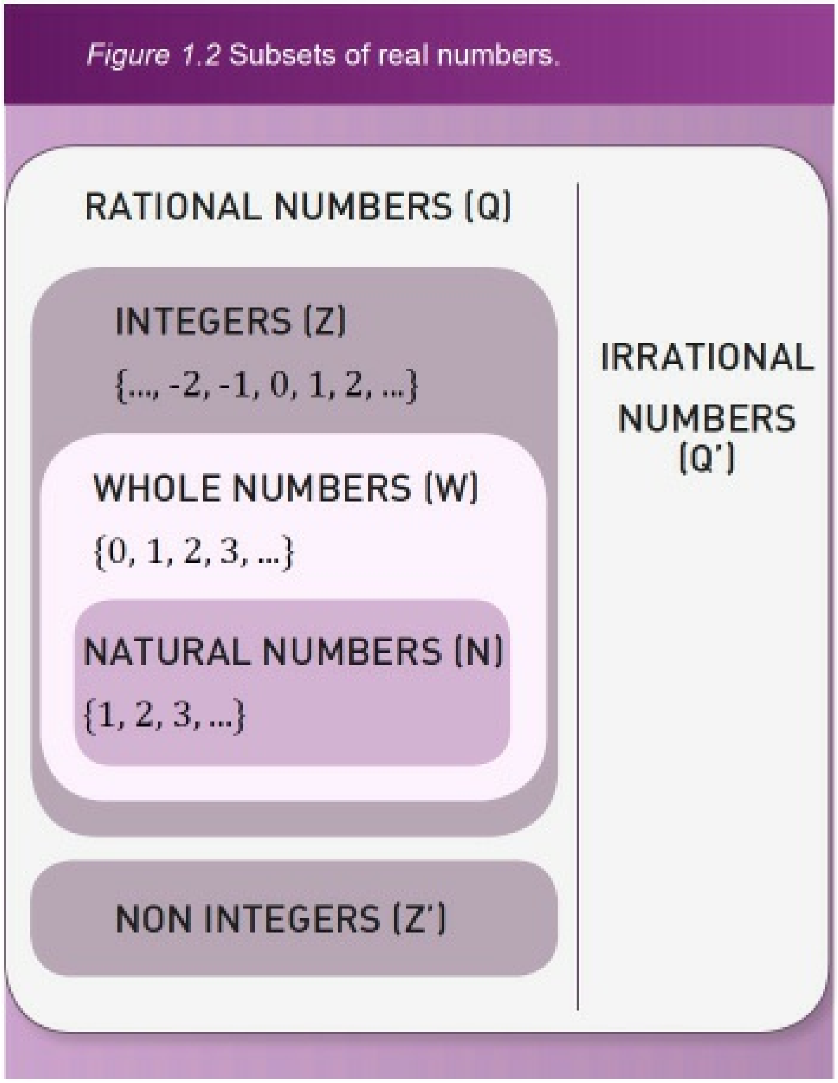


Figure 1.2 Subsets of real numbers.

Examples

Let

A= $\{-2, 5, \frac{3}{4}, -\frac{7}{5}, 0, \sqrt{7}, -\sqrt{49}, -2.56734\pi\}$.

Match the type of set of numbers with their corresponding list of elements from set A.

RESOURCES

- Solution

[Example A - 1.2.1 Real numbers and field properties]

The real number system is considered a field. A field is a set with two operations (addition and multiplication) that satisfies the field axioms shown in Table 1.3 (where ?, ? and ? are real numbers).

Table 1.3 *Field axioms*

Field axiom	Description	Of addition	Of multiplication
Closure properties	The addition, subtraction, multiplication or division of any two real numbers (a and b) will give as a result also a real number	$a+b =$ real number	$a \cdot b =$ real number
Commutative properties	The addition or multiplication of two real numbers (a and b) can be done in either order	$a+b = b+a$	$a \cdot b = b \cdot a$
Associative properties	Three real numbers (a, b and c) can be regrouped when performing an addition or a multiplication	$(a+b)+c = a+(b+c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity properties	The addition of zero to a number (a) or the multiplication of a number (a) by 1 will result in the same number (a).	$a+0 = 0+a = a$	$a \cdot 1 = 1 \cdot a = a$
Inverse properties	If the sum of two numbers is 0, then the numbers are called additive inverses (negatives), while if the product of two numbers is 1, then the numbers are called multiplicative inverses (reciprocals)	$a+(-a) = -a+a = 0$	$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1$
Distributive property of multiplication over addition	Tells how to multiply a number (a) by the sum of two numbers (b and c)	$a \cdot (b+c) = (a \cdot b) + (a \cdot c)$	

The properties of real numbers are useful when performing operations of addition, subtraction, multiplication and division of real numbers and to write expressions in equivalent forms, allowing working with simpler expressions.

Examples

For each operation identify the property that takes place.

- $(3 \cdot 4) \cdot 5 = 3 \cdot (4 \cdot 5)$
- $2+2\,(-2) = -2+2=0$
- $7+0 = 0+7 = 7$
- $5 \cdot (6+7) = (5 \cdot 6)+(5 \cdot 7)$
- $3 \cdot 4 = 7$
- $8+3 = 3+8$
- $9 \cdot 1 = 1 \cdot 9 = 9$

RESOURCES

- Determine whether each number is rational or irrational.

Exercise 1.2 Number Systems

1.2.2 Rational and Irrational Numbers

A rational number can be any ordinary number of arithmetic, that is, any whole number, fraction, mixed number or decimal, along with its negative image. One of the main characteristics of a rational number is that it can be expressed as a ratio of integers.

Then a number "n" is rational if there are two integers p and q, where $q \neq 0$, such that $n = \frac{p}{q}$.

Communication is a very important part of algebra, so it is important to distinguish and classify numbers by sets, such as rational, irrational and complex number sets, terms that should became part of your vocabulary. Some people might think of algebra as a collection of rules, but algebra is more than that since it is a language that can be used to answer many different problems and questions about real-life situations.

By the other hand, there are the irrational numbers, which cannot be expressed as a ratio of integers and they are non-periodic infinite decimals. One can determine their decimal approximations by using a calculator.

The combination of both rational and irrational numbers will give as a result the set of real numbers.

Solution

-2,3,125, 5/2, 19/13, 34/5, 0.5 and 1.33333…

All of the previous examples are rational numbers. The first three numbers (-2,3 and 125) are integers, and since every integer can be written as a fraction with denominator of 1, then they all are rational numbers. The next three numbers (5/2, 19/13, 34/5) are already expressed as a ratio of integers so they are all rational numbers.

And since 0.5 can be written as the fraction 1/2, and 1.33333… can be written as the fraction 4/3, they both are rational numbers.

A decimal that has a finite number of decimals after the decimal point, such as 0.5, is considered rational.

Numbers such as 1.33333…, which has numbers that repeat indefinitely, are called periodic decimals and they all belong to the rational numbers.

It is important to remember that the division by zero (0) is undefined, and that's why expressions such as 2/0 and 10/0, do not represent any number.

To check out if a number is rational, it is also important to analyze the simplified number. So, when having for example a square root, it is important to simplify it if possible to prove that it belongs to a rational number.

Examples

Determine whether each number is rational or irrational.

1- $\sqrt{25}$

2- $\sqrt{81/16}$

3- $-2\sqrt{4}$

4- $\sqrt{(-6)^2}$

Solution-

1- $\sqrt{25} = 5$

2- $\sqrt{81/16} = 9/4$

3- $-2\sqrt{4} = -2(2) = -4$

4- $\sqrt{(-6)^2} = \sqrt{36} = 6$

Determine whether each number is rational or irrational.

1) $\sqrt{3}$

2) π

3) $\sqrt{2.5}$

4) $\sqrt{5/25}$

Solution

1) $\sqrt{3} = 1.72305$

2) $\pi = 141592654$

3) $\sqrt{2.5} = 1.58113$

4) $\sqrt{5/25} = \sqrt{5}/5$

The previous examples cannot be written as fractions with integers, therefore they are called irrational numbers.

RESOURCES

- Determine whether each number is rational or irrational.

Exercise 1.2 Number systems

1.3 Types of Sets and Subsets

There are some basic concepts that are important to know when working with sets and there are specific symbols that are necessary to understand those concepts. The knowledge about the use of sets and its symbols will allow understanding later topics such as Venn diagrams and Mathematical logic, which at the same time will clarify the understanding of Probability.

Table 1.4 shows some of the different symbols used in set theory with their corresponding name and description along with an example to understand each symbol better.

