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Relations of the financial and energy markets in BRIC economies

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Thank you God for all the blessings received, for allowing me to achieve this dream and for inspiring and leading me along this way. May the knowledge and experiences that I have acquired in this process and continue searching, put them at the service of others. I ask You to always give me the desire to learn from others and to let me share what I have learned. Lord Jesus, You are the source of light and inspiration; Rock and Refuge; Beginning and End; Truth, Beauty and Goodness.

I beg You to guide the beginning of my work, direct its progress, and bring it to successful completion. I ask You, Lord, through the intercession of Virgin Mary, Seat of wisdom.

Amen.

Dedication

I dedicate my dissertation work to my family and friends. A special feeling of gratitude to my parents, Mariano and Silvia Susana, whose prayers and words of encouragement have always accompanied me. Thank you for your love and care in every moment.

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By

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Abstract

BRIC acronym has been widely used since its creation by Jim O'Neill in 2001. Numerous studies and research have emerged from the a priori integration of these four nations: Brazil, Russia, India and China, not mentioning the derived denominations after their origin where some other countries are added to the originals. The initial research question and that marked the guideline of our entire study was to reflect on the existence of a related group that can be studied and treated as a block, or, if on the other hand, we were facing 4 different economies, grouped initially just by sharing certain characteristics at a specific time.

In this study, we resolved to answer this question in a specific context: dealing with market index and oil mix variables in periods that include volatility and crisis events.

To approximate an answer, we fitted the series of study variables to Normal Inverse Gaussian distribution. Additionally, we calculated risk measures and built investment portfolios through Markowitz theory assuming different combinations of our research financial instruments.

The evidence showed possibilities for portfolio optimization through diversified instruments from the BRIC countries. Therefore, in our particular field of study, we conclude that the term of the BRIC block should be limited as a group of related countries.

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Chapter 1. Introduction

The BRIC acronym created by the British economist O'Neill in 2001, to group four nations that, according to the author, shared common characteristics about their growth, population, land area and that would have a preponderant participation in the coming years. Since this term was coined, the countries of these four nations (and from 2011, after having included South Africa) have held 11 summits. As of today, there are thousands of articles and literature that take this group of economies (including South Africa or not) for granted and cover their research presupposing the unity of these countries for studies of all kinds.

In the first instance, it is important to know the context in which this term was created. Goldman Sachs Head of Global Economic Research Jim O'Neill, published an essay in 2001 entitled "Building Better Global Economic BRICs", playing a pun between the term originating from Brazil, Russia, India and China, and the word "brick". With this, the economist and the firm wanted the BRIC acronym to characterize the four largest emerging countries by their economic growth, expressed in terms of GDP, trade balance, population and territory. Although Goldman Sachs' attempt was not to stimulate the political alliance of these four countries, it did suggest incorporating them into the G7 group.

Associated by the common quality of rates of return with expectations above the average, O'Neill developed that first essay based on the position and macroeconomic perspective of the four countries and the potential development that those nations would have and could realize in the coming decades to obtain the first places in the dominant economies. In 2003, Dominic Wilson and Roopa Purushothaman, colleagues of O'Neill, doubled the expectation in their work entitled "Dreaming with BRICs: the path to 2050", in which they presented the results of the future projections of the BRICs, affirming that in 2025 they will represent more than half the size of the G7. In 2050, only the United States and Japan would be the current members of the group that would remain in it, given that the aggregate economy of the BRICS would be superior to the G7.

In 2009, after the global financial crisis, the first meeting of the BRIC member countries was held in Yekaterinburg, Russia, where the presidents of Brazil, Russia, India and China laid the foundations for a group with direct implications in the international economic policy, an unforeseen event by O'Neill when the BRIC block was still a conceptual term for investors.

From then on, four meetings were held, agreeing on various political and financial projects, including South Africa as part of the group, in 2011.

In his first study on BRICs in 2001, O'Neill considered four different scenarios for the economies of the countries that made up the block. In any of the four options, based on different projection methods, the Irish economist argued that China had the potential to be as large as Germany based on its GDP, while Brazil and India would be close to Italy's GDP. In turn, Russia would remain in 11th place of all measurements, except the one corresponding to GDP in terms of Purchasing Power Parity. The relative weight of the BRIC economies would fluctuate, according to the scenarios, between 9% and 27%.

Ten years after this first paper on the potential of the BRICs, the figures seemed to confirm O'Neill's prediction. In 2011, China positioned itself as the second largest economy in terms of nominal GDP, according to the International Monetary Fund and the World Bank. Thus it surpassed Japan and Germany, which were in third and fourth place, respectively. The same positions were maintained considering the GDP data in PPP terms. The BRIC countries reached the top ten in the statistics of both international organizations, either measured in nominal GDP or in GDP in terms of PPP, except for Russia, which ranked 11th in the ranking (nominal GDP according to the World Bank) in 2011.

In the work published by Goldman Sachs in December 2011, "The BRICs 10 years on: Halfway Through the Great Transformation", they maintained the

predictions for the 2050 horizon, but with reservations, warning of the presence of different factors that could affect the contribution of BRICs to the global economy, pointing out that it could have already reached its maximum peak in 2011, in addition to the appearance of new emerging markets (such as the N-11 Next Eleven, a group promoted by the same financial institution as those economies with greater possibilities to accompany the BRICs in the economic leadership of this century).

Through the agreements reached at the BRIC summits, the emerging group seeks to occupy a more active place in politics and the international economy. Brazil, Russia, China and India have met 11 times in the Summits named in the same way as their acronym, having as headquarters: Yekaterinburg, Russia (2009); Brasilia, Brazil (2010), Sanya, China (2011), New Delhi, India (2012), Durban, South Africa (2013), Fortaleza, Brazil (2014), Ufa, Russia (2015), Benaulim, India (2016), Xiamen, China (2017), Johannesburg, South Africa (2018) and Brasilia, Brazil (2019).

The first summit held in Russia was held after the 2008 economic crisis. Topics such as cooperation actions for social development, disaster mitigation and food security were discussed, as well as the promotion of development research to advanced technologies. In general, the aim was to promote dialogue and cooperation within the BRIC in an increasing, proactive, pragmatic, open and transparent manner (Navarrete, 2011).

During this meeting, relevant topics such as sustainability and global warming were discussed, a relevant topic in which it was sought to place special emphasis because the four countries that make up the group stood out for their industrialization without a clear commitment to the rest of the world. The food crisis also figured and as they were countries with large populations, strategies were addressed to begin to face this problem.

The second summit was organized in Brasilia, Brazil. South Africa was not yet part of the group at this summit, although its possible accession was discussed. In this second summit, among other topics, the following topics were discussed:

- The contribution of the BRIC, to the increase of resources available to the IMF;
- The need for a capital increase from the World Bank and the International Finance Corporation, under the principle of burden sharing; and,
- The urgency of eliminating, in the course of 2010, the legitimacy deficit in the Bretton Woods institutions, through a substantial change in the voting power of emerging and developing economies, in line with their relative weight in the World economy.

The relevance of this summit was the increased participation of these four countries that seek a stronger position and with greater participation in decision-making at the global level and especially within financial institutions such as the IMF.

Among the priority topics on the agenda are the strategies in the agricultural sector of these nations, the agreements regarding research and technology and the energy that these countries have in a vast way and are fundamental in their economies.

The third summit was held in Sanya, highlighting the inclusion of the fifth member, South Africa, thus forming the new acronym BRICS. At this third summit, international issues such as conflicts unfolding in the Middle East, North Africa and West Africa were addressed, urging the international community to abide by the 1973 resolution to protect the civilian population and calling for peace. international.

Economic issues were high on the agenda and a meeting of the National Development Banks of the five countries was held in parallel, institutions that, since

the Brasilia summit, have acted as promoters and managers of intra-BRIC cooperation.

The venue for the fourth summit was New Delhi, India. The leaders of each of the nations met in this country to follow up on the agreements made in the previous summits and give rise to new agreements and treaties related to trade. The Presidents of the Development Banks of the five BRICS countries (Brazil, Russia, India, China and South Africa) signed agreements that would facilitate economic relations between the emerging powers in their own currencies. One of them, the Master Agreement, or framework agreement, establishes the general principles for the concession of credit lines in local currency, and the Letter of Credit Agreement that defines the rules for the confirmation of credits in export operations between the five countries. The five signatory institutions may grant loans in local currency in order to expand financial cooperation and the expansion of commercial exchange and investments among the countries of the block, reducing exchange risks for companies with international operations.

The foregoing in order to promote investment attraction between these countries and give competitive advantages compared to other economies interested in investing in these countries.

At the sixth BRICS Summit, the Contingent Reserves Arrangement and the creation of the New Development Bank stand out. The first is aimed at solving liquidity problems in a crisis context and at contributing to international financial stability. The second, based in Shanghai, aims to mobilize resources for infrastructure and development projects in BRICS and other emerging economies and developing countries.

The agreement creating the new development bank establishes that it will complement the work of multilateral and regional financial institutions. However, it is

clear that the BRICS seek their own sphere of action, apart from those, defending their particular interests and their ability to influence.

All in all, it is not surprising that the BRICS have chosen to develop the formula of selective multilateralism that moves away from multilateral institutions in which they are not a priority and from other global governance schemes such as the G-7. Without a doubt, this is the message that emerges from the sixth BRICS summit.

The seventh BRICS summit was held in the Russian city of Ufa in Bashkortostan. The summit coincided with the entry into force of the agreements establishing the New Development Bank and the BRICS Contingent Reserve Agreement. Among other things, Russia called on the other BRICS countries to cooperate in the field of information and communication technologies, opening their respective markets to the rest of the block to challenge the US monopoly in the sector.

Likewise, the first meeting of the Governing Council of the New BRICS Development Bank took place, discussing a strategy of direct economic association among its members.

During the summit, Russian President Putin and his Iranian counterpart Rohani agreed to collaborate closely in the military field, being able to share ports between the two countries.

The BRICS Summit Meeting of the year 2016 was the eighth and was held in the Indian city of Benaulim in Goa. Before the meeting began, several agreements were signed between Russia and India so that the latter could acquire anti-aircraft missiles and nuclear reactors, as well as permission to build Russian helicopters in Indian factories.

The creation of a rating agency and an emergency reserve agreement (CRA) was announced with a capital of \$ 10 billion, of which China contributed \$ 41 billion; Brazil, India and Russia 18 billion each and South Africa the remaining 5 billion, following the original ideal of being an alternative to the IMF and the World Bank.

During the IX BRICS Summit held in 2017 in Xianmen, China, the expansion of the organization was considered, which would adopt the name of BRICS Plus and would welcome new developing countries. Likewise, both Xi Jinping and Vladimir Putin, the President of Russia, showed their rejection of the nuclear tests carried out by the North Korean regime led by Kim Jong-un.

South Africa hosted the X BRICS Summit in 2018 and the event culminated with the signing of a communiqué - the Johannesburg Declaration - in which the leaders of the BRICS countries ratified that within the framework of the rules of the World Trade Organization, support and approve transparent and multilateral trade.

The XI BRICS Summit in Brazil, in 2019 continued to defend multilateralism, the denuclearization of North Korea and Brazil appeared more willing to expand and diversify relations with the countries of the block.

Derived from the objectives and resolutions of summits, as well as from the variations in the indicators of expected growth, uncertainties and assumptions arise in the thesis of BRIC group that question its homogeneity.

1.1 Motivation

The primary motivation of our study is to know if the block is actually a group with related elements that allow them to be amalgamated, or is it just a linguistic construct of nations that share or, rather, shared characteristics when the term was coined. To answer this question, we have focused the study on the energy sector and on the stock market of these emerging economies.

There are presently various studies concerned with the relationship between the oil price and economic variables, including: Akram, 2004; Chen and Chen, 2007; Cunado y Gracia, 2005; Davis and Haltiwanger, 2001; Hamilton, 1983,2003; Hamilton and Herrera, 2004; Hooker, 2002; Huang & Guo, 2007; Lee, Lee and Ratti, 2001; Lee and Ni, 2002; Nandha and Hammoudeh, 2007, among others.

As a result of the appearance and evolution of oil financial instruments such as futures and derivatives in the past decades of this last century, the commodity adopted financial characteristics that conducted to the research of its links with the financial market (Alquist and Gervais, 2013; Ma, Ji and Pan, 2019; Zhang et al., 2015; Zhang and Wang, 2015).

Although authors such as Fang and You (2014) signaled that there are fewer researches which have dealt with the relationship between the price of oil and financial markets, there is a wide range of literature which proves the interest and relevance of the link between these variables.

There is also some investigation on the subject of oil-stock oriented to BRIC emerging economies. Naming just some of them, Tiwari, et al, 2019; Fang and You, 2014; Aloui, Nguyen, and Njeh, 2013.

1.2 Problem Statement and Context

The present investigation focuses on modeling and finding out the relationship of the BRIC economies, specifically in the movement of stock market indexes and crude oil mixes: prices and equities of producing companies, in those countries. For this purpose, we use the Normal Inverse Gaussian distribution that has verified to be a viable option for modeling financial series due to its characteristics of heavy-tailed and being closed under convolution.

1.3 Research Question

The study will seek to answer the following question:

Considering variables in the energy sector and financial stocks in periods that include instability in the countries of the BRIC block, we question if there is a possibility of creating investment portfolios through a multivariate distribution that adequately adjusts to the series of these assets.