Table 1.4 *Symbols used in set theory*

Symbol	Symbol Name	Description	Example
$\{\,\}$	Set	a collection of elements	$A=\{$blue, yellow, red$\}$
$x \in A$	Element of	set membership where x is a real number	$A=\{1,2,3\}$, $3 \in A$
$x \notin A$	Not element of	no set membership where x is a real number	$A=\{1,2,3\}$, $4 \notin A$
$A \subset B$	Proper subset	subset A has less elements than the set B	If $B=\{1,2,3,4\}$ and $A=\{1,2\}$, then $\{1,2\} \subset \{1,2,3,4\}$
$A \subseteq B$	Improper subset	subset A has less elements or equal to the set B	If $B=\{1,2,3,4\}$ and $A=\{1,2,3,4\}$, then $\{1,2,3,4\} \subseteq \{1,2,3,4\}$
$A \not\subset B$	Not subset	set A not a subset of set B	If $B=\{1,2\}$ and $A=\{3\}$ then $\{3\} \not\subset \{1,2\}$
U	Universal set	set of all possible values	
$\emptyset$	Empty set	$\emptyset = \{\,\}$	$A=\{\,\}=\emptyset$
$\lvert A\rvert=\#A=n(A)$	Cardinality	the number of elements of set A	$A=\{4,5,6,7,8\}$, then $\lvert A\rvert=\#A=n(A)=5$
$\aleph$	Aleph	infinite cardinality	$A=\{x \in R \mid x>2\}$ $\lvert A\rvert=\#A=n(A)=R$
$A = B$	Equality	both sets have the same members	$A=\{1,2,3\}$, $B=\{1,2,3\}$, then $A=B$
$A \sim B$	equivalence	both sets have the same number of elements, or thesame cardinality	$A=\{1,2,3\}$, $B=\{4,5,6\}$, $n(A)=n(B)=3$, then $A \sim B$
$A \cup B$	Union	objects that belong to set A or set B	If $A=\{1,2,3,4\}$ and $B=\{5,6\}$ then $A \cup B=\{1,2,3,4,5,6\}$
$A \cap B$	Intersection	objects that belong to set A and set B	If $A=\{1,2,3,4\}$ and $B=\{3,4,5\}$ then $A \cap B=\{3,4\}$
$A'=A^C$	Complement	all the objects that do not belong to set A	If $U=\{x \mid x$ is a digit$\}$ and $A=\{2,3,4,5,6\}$ then $A'=A^C=\{0,1,7,8,9\}$
$A - B$	Difference of sets	objects that belong to A and not to B	$A=\{1,2,3\}$, $B=\{3,4,5\}$, then $A-B=\{1,2\}$

Note: uppercase letters, in this case A and B, are used to denote sets.

Set membership of elements requires the use of specifics symbols, such as $\in$ and $\notin$.

The relation that an element x belongs to a set A is denoted by $x \in A$ (Can be read as: "x is an element of A").

If the element x is not a member of set A, then it is denoted by $x \notin A$ (Can be read as: "x is not an element of A").

Examples

Complete the following relations as $\in$ or $\notin$.

1.	2		A	if $A=\{2,4,6,8,\}$
2.	6		B	if $B=\{x\|x$ is an odd number$\}$
3.	$\sqrt{5}$		Q	
4.	0		N	
5.	0		W	
6.	-3		Z	
7.	$5/6$		Q	
8.	4.567		R	
9.	Q		R	
10.	A		B	if $A=\{2,4,6,8,\}$ and $B=\{x\|x$ is an odd number$\}$

RESOURCES

- Solution

Example A - 1.3 Types of Sets and Subsets

When a set is defined, pieces can be taken of that set to form what is called a subset.

B is a subset of A if and only if every element of B is in A, even though not all elements of A belong to B. And can be written as $B \subset A$ (read as "B is a subset of A") and $A \not\subset B$ (read as "A is not a subset of B").

Examples

Let $A = \{1, 2, 3, 4\}$. Determine if the following statements are true or false.

1. If $B = \{1, 2, 4\}$then $B \subset A$ _____
2. If $C = \{3, 4\}$ then $C \not\subset A$ _____
3. If $D = \{4\}$ then $D \subset A$ _____
4. If $E = \{2, 6,\}$ then $E \not\subset A$ _____
5. If $F = \{1, 2, 3, 4\}$ then $A \subset F$ _____

RESOURCES

- Solution

Example B - 1.3 Types of Sets and Subsets

1.3.1 Universe and Empty set

The universe, or the universal set, is the set of all elements involved in a specific situation, and it is usually denoted by the letter U.

There are some universal sets, which are infinite, such as the set of real numbers. But, there are other which are finite, such as for example the set of digits $D = \{0,1,2,3,4,5,6,7,8,9\}$.

A set with no elements, is called an empty set or null set. It is denoted either by $\{\}$ or $\emptyset$. A common error is to express an empty set as $\{\emptyset\}$, which is not properly denoted. An empty set has cardinality equal to zero, denoted as $n(\emptyset) = 0$.

1.3.2 Cardinality of a Set

The cardinality refers to the number of elements that belong to the set and there are different ways to express it. If A is the letter to denote the set, then the cardinality will be expressed either as $|A|$, $\#A$, or $n(A)$.

Examples

Determine the cardinality for each of the given sets

1. If $A=\{x|x \text{ is a season of the year}\}$
2. If $B=\{2,3,4,5,6,7\}$
3. If $C=\{x \in W|x-2 \leq 2\}$

Solution

1. If $A=\{x|x \text{ is a season of the year}\}$
   The set can be expressed as roster form as $A=\{\text{Fall, Winter, Spring, Summer}\}$
   Then the number of elements that belong to set A is equal to four.
   The cardinality of set A is expressed as $n(A)=4$.
2. If $B=\{2,3,4,5,6,7\}$
   There are six elements that belong to set B.
   The cardinality of set B is expressed as $n(B)=6$.
3. If $C=\{x \in W|x \leq 4\}$
   x belongs to the whole numbers $(W=\{0,1,2,3,4,...\})$, then it is necessary to solve for x in the inequality so that $x \leq 2+2$ will give as a result $x \leq 4$, which means less than or equal to four, then expressing set C in roster form $C=\{0,1,2,3,4\}$.
   The cardinality of set C is expressed as $n(C)=5$.

There are cases where the cardinality is infinite, so it's denoted with the symbol ℵ, which is the first letter in the Hebrew alphabet, and is pronounced "aleph".

REVIEW ACTIVITIES

- Determine the cardinality of the following sets.

Exercise 1.3.2 Cardinality of a set

1.3.3 Disjoint, equal and equivalent sets

In Figure 1.4, the differences between disjoint, equal and equivalent sets are illustrated.

Two sets are **disjoint** if they do not have a common element. Their intersection is the empty set.

Two sets are **equivalent** $(A=B)$ if both sets have the same number of elements, that is if they have the same cardinality $n(A)=n(B)$ or $|A|=|B|$.

Two sets are **equal** $(A=B)$ if and only if they have the same elements. That is if A is a subset of B and B is a subset of A.

Example

Determine if the given sets are disjoint, equal or equivalent.

1. $A=\{x \in Z | 3<x<11\}$ and $B=\{x | x$ is a day of the week$\}$
2. $A=\{1,2,3,4,5\}$ and $B=\{6,7,8,9,10\}$
3. $A=\{0,1,2,3,4,5,6,7,8,9\}$ and $B=\{x | x$ is a digit$\}$

Figure 1.4 Disjoint, equal and equivalent sets. Own work.

Disjoint	Equal	Equivalent
» No elements in common	» Exactly same elements	» Same number of elements
» $A \cap B = \emptyset$	» $A = B$	» $n(A) = n(B)$
Examples:	Examples:	Examples:
» $A = \{a,e,i,o,u\}$	» $A = \{a,e,i,o,u\}$	» $A = \{a,e,i,o,u\}$
» $B = \{1,2,3,4\}$	» $B = \{x \mid x$ is a vowel$\}$	» $B = \{1,2,3,4,5\}$

REVIEW ACTIVITIES

- Solution

Example 1.3.3 Disjoint, equal and equivalent sets

1.3.4 Finite and Infinite Sets

A set is **finite** when the number of elements in the set can be counted in a given situation. That means that its cardinality can have a specific value.

A set is **infinite** when the elements in the set cannot be counted. That means that its cardinality is infinite.

Examples

Determine if the given sets are finite or infinite.

1. $A = \{x \in N \mid x < 23\}$

2. $B = \{x \in Q \mid x > 0\}$

1.3.5 Singleton set

> A singleton set is when there is a single element that belongs to the set. Therefore its cardinality is always equal to one.

Example

If $A=\{x \in W | x<1\}$ determine if set A is a single set or not.

1.3.6 Proper Subset and Improper Subset

A set A is a proper subset $(\subset)$ of a set B if and only if everything in A is also in B and they are not equal. This case can be represented as $A \subset B$ and $A \neq B$.

A set A is an improper subset $(\subseteq)$ of a set B if both sets have exactly the same elements. This case can be denoted as $A \subseteq B$.

> An **empty set** is a proper subset of any other set.

Examples

Determine if A is a proper or an improper subset of B.

1. $A=\{1,3,5\}$ and $B=\{x|x$ is a digit$\}$
2. $A=\{x \in \mathrm{N} | x<3\}$ and $B=\{1,2\}$
3. $A=\emptyset$ and $B=\{ x \in Q | x>0\}$

REVIEW ACTIVITIES

- Solution

Example 1.3.4 Finite and infinite sets

Example 1.3.5 Singleton set

Example 1.3.6 Proper subset and improper subset

1.4 Venn Diagrams and Set Operations

Just as there are operations in arithmetic, there are also operations between sets. In the following sections, four operations with sets are explained: union, intersection, complement and difference; each of them along with their Venn diagram representation.

Venn diagrams originated from a branch of mathematics called set theory and were developed by John Venn around 1880 to show relationships between sets.

> A Venn diagram is composed by two or more overlapping circles and it is often used in mathematics to illustrate the similarities, differences, and relationships between sets. The use of this kind of diagrams allows students to visualize better a problem as in logical reasoning and be able to establish relationships between different sets.

1.4.1 Union and intersection

The union $(\cup)$ of two sets, A and B, is denoted by $A \cup B$, and it refers to the set of all elements that belong to set A or to set B, or in other words, all elements of A and all elements of B. Both sets can be “added” together.

The union of two sets can be represented as:
$A \cup B = \{x | x \in A \text{ or } x \in B\}$

The orange shaded area of the Venn diagram shown in Figure 1.5 illustrates the union between set A and set B.

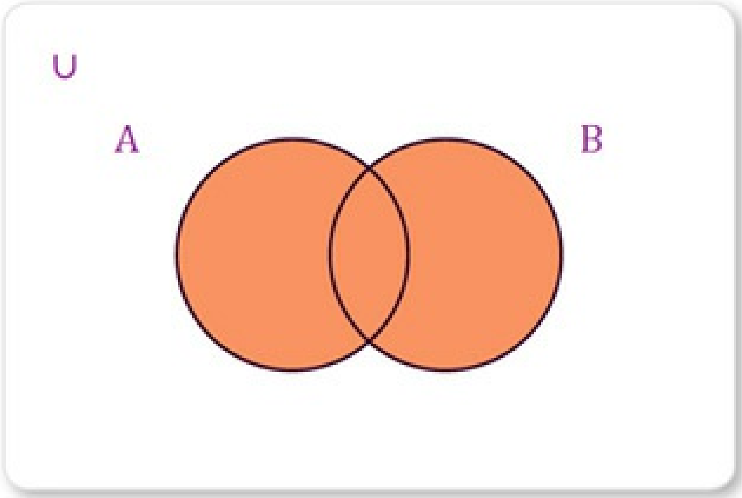


Figure 1.5 Union between sets A and B.

RECOMMENDED LINKS

- In this site you can find an interactive exercise on shading areas of the Venn diagram according to the given set operations. You can do the exercise by hand and then check the correct answers by moving your mouse pointer over the blank Venn diagram given in the web site.
  Venn diagram self test
- In this site you can find an interactive exercise on Venn diagram counting, where you can test your knowledge. It gives you immediate feedback of your results and can help you to practice.
  Venn diagram counting

Examples

Perform the union between the given sets and express the solution set.

1. $A=\{1,2,3,4\}$ and $B=\{2,4,6,8\}$
2. $A=\{1,3\}$ and $B=\{1,2,3\}$
3. Set of all rational numbers (Q) and the set of all irrational numbers (H).

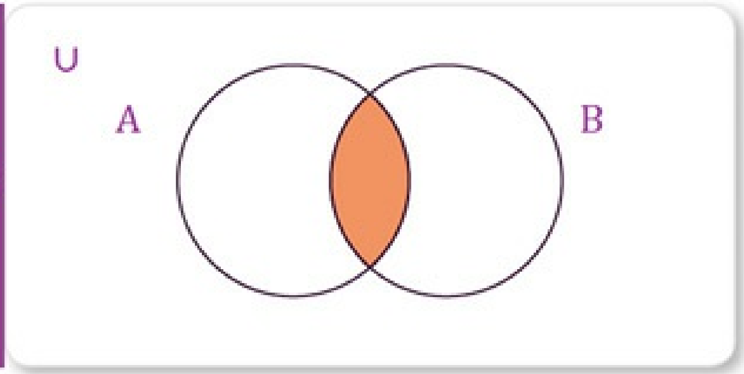
The intersection $(\cap)$ of two sets, A and B, is denoted by $A \cap B$, and it refers to the set of all elements that belong to both sets, A and B, or in other words, all elements that both sets have in common.

The intersection of two sets can be represented as:

$$A \cap B = \{x | x \in A \text{ } or \text{ } x \in B\}$$

The orange shaded area of the Venn diagram shown in Figure 1.6 illustrates the intersection between set A and set B.

Figure 1.6 Intersection between sets A and B.



Examples

Perform the intersection between the given sets and express the solution set.

1. $A = \{1,2,3,4\}$ and $B = \{2,4,6,8\}$
2. $A = \{1,2,3,4\}$ and $B = \{5,6,7,8\}$

1.4.2 Complement

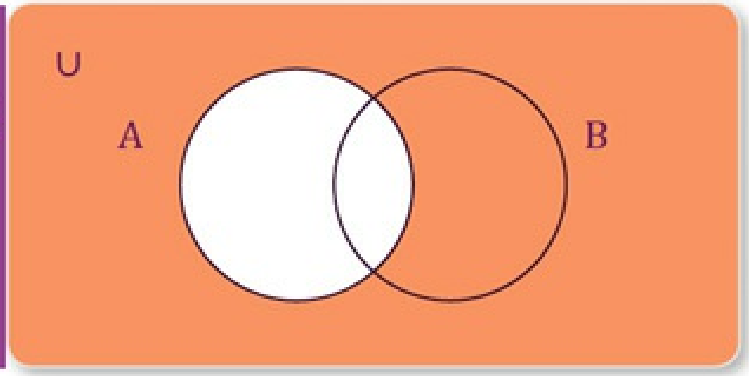
The complement of a set A, can be denoted as A' or A^C, and it refers to the set of all elements that belong to the universe (U) but do not belong to set A. Then, it is important to know the elements from the universe so that the complement can be identified.

It can be represented in set builder form as:

$$A^c = \{x \in U \mid x \notin A\}$$

The orange shaded area of the Venn diagram shown in Figure 1.7 illustrates the complement of set A.

Figure 1.7 Complement of set A.



REVIEW ACTIVITIES

- Solution

[Example A - 1.4.1 Union and intersection]

[Example B - 1.4.1 Union and intersection]

Examples

1. If $U=\{1,2,3,4\}$ and $A=\{1\}$, determine A'.

2. If $U=\{x|x=R\}$ that is that the universe is {real numbers} and $A=\{x|x=Q\}$ which means that set A is {rational numbers}, then determine A'.

3. If $U=\{1,2,3,4,5,6\}$, $A=\{2,3\}$, and $B=\{5,6\}$, determine the solution set of $A' \cap B'$.

4. If $U=\{1,2,3,4,5,6\}$, $A=\{2,3\}$, and $B=\{5,6\}$, determine the solution set of $(A \cup B)'$.

Solution

1. $A'=\{2,3,4\}$, since $2,3$ and 4 belong to the universal set (U) but do not belong to set A.
2. $A'=\{x|x=H\}$,which means that the complement of set A would be {irrational numbers}.
3. In this case the recommendation is to start solving by steps.

If $A=\{2,3\}$, then $A'=\{1,4,5,6\}$

If B={5,6}, then B'={1,2,3,4},

since the required operation is of intersection then we have to check which elements A' and B'

have in common, and that is numbers 1 and 4.

Then the solution set is: $A' \cap B' = \{1,4\}$

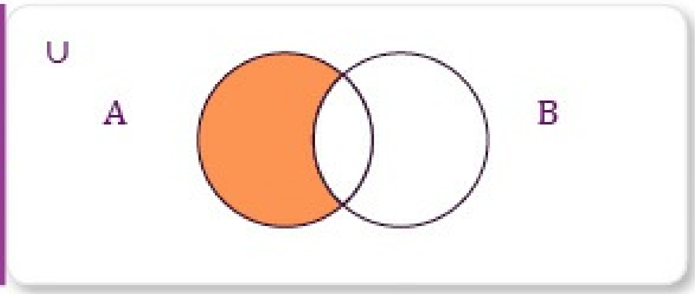
1. First perform the operation of $(A \cup B)$, that leads to {2,3,5,6}, then the complement of that union will be the solution set $(A \cup B)' = \{1,4\}$

1.4.3 Difference

If a set B is subtracted from set A, the resulting difference set consists in getting rid of elements of set B, which are in set A, in other words, it is the set of all elements of "A", which do not belong to "B". The difference between two sets can be represented as: "A-B"

The orange shaded area of the Venn diagram shown in Figure 1.8 illustrates the difference between set A and set B or set A minus set B.

Figure 1.8 Difference between sets *A* and *B* or *A*-*B*.



Examples

Perform the difference between the given sets and express the solution set.

1. Determine A-B if A ={1,2,3,4} and B={2,4,6,8}
2. Determine B -A if A ={1,3} and B={1,2,3}
3. Determine A-B if A ={5,6,7} and B={4,5,6,7,8}

RESOURCES

- Solution

Example 1.4.3 Difference

1.4.4 Combination of set operations

The operations performed previously (union, intersection, complement and difference) can be combined in a single operation when working with sets.

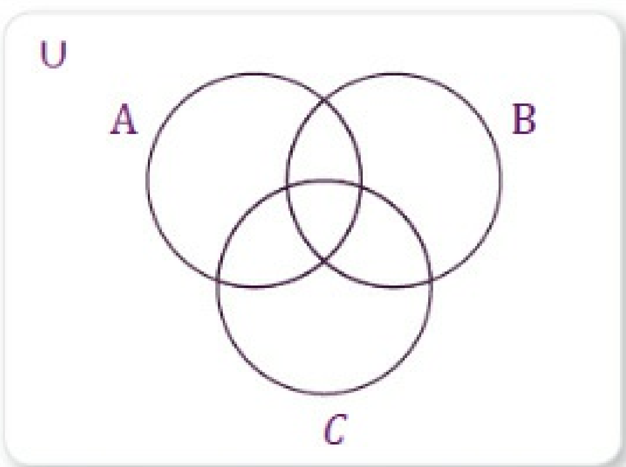
> One way to solve a combination of set operations is by enumeration or construction, and the other is by using Venn diagrams, which help to visualize better the problems.

When using Venn diagrams, sets are represented by circles that are included in a rectangle that represents the universal set. The solution set of the operation is represented by the shaded area in the diagram.

One way to solve an operation that involves a Venn diagram is to enumerate each section of the diagram, follow the operation by enumeration and then the numbers that belong to the solution set will be the ones that should be shaded in the diagram.