1.4 Solution overview

To address the research question, we will first review the adjustment of the main market indexes and variables of crude oil mixes in Brazil, Russia, India and China, to a distribution that allows capturing periods of instability or crisis, to later, look for possibilities of diversification, through the construction of efficient portfolios. This is intended to be achieved through:

- I. Modeling the returns of market indexes of BRIC countries through a Normal Inverse Gaussian distribution
- II. Obtaining a risk minimization in variables of the oil sector using a Normal Inverse Gaussian distribution
- III. Finding diversification opportunities in oil and stock indexes of BRIC countries

Chapter 2. Modeling market indexes of the BRIC countries through a Normal Inverse Gaussian distribution

2.1. Introduction

One of the main concerns regarding financial analysis, derives from deficient study of the returns of different financial variables; since it is essential to model the dynamics of these returns. With the constant advancement of mathematical tools as well as computational capacity, different approximations have attempted to improve model adjustment by blending numerical and analytical frameworks. In particular, the random processes modelling returns have been the core of the researching agenda development since the thesis proposed by Bachelier in 1900, premise in which normality is assumed and has been proven to be invalid by application of diverse statistical tests.

In this regard, multiple studies have investigated for better approximations to empirical returns density, and have also demonstrated that the members of the Generalized Hyperbolic family have a better fit to observations, like Hyperbolic distribution (Eberlein and Keller, 1995; Eberlein and Prause, 2002). One of the main properties regarding this distribution set is the presence of heavier tails than those of Normal distribution; which is a significant property of returns' behavior (Cont, 2001). Another property is the flexibility of the functions, since these distributions are influenced by five parameters. The adjustment has a better opportunity to reflect the nature of the empirical data; property that according to Cont (2001), is necessary to approach this kind of distribution.

Despite the fact that the Normal Inverse Gaussian (NIG) distribution has proved to be an excellent selection for stock returns distribution, it may also be used inclusively for index returns of financial markets (Barndorff-Nielsen, 1997; Barndorff-Nielsen and Shephard, 2001).

In specific, this chapter sets its focus on the probability distribution model of the main indexes of the BRIC economies in different periods of financial conditions (the case of South Africa is excluded because it was not until 2010, that it became part of this group).

By dividing this study into three time stages, the period from 2002 to 2015, we went over the capability of NIG to fit distribution returns even in extreme periods, such as the downturn of the financial crisis. By doing so, it may be possible to use Monte Carlo simulations with the proper distribution and parameters so as to obtain a better price of derivatives which relies on index as an underlying asset.

The goal of this chapter is to demonstrate that NIG distribution is statistically acceptable to model the returns of the market indexes of BRIC economies; distribution that we will use in next chapters to adjust the variables of crude oil mixes and seek the construction of optimal investment portfolios, through the Markowitz theory (1952), as we will see later.

The organization of this chapter follows the next order: in section 2.2, we discuss the Generalized Hyperbolic family and, in particular, the Normal Inverse Gaussian distributions. The methodology applied to the data is presented in section 2.3; then, section 2.4 demonstrates obtained results. Finally, the conclusions are stated in section 2.5.

2.2. Literature review

The Generalized Hyperbolic family was initially set by Barndorff-Nielsen in 1977. This class of distributions is defined by five parameters; by fixing the parameter $\lambda = -1/2$, the NIG distribution is obtained. Barndorff-Nielsen exposed the capability of the NIG distribution to model heavier tails than those of the Normal distribution, a fact which is frequently found in returns data series. These characteristics became of interest due to the consequences in risk management applications and other branches of finance. Particularly Eberlein & Keller used the DAX index in a three-year period from 1989 to 1992 to perform statistical tests to compare the fit of empirical data with adjusted Normal and Hyperbolic distributions (Eberlein and Keller, 1995). Their results concluded that, for the studied period, the Hyperbolic distribution is a better option to model returns.

However, in later studies Barndorff-Nielsen (1995) used NIG distributions as they are better to model heavy tailed observations. Under this assumption Barndorff-Nielsen used statistical tests as well, in order to compare how Hyperbolic and NIG distributions fit empirical data. Finally, he concluded that NIG performs better as a model for the same data used by Eberlein and Keller (Barndorff-Nielsen, 1995). Later, Rydberg (1999) proved, using data from Denmark, Germany and United States' stock markets, that the NIG distribution is a more fitted function to model the returns.

Trejo, Núñez and Lorenzo (2006) developed a study around the usage of NIG to model the stock return distribution in the Mexican market, as well as the IPC and S&P500 –indexes from Mexico and United States respectively–. By studying the Mexican market, they were able to make evident that NIG distribution has a better fit for stocks and index return distribution than a Normal one; therefore, tools like the Brownian Process are not the best alternative to simulate Mexican financial series.

Recent studies concerning the Generalized Hyperbolic family of returns in prices of commodities such as gold and petroleum have been done. Mota and Mata (2016) used historical prices from Brent, WTI and Mexican mixes, to adjust the parameters of a HG distribution. For this purpose, the authors took two-time period intervals to determine if those fits can be used in different states of nature; using the period from 2010 to 2013, where it was observed a higher price; and from 2014 to 2015, a period in which the international price plummeted. The results showed that these types of distributions are able to better fit empirical returns of such commodities.

Relative to gold return, the studies have been conducted around the possibility to model distribution in order to obtain a better Value at Risk (VaR) measure (Shen et al, 2017; Shen, Meng, and Meng, 2017). Using the time interval from 1991 to 2017, the studies use different classes of the Generalized Hyperbolic family, in order to compare them within risk management applications and techniques. Risk measures and their relationship with the adjustment of the concern variables to NIG distribution will be addressed in this study in Chapter 3.

2.3. Methodology

For the purpose of this study, we used the daily data gathered from Bloomberg of the BRIC indexes: IBOV, NIFTY50, SHCOMP and RTSI. Through the selection of data series, we intended to represent three periods of time, in a way to demonstrate that NIG distribution could fit indexes return data series in all nature states; the periods were named as pre-crisis (2002-2006), crisis (2007-2010) and post-crisis period (2011-2015). These representing periods were carefully chosen in order to shelter against other impacts not considered as part of the study. In particular, the crisis of 2001 emerged after the terrorist attack of the Twin Towers, and 2016 as the year in which capital flows returned to developed countries –mainly the continental United States– with the imminent interest rate normalization period (Neely, 2004 and Clark, 2016). The nature of these shocks is distinct, but this work

places emphasis only on the crisis included in the period from 2007 to 2010, thus naming the studied periods.

It should be noted that only in this chapter the periods of study were separated, seeking to identify the adjustment in periods of crisis, as well as those that precede and follow the crisis. However, in subsequent chapters, the work was concentrated on a single study period that covered periods of crisis, as well as periods before, after, and stability.

From each index series we estimated the logarithmic return as daily data could be used as an approximation to a continuous series; the equation states as follows:

$$r_i = \ln P_i - \ln P_{i-\Delta t} \quad \forall i = \{1, \dots, n\}$$

where:

r_i is the return of the index on the day i .

P_i is the closing level of the index at day i .

P_{i-1} is the closing level of the index at day $i - 1$.

2.3.1. Descriptive Statistics

Skewness and Kurtosis were calculated for each index series for validating distributions which have higher values of skewness, whether positive or negative; so we can expect that the empirical data does not correspond to a Normal distribution.