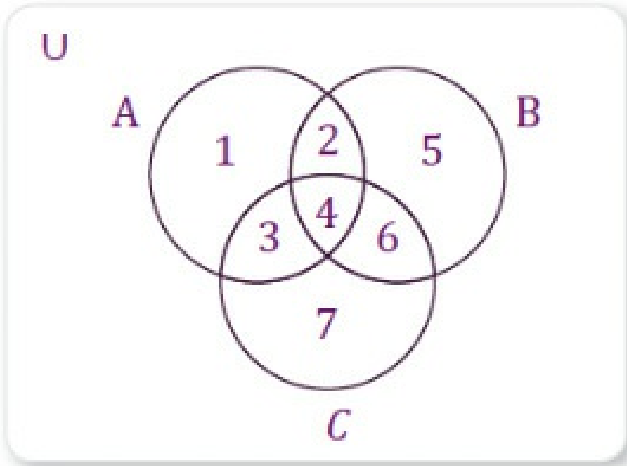
Example

Shade the area in the Venn diagram shown that corresponds to the set expression $(B \cap C) \cup A^c$



Solution

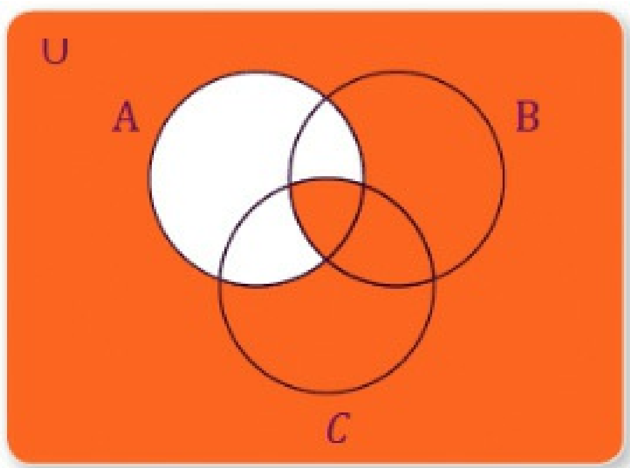
If it comes difficult to solve the operation mentally, then a recommendation is to enumerate each section of the diagram (the order or location of numbers is indifferent since at the end the shaded are should be the same).



Then the operation can be done by enumeration or construction step-by-step.

$$(B \cap C) = \{4,6\}$$
$$A^c = \{5,6,7,8\}$$

Set $(B \cap C) \cup A^c = \{4,5,6,7,8\}$, those are the numbers of the regions that should be shaded in the diagram.



The other way to do it is step by step following the order of the operation and start shading the areas step-by-step.

REVIEW ACTIVITIES

- Match the given operations with their corresponding solution set.
  Exercise A - 1.4.4 Combination of set operations

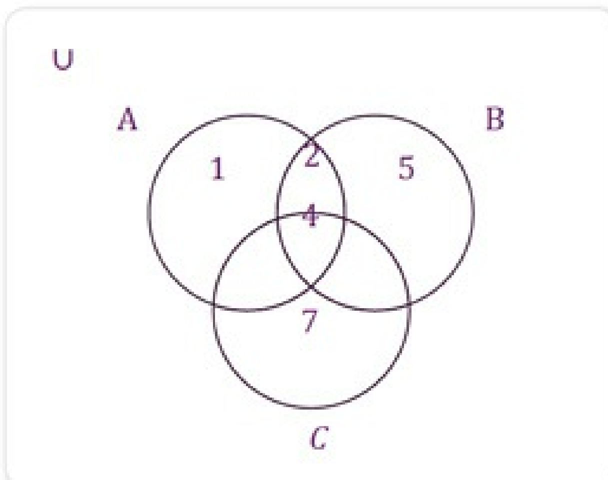
- Match the correct set expression with its corresponding Venn diagram.
  Exercise B - 1.4.4 Combination of set operations

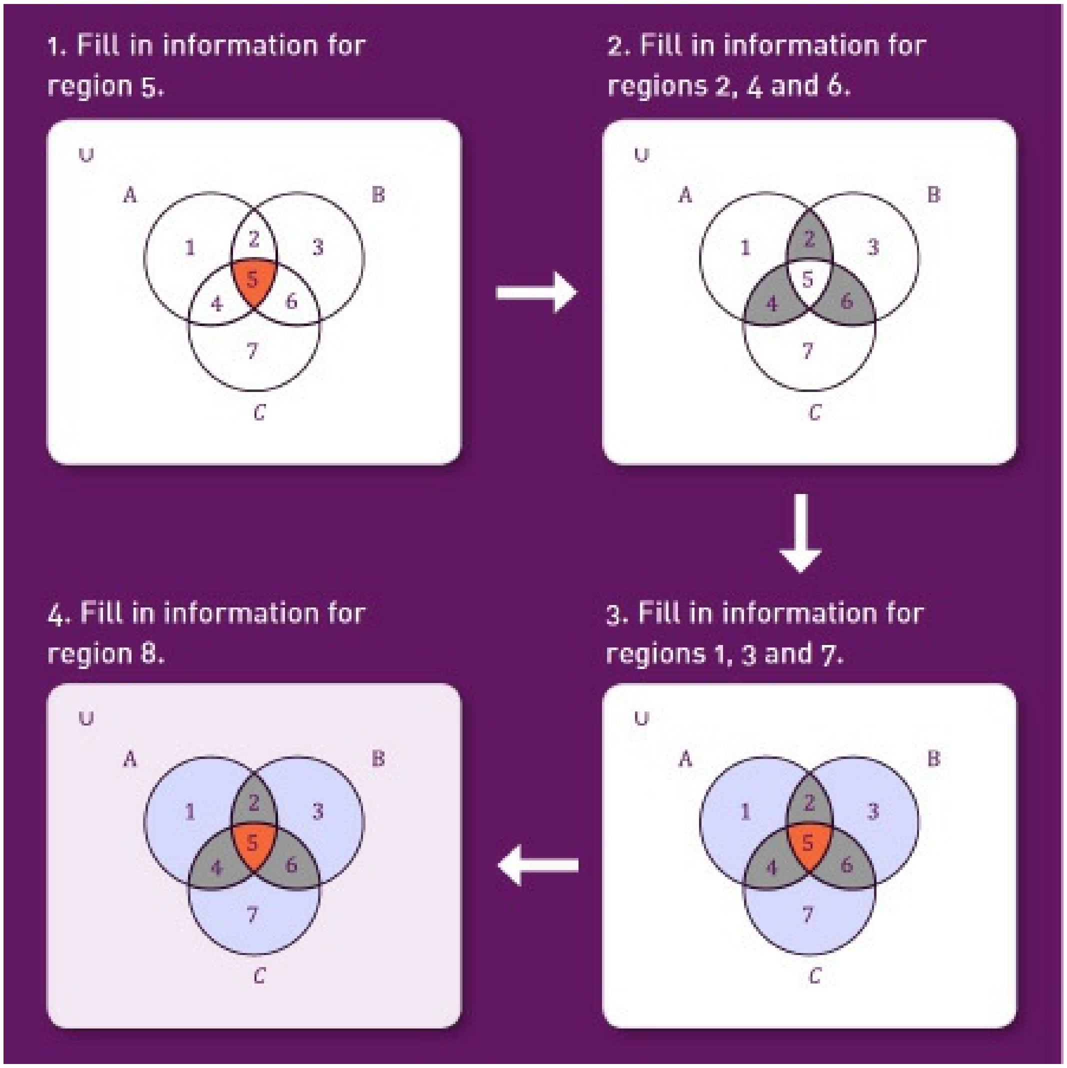
1.4.5 Word Problems Solution

The set operations and Venn diagrams can be used to solve counting word problems. Venn diagram word problems generally give two or three classifications and specific information of numbers. The remaining information can be figured out by using the given information to populate the diagram.

When having a 3-set Venn diagram as the one shown in Figure 1.9, the information must be filled inside-out through each of the regions:

Figure 1.9 A 3-set Venn diagram





Example

1. There were 125 customers attended in a travel agency during the last month of the year. There were three promotions offered by the agency to three different destinations: Toronto, New York and Miami. There were18 customers who got the three promotions, 23 got Toronto and Miami, 26 got New York and Miami, 34 got Toronto and New York, while 68 got Toronto, 53 got New York and 47 got Miami.

   a. How many customers didn't have any of the three promotions?

   b. How many customers got only the promotion for Toronto?

   c. How many customers got New York and Miami but not Toronto?

   d. How many customers got Toronto or Miami but not New York?

   e. How many customers got a single destination?

2. Out of 35 students, 16 are taking Mathematics and 21 are taking Chemistry. If six students are in both classes,

a. How many students are in neither class?

b. How many students are taking only Mathematics?

c. How many students are taking only Chemistry?

Venn diagrams are then a type of graphic organizer, which can organize complex relationships visually. They allow abstract ideas to be more visible and are now used across many disciplines, such as in science, where they can be helpful for classification. They are also used to teach basic set theory, as well as to illustrate simple set relationships in statistics, probability, linguistics, logic, and computer science, so its use is very important to understand a wide variety of topics.

RESOURCES

- Solution
  [Example 1.4.5 Word problems solution]

1.5 Elements of Mathematical Logic

Mathematics is like learning a foreign language and it is very useful to describe and comprehend the world around us. Like any other language, to understand it, it is important to learn the vocabulary, put it into practice and know how to express ideas with that vocabulary. Mathematical logic will allow constructing the bases of the structure necessary to describe concepts in terms of mathematics and will help to understand the fundamentals of mathematics. Logic is an area of mathematics that has its roots in philosophy and that studies formal reasoning based on propositions or statements.