The statistics of these series are presented in the following table:

Time Period	Index	Mean	Variance	Skewness	Kurtosis
2002-2005	IBOV	0.0009	0.0003	-0.2645	0.6944
	NIFTY50	0.0011	0.0002	-1.0271	9.0648
	SHCOMP	0.0004	0.0002	0.6407	4.6071
	RTSI	0.0015	0.0003	-0.6751	4.1838
2006- 2010	IBOV	0.0006	0.0004	-0.0031	6.1839
	NIFTY50	0.0004	0.0004	0.0780	7.1262
	SHCOMP	0.0000	0.0004	-0.2815	2.1921
	RTSI	-0.0001	0.0007	-0.3191	10.7541
2011- 2015	IBOV	-0.0004	0.0002	-0.0468	1.5607
	NIFTY50	0.0002	0.0001	-0.1697	1.5807
	SHCOMP	0.0002	0.0002	-0.8859	6.2777
	RTSI	-0.0007	0.0004	-0.3010	6.7127

Table 2.1. Descriptive statistics of BRIC market index returns

By analyzing the excess of kurtosis, a different behavior from Normal distribution was distinguished due to the presence of heavy tails.

It should also be noted that Kurtosis changes in time intervals were different for each index. In the case of Brazil (IBOV) and Russia (RTSI) indexes, a growth behavior of Kurtosis was observed during the crisis and a decreasing effect in the post-crisis period. However, in the Indian index (NIFTY50), a lessening effect was observed in both crisis and post-crisis periods. About the Chinese index (SHCOMP), the value of Kurtosis decreased in crisis, and increased considerably in post-crisis.

Regarding the variance, the behavior was intuitive in all indexes: an increase during the crisis period and a decrease in post-crisis.

The behavior of these moments for every index is interesting, and since it is not part of the scope of this research, we propose to be deepened in a future work.

2.3.2. Normality Test

Using the Anderson-Darling and Shapiro-Francia normality test, it is possible to reject the null hypothesis of normality. In this case the presented NIG distribution becomes a solid candidate to fit the empirical data. Both normality tests are assumed as follows:

H_0 : Sample resulting from a normal distribution is confirmed.

H_a : Sample which does not come from a normal distribution, H_0 is rejected.

For the non-rejection of the null hypothesis, the p-value of each of the data series was obtained using both tests (Anderson-Darling and Shapiro-Francia), with a level of significance of 0.05, so that, if p-value \geq 0.05, the null hypothesis is not rejected, otherwise, we could affirm, with this level of significance, that the sample does not come from a normal distribution.

2.3.3. Shapiro-Francia Test

The normality test developed by Shapiro and Francia (1972) as an approximate and simplified version of the Shapiro Wilk test to prove the normality of a larger series of data. The test parameter is obtained by calculating the slope of the regression line by simple least squares, i.e.,

$$W = \left(\sum_{i=1}^n b_i y_i \right)^2 / \sum_{i=1}^n (y_i - \bar{y})^2$$

2.3.4. Anderson Darling Test

The Anderson-Darling (1954) criteria is used to test the hypothesis that a series of data comes from a population that adheres to a continuous Cumulative Distribution Function (CDF). The test is performed as follows:

$$W_n^2 = -n - \frac{1}{n} \sum_{j=1}^n (2j-1) [\ln(u_j) + \ln(1-u_{n-j+1})]$$

2.3.5. Normal Inverse Gaussian Distribution

As mentioned before, the NIG distribution has been proven in multiples studies to fit the financial series. This kind of distribution is defined by Barndorff-Nielsen as follows:

$$g(x; \alpha, \beta, \mu, \delta) = a(\alpha, \beta, \mu, \delta) q\left(\frac{x-\mu}{\delta}\right)^{-1} K_1\left\{\delta \alpha q\left(\frac{x-\mu}{\delta}\right)\right\} \exp(\beta x)$$

where

$$a(\alpha, \beta, \mu, \delta) = \pi^{-1} \alpha \exp\left(\delta \sqrt{\alpha^2 - \beta^2}\right) - \beta \mu$$

and

$$q(x) = \sqrt{1+x^2}$$

where K is the modified Bessel function of third order and index 1. Also, α, β, μ and δ , are parameters, satisfying $0 \leq |\beta| \leq \alpha, \mu \in R$ and $0 < \delta$.

The parameters α and β determines the shape, and μ and δ scale the distribution. Parameter α , which takes nonnegative values, denotes the flatness of the density function, i.e. a high value of α means a greater concentration of the

probability around μ . The parameter β defines a kind of skewness of the distribution. When $\beta = 0$, the NIG distribution is symmetric around the mean. A negative value represents a heavier left tail. The parameter δ describes the scale of the distribution and the parameter μ is responsible for the shift of the distribution density (Barndorff-Nielsen, 1997).

2.3.6. Goodness of Fit Tests

By simulating a vector with the obtained parameters, we test the similarity of both distributions with Kolmogorov-Smirnov (Massey, 1951) and Anderson-Darling criteria in which the p-values correspond with the non-rejection region of the null hypothesis. So, derived from the statistical tests, it is possible to state that NIG distribution is capable of modeling the returns even during a period of economic crisis.

2.4. Results

Through the results obtained by calculating the descriptive statistics of the series, we concluded that all series have heavy tails and high skewness values, which point out that the series could be fitted with a member of the Hyperbolic Generalized Family; as the NIG distribution. Previously we proceeded to estimate the NIG parameters, we applied the Normality Test (Anderson-Darling, Shapiro-Francia), to confirm that the series were not Normal.

Applying the Normality Test to the data, it was possible to reject the null hypothesis of normality in every case. This result is consistent with the stylized features of financial series where higher values of kurtosis do not correspond to the Normal distribution.

Time Period	Index	Anderson-Darling	Shapiro-Francia
2002-2005	IBOV	5.783e-06	1.55e-05
	NIFTY50	< 2.2e-16	< 2.2e-16
	SHCOMP	< 2.2e-16	< 2.2e-16
	RTSI	< 2.2e-16	< 2.2e-16
2006- 2010	IBOV	< 2.2e-16	< 2.2e-16
	NIFTY50	< 2.2e-16	< 2.2e-16
	SHCOMP	< 2.2e-16	1.62e-14
	RTSI	< 2.2e-16	< 2.2e-16
2011- 2015	IBOV	4.85e-09	3.502e-09
	NIFTY50	8.357e-09	5.173e-09
	SHCOMP	< 2.2e-16	< 2.2e-16
	RTSI	< 2.2e-16	< 2.2e-16

Table 2.2. Normality Test (p-value) for BRIC market index returns. Both Tests assume a significance level of 0.05, that means that if $p\text{-value} \geq 0.05$ null hypothesis is not rejected, otherwise is refused and the alternative hypothesis is confirmed. Own elaboration, data processed in R Software.

Having proven the non-normality of indexes return distribution, and the excess kurtosis obtained from the descriptive statistics of the series, the NIG could be used to shape the empirical data in order to obtain a distribution which better describes the empirical data series.

For the estimation of the NIG parameters we applied Maximum Likelihood Estimation (MLE). Although other methods could have been used, the selected algorithm solves the maximization problem by numerical methods. The parameters are shown in the table below. Parameters were estimated using R Software.

Time period	Index	α	β	δ	μ	n
2002-2005	IBOV	115.3831	-20.6768	0.0325	0.0068	1043
	NIFTY50	79.7261	.19.2693	0.0136	0.0044	1258
	SHCOMP	56.8119	5.7093	0.0097	0.0006	1303
	RTSI	47.6604	-7.6627	0.0146	0.0039	1303
2006- 2010	IBOV	31.8151	-3.1540	0.0131	0.0019	1304
	NIFTY50	35.9781	-2.9873	0.0145	0.0016	989
	SHCOMP	40.5741	-9.4047	0.0175	0.0042	1044
	RTSI	19.5138	-2.9695	0.0139	0.0021	1044
2011- 2015	IBOV	99.0900	5.1516	0.0199	-0.0014	1303
	NIFTY50	124.5000	-3.9450	0.0141	0.0007	1238
	SHCOMP	37.4145	-1.1891	0.0081	0.0004	1303
	RTSI	44.6705	-0.9362	0.0158	-0.0003	1303

Table 2.3. NIG Parameters obtained for each index series at different period times. Own elaboration, data processed in R Software.

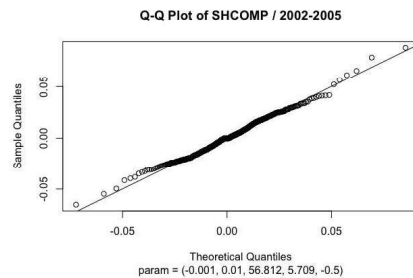
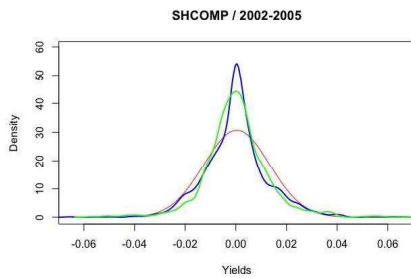
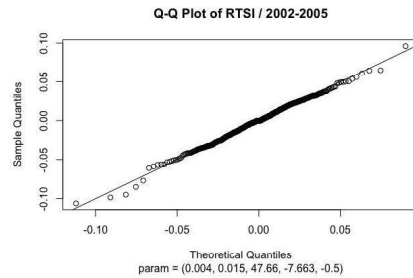
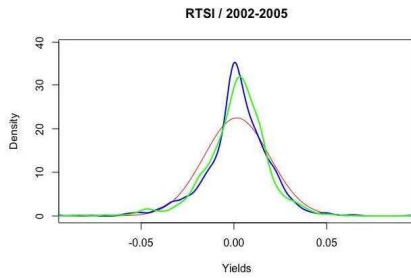
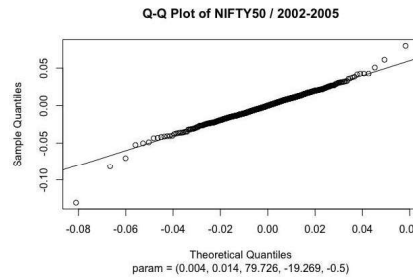
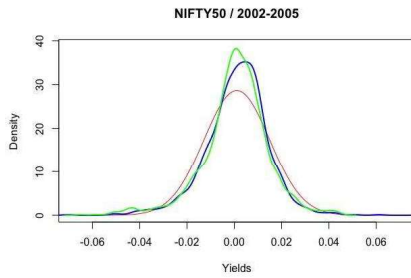
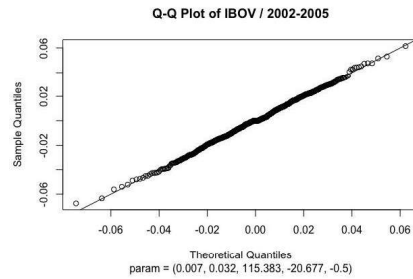
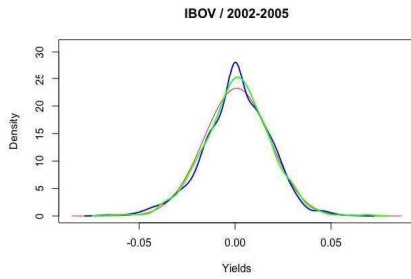
With the estimated NIG parameters, a series with a particular NIG distribution was emulated so as to make a statistical analysis using Log-likelihood Test to compare the similarity of the empirical data series with the simulated data series. To complete this task, Kolmogorov-Smirnov and Anderson-Darling tests were carried out.

The results did not reject the null hypothesis –the statistical similarity of distributions– in all cases except for the SHCOMP index in the pre-crisis period under the Kolmogorov-Smirnov test. However, the Anderson-Darling, considered a more precise test, confirmed similarity in every case. Therefore, according to statistical criteria, the NIG distribution can fit the indexes return distribution in the three states of nature; defined as pre-crisis, crisis and post-crisis.

Time Period	Index	Kolmogorov-Smirnov	Anderson-Darling
2002-2005	IBOV	0.8758	0.7774
	NIFTY50	0.2142	0.3006
	SHCOMP	0.0465	0.1121
	RTSI	0.0863	0.2552
2006- 2010	IBOV	0.3400	0.5358
	NIFTY50	0.6029	0.8094
	SHCOMP	0.0890	0.3554
	RTSI	0.4937	0.3279
2011- 2015	IBOV	0.2711	0.5014
	NIFTY50	0.7713	0.7679
	SHCOMP	0.1961	0.2523
	RTSI	0.0681	0.1423

Table 2.4. Likelihood Test (p-value) for BRIC indexes. Both tests assume a significance level of 0.05, that means that if $p\text{-value} \geq 0.05$ null hypothesis is accepted otherwise is rejected and the alternative hypothesis is confirmed. Own elaboration, data processed in R Software.

Quantitative results of Likelihood between NIG simulation distribution and the empirical data distribution obtained through the analysis could be observed graphically. Qualitative comparison of distributions considered Normal Distribution (red), empirical data distribution (blue) and simulated NIG distribution (green), as well as the QQ Plot is presented in the graphics bellow.



Distribution graphics show Normal Distribution (Red), Empirical Data Distribution (Blue) and NIG simulated Distribution (Green), for interpretation purposes. Graphics were generated in R software.

Figure 2.1. Normal, Empirical and NIG data series distribution at pre-crisis scenario.