1.5.1 Propositions, not Propositions and Open Propositions

Statements can be classified according to the diagram shown in Figure 1.10.

A proposition is a declarative statement that can be either true or false, but it cannot be both. Usually, letter T or number 1 are used to denote a true proposition, while letter F or number 0 are used to denote a false proposition. T and F are called the truth values of a proposition.

When the sentence is a question, command, opinion or exclamation, it is a not proposition.

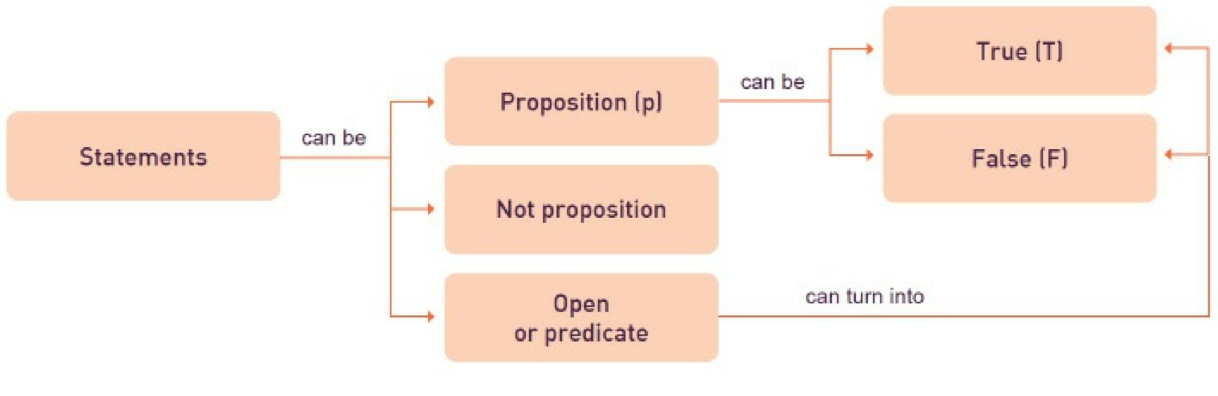


Figure 1.10 Classification of statements.

An open statement or predicate is a statement that involves a pronoun or a variable that does not have a specific truth value and therefore it is not a proposition. However, once the pronoun or variable is specified, the statement becomes a proposition.

Lowercase letters are used to represent a proposition. The most common variables used are p, q and r and an additional variable, such as $p(x)$. In the case of open statements, a domain must be specified for "x", so that it becomes a proposition. In the case of sets, the domain can be defined by the universal set. The domain is usually specified after the statement as follows:

$$p(x)\text{: "proposition"; } x \in \text{domain}$$

To find the solution set of open statements, the values of the solution set should be inside the domain and must satisfy the conditions of the statement.

1.5.1.1 Compound propositions

Two or more propositions (p, q or r) can be combined together to make compound propositions using logical connectives or logical operators.

The logical connectives used to form compound propositions are shown in Figure 1.11.

Conjunction	Disjunction
» Symbol ∧	» Symbol ∨
» Keyword "AND"	» Keyword "OR"
» Operation of intersection ∩	» Operation of Union ∪

Figure 1.11 Conjunction and disjunction.

A conjunction is a compound proposition, which combines two propositions with the connective "and" or "

", so if both propositions are true then the conjunction will be true, while any other combination will result on a false conjunction, just as shown in Table 1.5. A truth table is a table with a row for each possible truth value for a specific simple or compound proposition.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Table 1.5 *Truth conjunction table*

Note: A conjunction is true only when BOTH statements are TRUE. A conjunction is false when either statement is false.

Table 1.5 can be entered into an Excel Spreadsheet by using the Excel logical function or formula AND as shown in Figure 1.12.