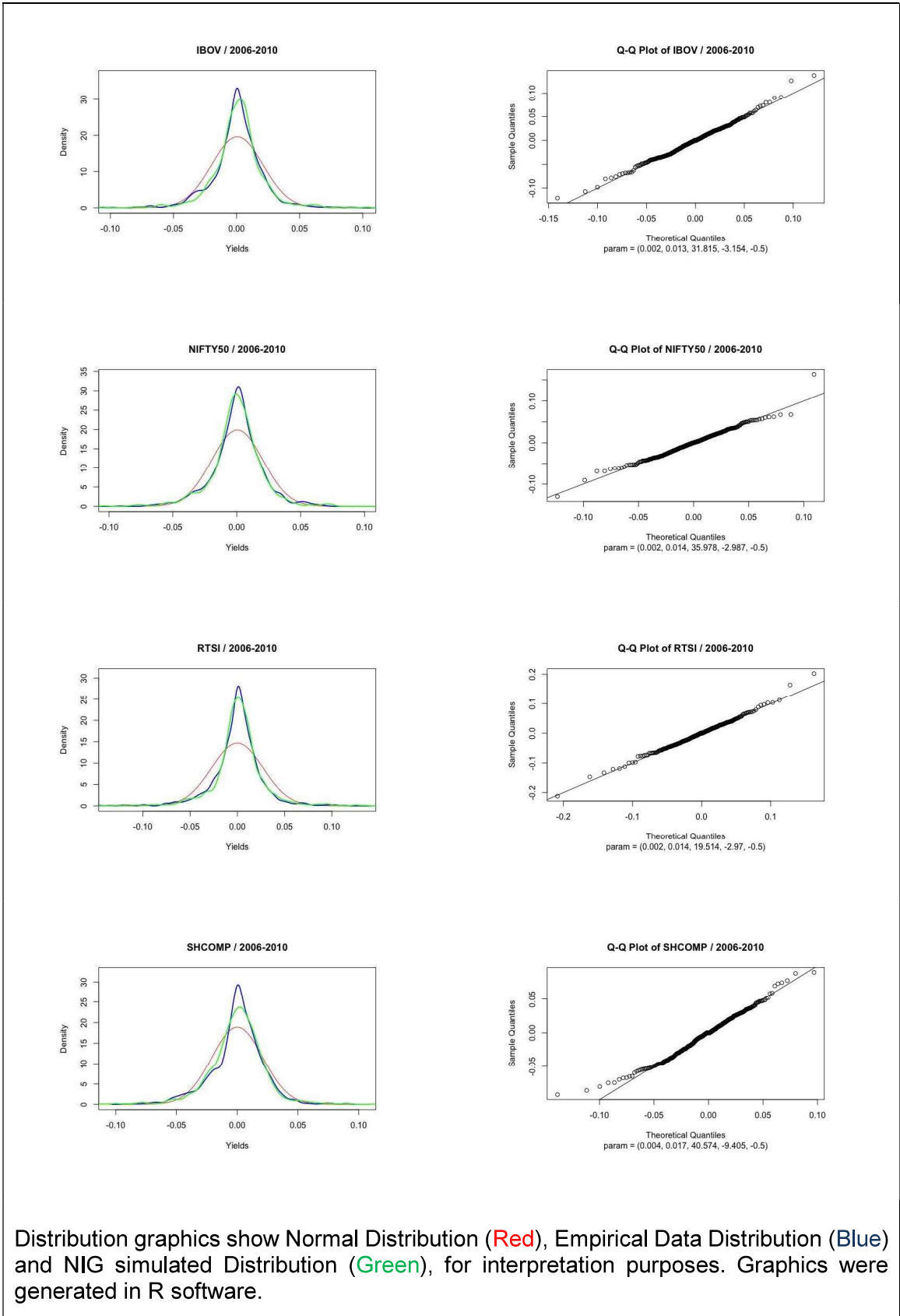
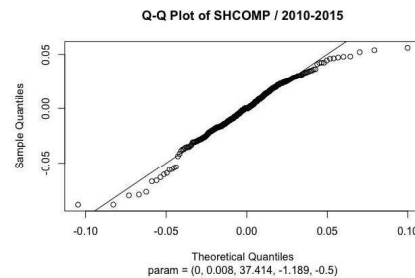
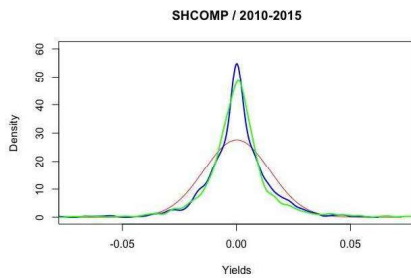
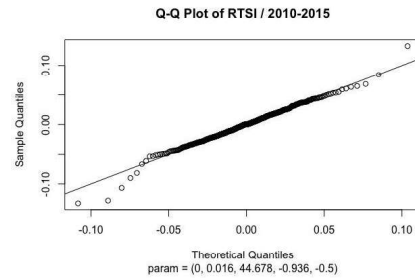
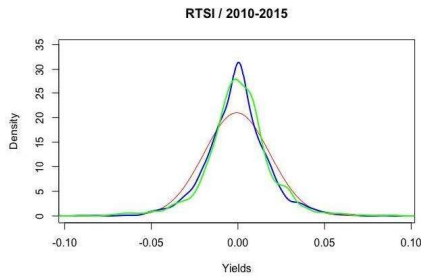
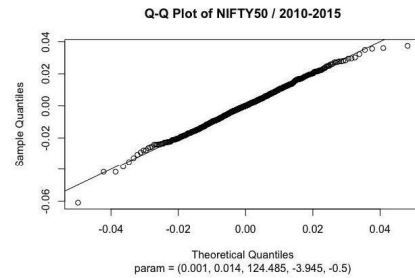
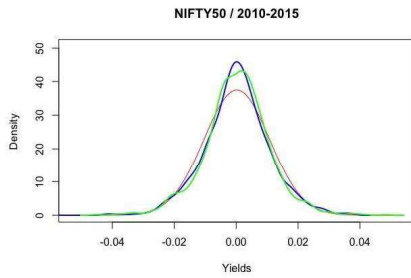
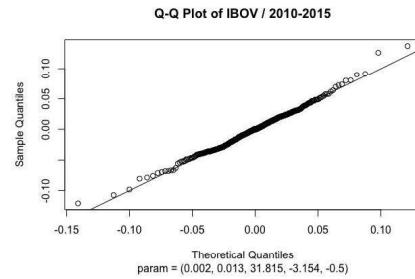
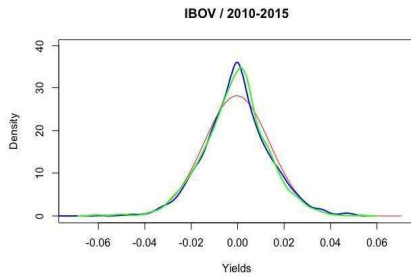


Figure 2.2. Normal, Empirical and NIG data series distribution at crisis scenario.



Distribution graphics show Normal Distribution (Red), Empirical Data Distribution (Blue) and NIG simulated Distribution (Green), for interpretation purposes. Graphics were generated in R software.

Figure 2.3. Normal, Empirical and NIG data series distribution at post-crisis scenario.

2.5. Conclusions

Through this analysis, it can be noted that financial returns are reasonably adjusted by the NIG distribution. We proposed to use the NIG as a distribution that has proven to be a suitable method to fit distribution of stock returns. We have demonstrated to fit the returns of indexes of the BRIC economies as well. Under this basis the indexes return from these economies were analyzed in three periods divided according to different states of nature. Those are pre-crisis, crisis and post-crisis scenarios. These results provide evidence that NIG should be used, or at least tested to model indexes returns in different scenarios, because distribution fits better to empirical data series, even at extreme periods like 2008 financial downturn, which deeply affected all of the world economies.