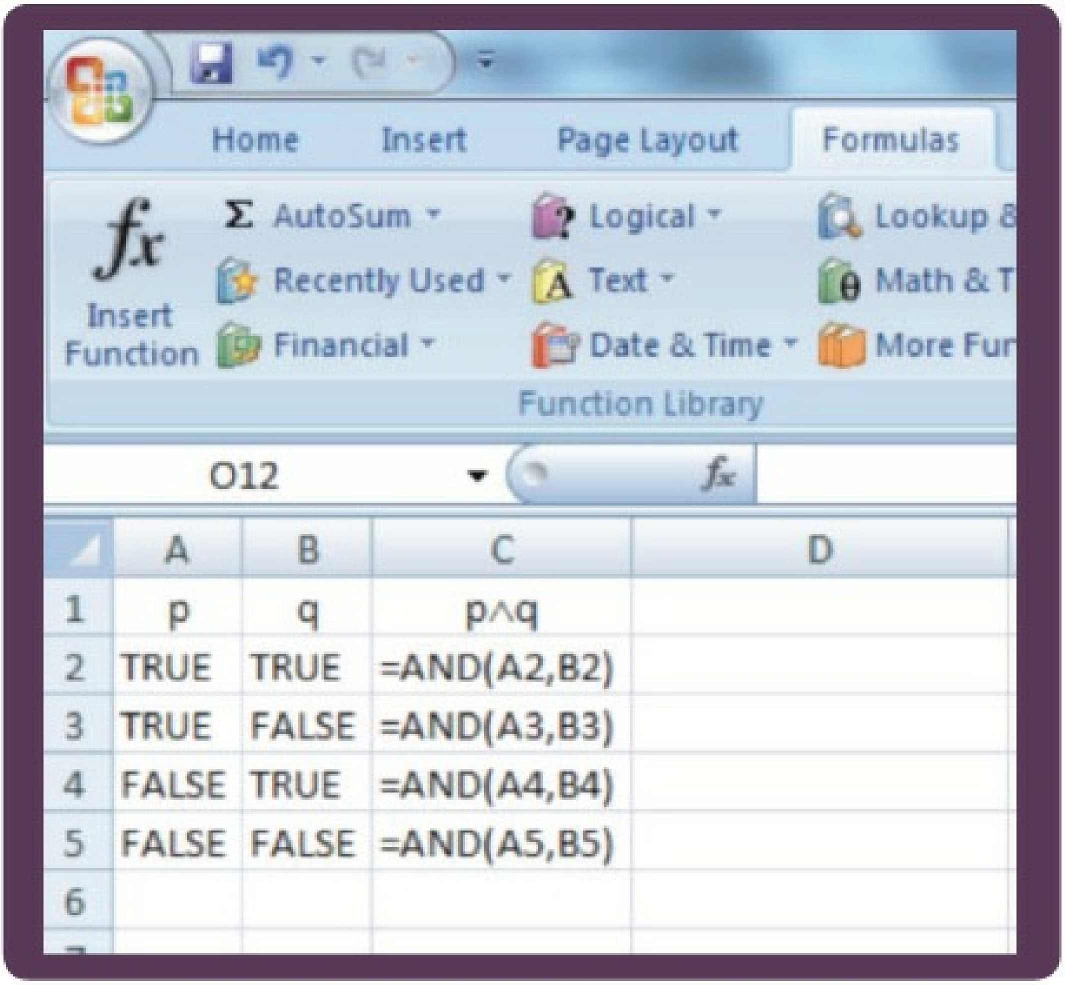


Figure 1.12 Excel function AND generates the outputs shown in figure 1.13

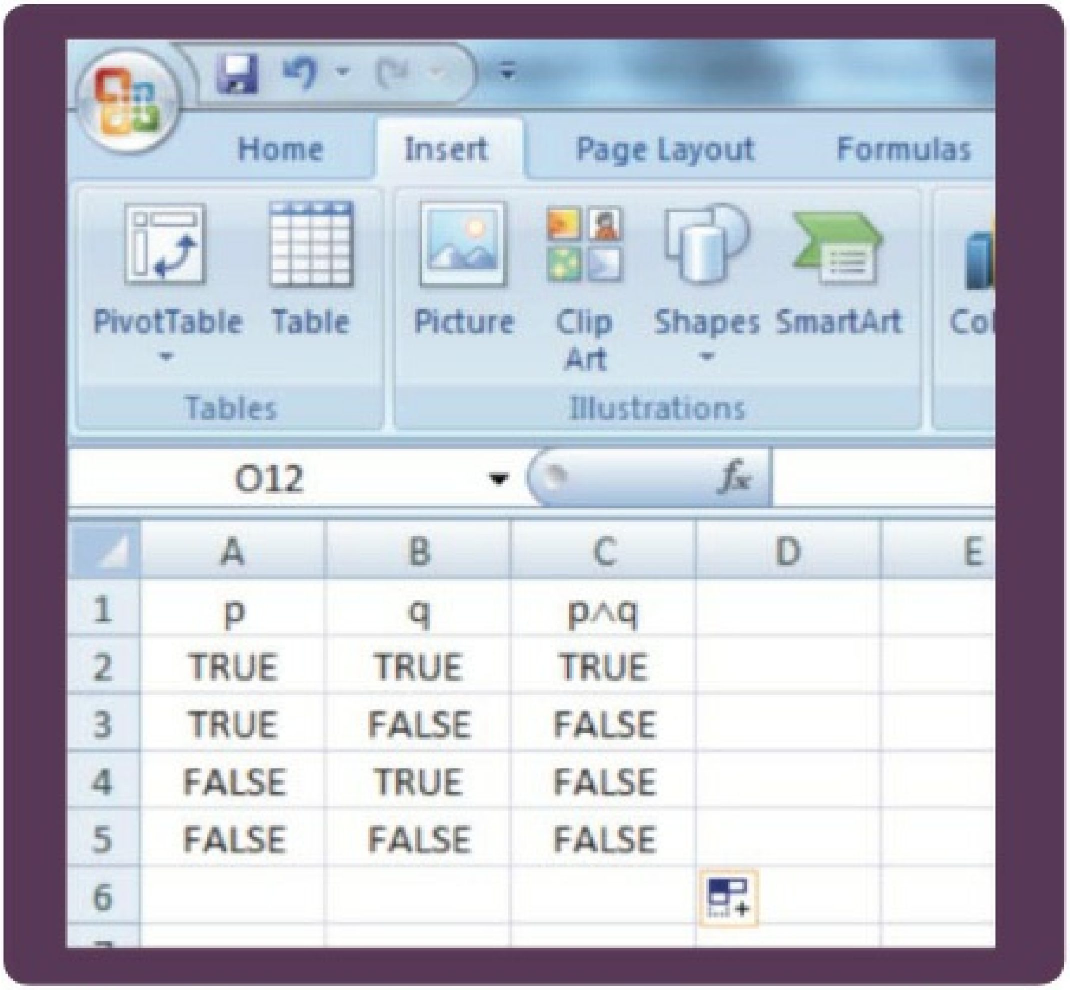


Figure 1.13 Conjunction formulas in Excel.

There are also open conjunctions, which are formed with the combination of open propositions, so to obtain the solution set of the conjunction, first the solution set for each proposition must be found. Once having the solution set for each open proposition, then the conjunction can be determined by performing an intersection $(\cap)$.

Examples

Determine the solution set for each of the given compound propositions.

1. $p(x) \wedge (x)$: "x is prime" ∧ "$x > 3$"; $x \in$ {Digits}
2. $q(x) \wedge (x)$: "x is odd" ∧ "$x > 4$"; $x \in \mathrm{N}$
3. $p(x) \wedge (x)$: "$x \leq 8$" ∧ "x is a multiple of 2"; $x \in \mathrm{W}$
4. $p(x) \wedge (x)$: "$3 \leq x \leq 15$" ∧ "x is even"; $x \in \mathrm{Z}$

A **disjunction** is a compound proposition, which combines two propositions with the connective “or” or “$\vee$”, so if both propositions are false then the conjunction will be false, while any other combination will result on a true conjunction, just as shown in Table 1.6.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Table 1.6 *Truth disjunction table*

Note: A disjunction is true when EITHER statement is TRUE. A disjunction is false only when both statements are false.

RESOURCES

- Solution

Example B - 1.5.1.1 Compound propositions

REVIEW ACTIVITIES

- For practice on this topic, go to Example A - 1.5.1.1 Compound propositions

Table 1.6 can be entered into an Excel Spreadsheet by using the Excel logical function or formula OR as shown in Figure 1.14.

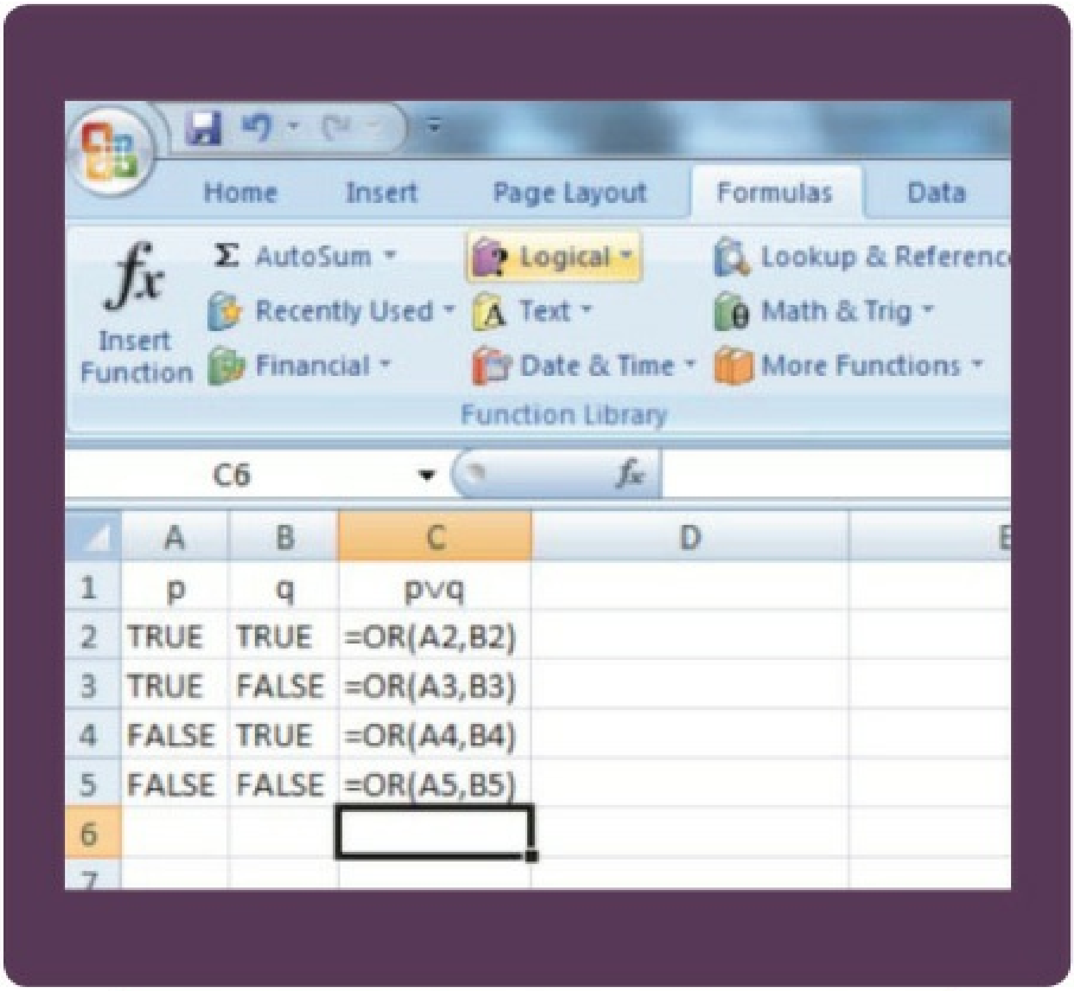


Figure 1.14 Disjunction formulas in Excel.

Excel function OR generates the outputs shown in Figure 1.15.

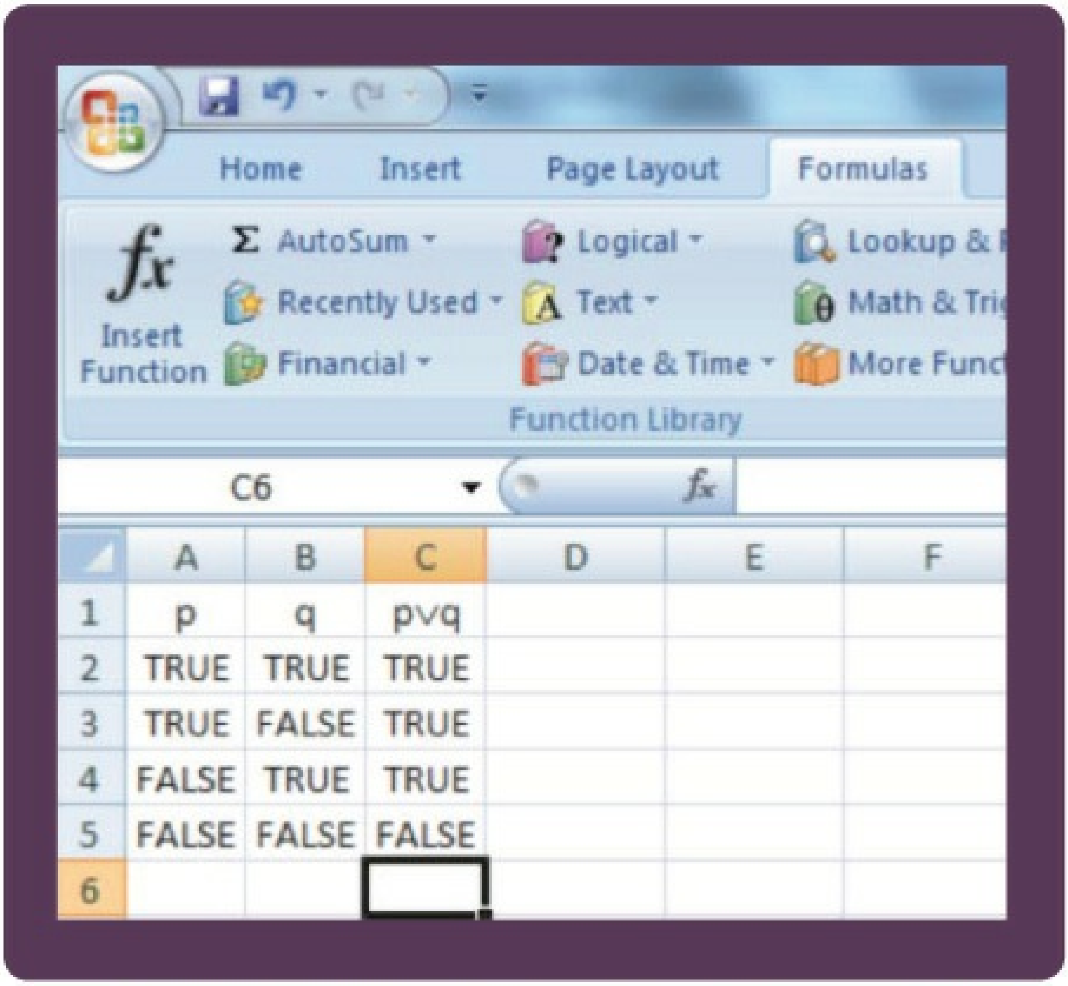


Figure 1.15 Disjunction results in Excel.