In the following chapters we will address the other set of variables of our interest: crude oil mixes, in our geographical framework: BRIC countries, and in the time context: integrated period that includes shocks and crisis events from 2004 to the most recent date with available data at the time of the study.

Chapter 3. Risk measures of oil sector variables using Normal Inverse Gaussian distribution

3.1. Introduction

As stated by Baffes, Kose, Ohnsorge and Stocker (2015), the oil sector has presented volatility in international oil prices in the past decades, which have been more illustrative since 2004, and respond to several factors such as the policies of the Organization of Petroleum Exporting Countries (OPEC), geopolitical tensions, disproportions between supply and demand in international markets and some others. Likewise, Sadorsky (1999) explains that extreme fluctuations in oil prices have an impact on economic activity and significantly affect movements in the stock market.

Therefore, how is it possible to find accurate models that allow better risk management when facing a scenario of uncertainty and volatility? The VaR as a standard for measuring and evaluating risk, and the CVaR as a coherent measure of risk, are used to estimate losses (maximum, probable and expected) given a level of confidence in a time horizon, and under normal market conditions that present adverse price movements. However, it has been shown that normality is not the best adjustment to financial instruments, resulting in better approximations of alternative distributions, as in the case of the Generalized Hyperbolic family, proposed by Barndorff-Nielsen in 1977. As mentioned before, this class of distributions is defined by five parameters, and fixing the parameter $\lambda = -1 / 2$, NIG distribution is obtained. Barndorff-Nielsen exposed the capability of NIG distribution to model heavier tails than in the case of Normal distribution, a fact commonly found in return data series (Barndorff-Nielsen, 1977, 1997).

In this environment, we will focus this study on VaR and CVaR estimates in BRIC oil companies, comparing the precision of both values and considering an

alternative distribution to Normal distribution: the NIG distribution. According to Núñez, Contreras-Valdez, Ramírez-García and Sánchez-Ruenes (2018), and detailed in the previous chapter, the adjustment of data series of the main indexes of the BRIC economies to NIG distribution has demonstrated to be a better option than Normal distribution in periods of volatility.

It should be noted that the election of distribution to calculate VaR and CVaR impacts the estimation of the quartiles that determine the risk. Moreover, a better adjustment of the empirical data to a specific distribution permits the construction of functions that more accurately estimate the risk given the conditions of uncertainty and volatility.

Recent studies have applied some distributions of the GH family to adjust the returns in prices of commodities such as oil and gold. This is the case of the work by Mota and Mata (2016) discussed in the previous chapter, who have adjusted the prices of various petroleum mixes to GH distribution in two periods: 2010 to 2013 and 2014 to 2015. The results demonstrated that the empirical data better fit this class of distributions.

Regarding gold returns, data (from 1991 to 2017) have been adjusted to some GH distributions to estimate the VaR, obtaining better approximations than with Normal distribution (Shen, Meng and Meng, 2017; Shen, Meng, Guo, Zhao, Ding and Meng, 2017).

In this chapter, our goal is to demonstrate a bigger estimation of risk through the quantification of VaR and CVaR in equity price returns of oil BRIC companies when assuming NIG distribution, compared to normal distribution in times of instability.

Throughout this chapter we will make a parametric estimate of VaR through the alternative NIG distribution of equity returns of oil companies in the BRIC economies, differing from most papers found in the oil sector, which focus on estimating the risk measures of crude oil mix prices and assuming traditional distributions as normal.

The present chapter is organized as follows: Section 3.2 presents and discusses the concepts and methods for estimating VaR and CVaR measures, as well as a description of NIG distribution. In section 3.3 we present the methodology applied to the data to estimate VaR and CVaR considering both distributions: Normal and NIG; and the results are shown in section 3.4. Finally, the conclusions are exposed in section 3.5.

3.2. Literature review

The Basel Committee on Banking Supervision (1996) requires that financial institutions, such as banks and investment firms, comply with the capital requirement based on the estimation of VaR. An incorrect estimation of the VaR results in a sub-optimal allocation of resources.

Yamai and Yoshihara (2005) studied and compared the VaR and CVaR risk measures. From that compilation, we will approach the concepts presented below.

Following Artzner, Delbaen, Eber and Heath (1997), VaR is defined as the maximum probable loss of a portfolio or financial instrument in a given time horizon, for a level of confidence under normal market circumstances because adverse movements in prices.

Definition 1. Given a time interval $[t; T]$, the relative change in the value of a portfolio in a given time horizon $\tau = T - t$ is defined as:

$$\Delta\Pi_t(\tau) = \Pi(t + r) - \Pi(t)$$

Where $\Pi(t) = \ln V(t)$, $V(t)$ represents the value of the portfolio at time t . If $X = \Delta\Pi_t(\tau)$, then $X: \Omega \rightarrow \mathbb{R}$, and $\Omega = \Omega_1 \times \Omega_2 \times \dots \times \Omega_n$ is a continuous random variable defined on the sample space Ω that represents the change in the value of the portfolio composed of n assets (portfolio return). Assuming that X is defined over a fixed probability space (Ω, A, P) , then the Value at Risk of X at level $1 - q$ is defined as the minimum of the upper bounds for a confidence interval of $(1 - q) \%$ such that:

$$P = \{X \leq VaR_{1-q}^X\} = q$$

i.e.:

$$VaR_{1-q}^X = -\sup\{x \in \mathbb{R}: P(X \leq x) \leq q\}$$

from which the following is derived:

$$VaR_{1-q}^X = -\inf\{x \in \mathbb{R}: P(X > x) \leq 1 - q\}$$

From VaR definition, it is possible to obtain the VaR given the cumulative distribution function of the portfolio returns:

$$q = F_X(-VaR_{1-q}^X) = \int_{-\infty}^{-VaR_{1-q}^X} f_X(x) dx$$

where $F_X(x) = P(X \leq x)$ is the cumulative distribution function of the portfolio returns over a period and $f_X(x)$ is the probability density function of X . Then:

$$-VaR_{1-q}^X = F_X^{-1}(q)$$

That is, the VaR is the quantile q of F_X . Therefore, the essence of the VaR calculation is the estimation of the lower quantiles of the cumulative distribution function of the portfolio returns, which in practice is unknown (Artzner, Delbaen, Eber and Heath, 1999).

VaR estimation methods suggest different ways of constructing this function. The most common are: parametric method, historical simulation and Monte Carlo simulation.

Lauridsen (2000) presented an overview of the main methods for estimation of VaR comparing their performance, advantages and drawbacks.

3.2.1. VaR estimation by historical simulation

In this method, historical financial data produces future behavior information. Therefore, it is thinkable to use the history of financial data to obtain significant predictions of future performance. Historical simulation is a relatively simple method, easy to implement and has the advantage that it is not necessary to assume that returns follow a normal distribution.