<u>Examples</u>

Determine the truth value for each proposition ? and ?, then perform a disjunction between the pair of propositions.

Disjunction	p	q	$p \vee q$
1. "Honolulu is the capital of Hawaii" or "Berlin is the capital of Germany".			
2. "The Moon is Earth's only natural satellite" or "The Sun is the star at the center of the Universe".			
3. "5+5=15" or "34 is greater than 25".			
4. "The sky is green" or "The grass is blue".			

There are also open disjunctions, which are formed with the combination of open propositions, so to obtain the solution set of the disjunction, first the solution set for each proposition must be found. Once having the solution set for each open proposition, then the disjunction can be determined by performing a union $(\cup)$.

Example

Determine the solution set for each of the given compound propositions.

1. $p(x) \vee q(x)$: "x is prime"∨" $x > 3$"; $x \in$ {Digits}
2. $q(x) \vee r(x)$: "x is odd"∨" $x > 4$"; $x \in \mathrm{N}$
3. $p(x) \vee q(x)$: "x is a multiple of 3 less than 19 "∨" $1 < x < 3$"; $x \in \mathrm{W}$
4. $p(x) \vee r(x)$: "x is factor of 6 "∨" $x < 4$"; $x \in \mathrm{Z}^+$

RESOURCES

- Solution

Example B - 1.5.1.1 Compound propositions

1.5.1.2 Negation of propositions

The negation of a quantified statement is obtained from the DeMorgan's law, which is a rule of inference that pertains to the NOT, AND, and OR operators and is used to distribute a negative to a conjunction or disjunction.

The **negation of a proposition** is an operation that can be formed by negating a proposition, in other words making the opposite claim of p.

If p is a proposition, then its negation is denoted by $(\sim p)$ read as "not p".Table 1.7 shows the truth values for the negation of p. $\sim p$

Proposition (p)	Negation of proposition ($\sim p$)	Explanation
T	F	If p is true (T), $\sim p$ is false (F)
F	T	If p is false (F), $\sim p$ is true (T)

Table 1.7 can be entered into an Excel Spreadsheet by using the Excel logical function or formula NOT as shown in Figure 1.16.

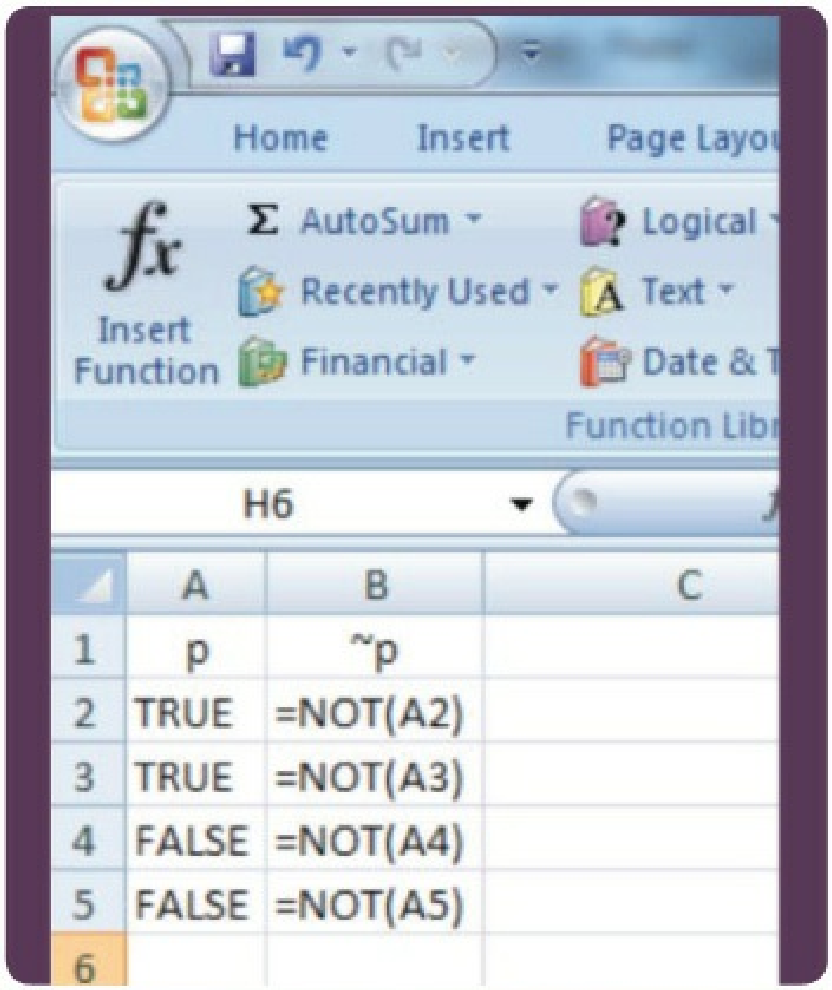


Figure 1.16 Negation formula in Excel.

Excel function does NOT generate the outputs shown in Figure 1.17.

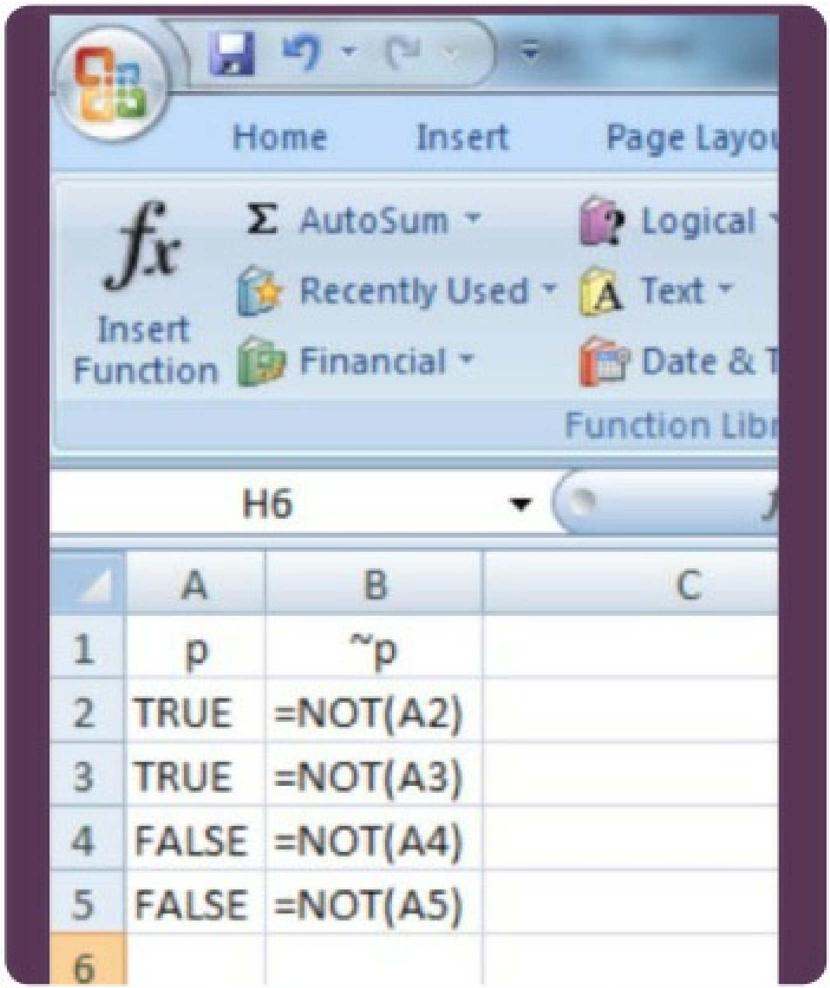


Figure 1.17 Negation results in Excel.

DeMorgan's law transforms also a proposition with a conjunction into a disjunction by negating each member of the expression. It also works in the same way transforming a disjunction into a conjunction by the negation of each member in the expression.

To express the negation of a simple or a compound proposition (conjunction or disjunction) the symbols shown in Table 1.8 should be negated.

Symbol	Negation of the symbol
=	≠
<	≥
>	≤
and	or

Note: The previous symbols can be negated the other way around as well, so that if the proposition has the connector "or" then its negation will be "and", or if the proposition has the symbol "≠" then its negation will be "=". If there are two propositions with a connector, then both propositions and the connector should be negated.

The solution of the negation of compound propositions can also be found. For doing so, the following steps should be followed:

1. Determine the negation of the compound proposition.
2. Determine the solution set for each negation statement.
3. Find the solution set of the negation by performing the operation.

If the solution set of $p(x)$ is P, then the solution set for $\sim p(x)$ will be P^c. That means that the solution for the negation will correspond to the complement of the solution of the proposition.

In the same way if the solution set of:

- $p \vee q$ is $P \cup Q$ then the solution set for $\sim(p \vee q)$ will be $(P \cup Q)^c$. That means that the solution for the negation of the compound proposition will correspond to the complement of the solution of the union between P and Q.

- $p \wedge q$ is $P \cap Q$ then the solution set for $\sim(p \wedge q)$ will be $(P \cap Q)^c$. That means that the solution for the negation of the compound proposition will correspond to the complement of the solution of the intersection between P and Q.

REVIEW ACTIVITIES

- For practice on this topic, go to [Exercise 1.5.1.2 Negation of propositions]

1.5.1.3 Truth tables

A truth table lists all possible combinations of truth values of two or more propositions. A truth table

can be constructed using the letters p, q and r, to represent the logical variables.

A single statement has two possible truth values: true (T) or false (F). Its truth table is shown in Table 1.9.

Table 1.9 *Truth table for a single statement*

p
T
F

Given two statements, there are four possible truth value combinations. So, there are four rows in the truth table. Its truth table is shown in Table 1.10.

Table 1.10 *Truth table for two statements*

p	q
T	T
T	F
F	T
F	F

Given three statements, there are eight possible truth value combinations. So, there are eight rows in the truth table. Its truth table is shown in Table 1.11.

Table 1.11 *Truth table for three statements*

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

In general, given n number of statements, there will be 2^n combinations (or rows) in the truth table. In Table 1.12 the relation between the number of statements and the number of rows is shown.

Table 1.12
Relation between number of statements and number of rows

n (number of statements)	2^n (number of rows)
1	2
2	4
3	8
4	16... etc.

Then, when having more than three statements, the same pattern can be followed:

1. Determine the number of statements (n).

2. Determine the number of rows (2^n) for the truth table.

3. For the first variable (in first column) the first half of the rows will be true (T) while the second half will be false (F).

4. For the second variable (in second column) the rows will be separated into 4 equal sections, alternating each section with true (T) and then the next with false (F).

5. For the third variable (in third column), the rows will be separated into 8 equal sections, alternating each section with true (T) and then the next with false (F), etc.

Examples

Construct a truth table for each of the following compound propositions.

Note: all these examples can also be solved in an Excel spreadsheet by using the corresponding Excel functions for the operators $\wedge$ (AND), $\vee$ (OR) and $\sim$ (NOT).

1. $(p \wedge q) \wedge r$

2. $[p \vee (p \vee q)] \wedge r$

3. $(p \vee q) \wedge (q \wedge r)$

4. $\sim p \vee q$

5. $\sim(p \wedge q) \wedge \sim r$

Knowing the basic truth table, other operations, such as conjunctions, disjunctions, and negations can be performed with truth values.

REVIEW ACTIVITIES

- For practice on this topic, go to Exercise 1.5.1.3 Truth tables

Solution

1. $(p \wedge q) \wedge r$

number of statements $(n) = 3$

number of rows $(2^n) = 8$

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

2. $[p \vee (p \vee q)] \wedge r$

number of statements $(n) = 3$

number of rows $(2^n) = 8$

p	q	r	$p \vee q$	$[p \vee (p \vee q)]$	$[p \vee (p \vee q)] \wedge r$
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	T	F
F	F	T	F	F	F
F	F	F	F	F	F

3. $(p \vee q) \wedge (q \wedge r)$

number of statements $(n) = 3$

number of rows $(2^n) = 8$

p	Q	r	$p \vee q$	$q \wedge r$	$(p \vee q) \wedge (p \wedge r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	F	F
T	F	F	T	F	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	F	F	F
F	F	F	F	F	F

4. $\sim p \vee q$ ...be careful since the negation is only for p

number of statements $(n) = 3$

number of rows $(2^n) = 4$

p	Q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T
F	T	T	T

5. $\sim(p \wedge q) \wedge \sim r$

number of statements $(n) = 3$

number of rows $(2^n) = 8$

p	q	R	$\sim r$	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \wedge \sim r$
T	T	T	F	T	F	F
T	T	F	T	T	F	F
T	F	T	F	F	T	F
T	F	F	T	F	T	T
F	T	T	F	F	T	F
F	T	F	T	F	T	T
F	F	T	F	F	T	F
F	F	F	T	F	T	T

1.5.2 Quantifiers

When working with open propositions, a domain should be specified to know which of the elements in the domain will make the proposition true. In order to express the truth value of the proposition with respect to a set of elements, then quantifiers are used. The quantifiers can tell for how many elements the proposition is true, either for all, for some, or for none.

There are two quantifiers used for logical variables:

- The universal quantifier, denoted as $\forall$, and is read as "for each", "for all", "for every", etc.

It can be represented as follows:

$\forall x \in A$, which is read as "for all elements x that belongs to set A"

- The existential quantifier, denoted as $\exists$,and is read as "for some", "for at least one", "there exists", etc.

It can be represented as follows:

$\exists x \in A$, which is read as "for some elements x that belongs to set A"

Examples

I. Determine the truth vale for the following propositions expressed with quantifiers

1. $\forall x \in Z^+; x > 0$

2. $\exists x \in N; x - 3 = 0$

3. $\forall x \in W; x^2 > 0$

4. $\exists x \in D; x - 2 \leq 5$

II. Determine the negation of $\forall x \in Z^+; x > 0$.

As in propositions, quantifiers can also be negated by using the DeMorgan's law. The negation of quantifiers can be done by applying the following negation rules:

Example

1. Negate the quantifier first, that is, replace $\exists$ by $\forall$ and vice versa, then

2. Negate the statement

Such that:

$\sim(\exists x, P(x)) = \forall x, \sim P(x)$ And $\sim(\forall x, P(x)) = \exists x, \sim P(x)$

> Set theory and mathematical logic language is not only about understanding statements in a textbook, but about developing the ability to reason and to get valid conclusions. These abilities are required in all areas of mathematics for problem solving, and to allow the understanding of many theoretical concepts. Being introduced to logic one can work with mathematical statements knowledgeably and can apply this knowledge in fields such as digital electronics, computer science, and other fields of engineering related to applied logic and math to reduce basic complex operations.

RESOURCES

- Solution

[Example 1.5.2 Quantifiers]

Chapter 1. Conclusion

Algebra is a language, and has its own vocabulary. Communicating ideas in mathematical language has its essence in the acquisition of the basic concepts that will allow transmitting abstract mathematical ideas and to understand important concepts that will allow solving problems related to real-life situations.

Classifying numbers within sets, using Venn diagrams, and learning to use the proper notation and symbols allows to organize mathematical information easier and to apply it in fields of knowledge such as the ones related to mathematical logic, probability, statistics among others.

Precision in algebra is required to explore, manipulate and communicate mathematical ideas unambiguously, and so it has been necessary to develop the codification of mathematical logic into symbols. Then, as expressing algebraic statements, complicated logic statements can be analyzed and rewritten with logic symbols following their corresponding rules. Symbolic logic is a powerful tool for analysis and communication in math, and it is important to manage it both verbally and mathematically to either express abstract ideas or to understand any mathematical expression. Just as following the road directions to get into a final destination, mathematical symbols and rules must be followed to get the final solution of a specific problem.

Acquiring the basics of sets and mathematical logic will open the door to the understanding of more deep areas that will allow submerging into the wide world of algebra.

Chapter 1. Review Activity

1.1 Set Representation

1.2 Number Systems

1.3.2 Cardinality of a set

A - 1.4.4 Combination of set operations

B - 1.4.4 Combination of set operations

1.5.1.1 Compound propositions

1.5.1.2 Negation of propositions

1.5.1.3 Truth tables

- Integrating exercise
- Practice more through the following activities. Additional Activities

Chapter 1. Resources

The following websites can help you to enrich your knowledge about the topics covered on the first chapter and practice your skills.

- McFarland. (2007). Venn diagram self test. *University of Wisconsin*. Department of Mathematical and Computer Sciences. Retrieved on August 6th 2011 from http://math.uww.edu/~mcfarlat/143venn.htm
  In this site you can find an interactive exercise on shading areas of the Venn diagram according to the given set operations. You can do the exercise by hand and then check the correct answers by moving your mouse pointer over the blank Venn diagram given in the web site.
- McFarland. (2007). PCs and Macs. *University of Wisconsin*.Department of Mathematical and Computer Sciences. Retrieved on August 6th 2011 from http://math.uww.edu/~mcfarlat/pc_mac.htm
  In this site you can find an interactive exercise on Venn diagram counting, where you can test your knowledge. It gives you immediate feedback of your results and can help you to practice.
- Miller, T. (2011, January 5). Sets, Venn Diagrams & Counting. *Arizona State University*. Retrieved on August 6th 2011 from http://www.asu.edu/courses/mat142ej/readings/Sets_and_Counting_bars.
  In this site there are useful notes related to set operations using Venn diagrams, along with some Venn diagram counting problems that can be helpful to understand better the topic.
- Radio, M. (2006). Sets and sets and numbers. *University of Maryland*. Retrieved on August 6th 2011 from http://www-users.math.umd.edu/~radio/math0305/sets.pdf
  In this site there are useful notes related to set notation, set symbols and sets of numbers that can enrich the set knowledge acquired on the first chapter.
- Ruoming, J. (2009). First order logic. *Kent State University*. Retrieved on August 6th 2011 from http://www.cs.kent.edu/~jin/Discrete10Spring/L01.pdf
  In this site there are useful notes related to propositional logic and truth tables, and they come from the Computer Science Department of Kent State University, so as you can see this topic is widely linked with computer systems.

- Seward, K. (2011, April 6).Virtual Math Lab. Intermediate Algebra. *West Texas A&M University*. Retrieved on August 6th 2011 from http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/inde
This site is very useful in topics related to intermediate algebra. The material is accessible to any person who wants a math online tutorial. Some of the topics that it covers from the first chapter are: sets of numbers and properties of real numbers.