In general, the historical method depends on the observed data, so if the sample size is not large enough, there is a greater chance that the VaR estimate lacks precision. For a detailed discussion on this approach, there are various articles such as Hendricks (1996) and Barone-Adesi et al. (2000).

3.2.2. VaR estimation by Monte Carlo simulation

Through this method, recently studied by Hong, Hu and Liu (2014), an approximation of the performance of the expected return of a portfolio or financial instrument, is obtained by means of simulations that generate random trajectories of the returns of the portfolio or financial instrument, considering certain initial assumptions about the volatilities and correlations of the risk factors.

3.2.3. Parametric VaR

The parametric technique is used under the assumption of observed data following some rules or models with unknown parameters. We use these data to obtain the parameter estimates applying the rule or the model established to calculate the VaR as Mentel (2013) pointed out. In this method, we have two approaches: the unconditional approach and the conditional approach.

3.2.4. Unconditional approach

In this method, the assumption is that the financial returns in each time of period are identical and independently distributed random variables (iid) that follow a multivariate Gaussian distribution. However, numerous investigations show that

multivariate Normal distribution cannot explain some properties of the empirical distribution of financial data. For example, Fama (1965), Hull and White (1998) point out that changes in several market variables (stock prices, prices of zero coupon bonds, exchange rates, commodity prices, etc.) are leptokurtic, that is, there are several values close to the average and too many in the extreme tails. These changes increase the probability of very large and very small movements in the value of the market variables and decrease the probability of moderate movements. Therefore, there is not enough evidence to support the Gaussian hypothesis.

3.2.5. Conditional approach

This approach admits that time series of financial returns depend on past information. Traditionally, the AutoRegressive Moving Average model (ARMA) describes a dependence of the series, where we obtain a stationary series. Nonetheless, the ARMA models assume that the variance is constant, and given that generally speaking the volatility of financial time series is not constant, these models are not suitable to use with them.

Several investigations try to find models that describe the volatility variable in time, a common characteristic of financial returns. The most popular is the AutoRegressive Conditional Heteroscedasticity model (ARCH) proposed by Engle (1982) where the variance conditioned on past information is not constant, and depends on the square of past innovations. Subsequently, Bollerslev (1986) generalizes the ARCH models when proposing the models of Generalized Conditional AutoRegressive Heteroscedasticity (GARCH). In these models the conditional variance depends not only on the squares of the perturbations, as in ARCH models, but also, of the conditional variances of previous periods. It is possible to combine both models with the ARMA model, obtaining the ARMA-ARCH and ARMA-GARCH models.

The standard GARCH model, widely used today, we assume that observed data fits a Gaussian distribution. However, for many series of financial returns, the Gaussian distribution is not adequate since it does not consider leptokurtosis.

3.2.6. Strengths and weaknesses of VaR

VaR is a popular risk measure since it is possible to quantify the market risk of a financial institution through the calculation of a unique numerical value. This value is the maximum possible loss of a portfolio or financial instrument in a period of given time, for a certain level of confidence (Barone-Adesi, Giannopoulos and Vosper, 2000). At the same time, one of its disadvantages is that it ignores, by definition, the losses whose probability of occurrence is less than that chosen as the level of confidence in the estimation. For these reasons, Bali affirms that the standard VaR provides an inadequate estimation of the losses during periods of high volatility such as those corresponding to financial crises (Bali, 2007).

On the other hand, Artzner, Delbaen, Eber, and Heath (1999) indicate that one of their main disadvantages is that it is not a coherent measure of risk and nonconvex. A measure of risk is coherent if it satisfies the axioms of monotonicity, subadditivity, positive homogeneity and invariance under translations.

3.2.7. Conditional Value at Risk

An alternative coherent risk measure is CVaR, introduced by Artzner, Delbaen, Eber, and Heath. We define this measure as the expected loss that exceeds the VaR. In general, the CVaR provides the conditional expectations of loss above the VaR. Some other names of CVaR are Expected Shortfall, Mean Excess Loss, Beyond VaR, Tail VaR, Conditional Loss and Expected Tail Loss.

The CVaR provides better adjustment and consistency in the estimation of the risk with respect to the VaR, since it complements p-value information provided by the VaR, becoming very useful when we have asymmetric distributions (Artzner, Delbaen, Eber, and Heath, 1997).

Rockafellar and Uryasev (2002), show that the CVaR is a coherent measure of risk and show the significant advantages of this methodology with respect to the traditional VaR. On the other hand, Kibzun and Kuznetsov (2003), mention that the CVaR, under normal conditions, is a convex function with respect to the positions taken, which allows the construction of an efficient optimization algorithm. Peña (2002) indicates that despite the advantages of the CVaR over the VaR, both are complementary measures, that is, if the objective of the VaR is to control market risk under normal conditions, the objective of the CVaR is to control the risks of market in extreme conditions. In addition, it is possible to establish a relationship between the VaR and the CVaR if a specific parametric distribution is assumed. For the specific case of normality, the VaR and the CVaR are similar.

Therefore, we can consider the use of the CVaR as a complement to the VaR, that is, if the objective of the VaR is to control the market risk under normal conditions, the objective of the CVaR is to control the market risks in extreme conditions.

Currently both risk measures are widely used in the field of research and in empirical studies and applications as well.

3.2.8. Portfolio optimization

Regardless of whether we apply the VaR or the CVaR to measure the risk of a portfolio or financial instrument, one of the natural objectives of risk management is the optimization of the portfolio. In Markowitz Portfolio Theory, the concepts of correlation and covariance are key elements of the model. In addition, it introduced the concept of diversification, that is, the addition of some assets to an investment portfolio, which derives in a risk decrease.

The objective of Markowitz (1952) was to find the optimal allocation for each investment instrument, minimizing the variance (considered as the risk measure) subject to a condition on the expected return; that is, the investor wants to obtain the highest return and he is adverse to the risk. For Markowitz, a portfolio is considered efficient if it provides the maximum possible return for a given level of risk, or equivalently, if it provides the lowest possible risk for a specific level of profitability.

Thus, we can say that an optimal portfolio is a combination of financial instruments that represents the return-risk ratio which maximizes investor satisfaction.

It is possible to replace the variance with some other risk measures such as VaR and CVaR. Nonetheless, since VaR is not a consistent measure, it is difficult to find the optimal portfolio that minimizes that risk measure. Gaivoronski and Pflug (2005) propose a very complex approach to VaR using a soft measure called SVaR (local minimizer) that filters irregularities locally.

On the other hand, the CVaR is a coherent and convex measure, so it is possible to find a unique global minimizer. Rockafellar and Uryasev (2002) point out a simple way to optimize a portfolio by minimizing CVaR, which allows transforming

the problem into a classic linear programming problem. Recently, various studies have appeared that extend this model for example, Andersson et al. (2001); Charpentier and Oulidi (2008); Glasserman et al. (2002); and Krokmal et al. (2001).

Nevertheless, all these authors use the historical approach to calculate the portfolio allocation. Therefore, if there is not enough historical data it is possible to underestimate the risk severely. Another drawback is that the authors previously mentioned do not address the dependence of the historical data series.

In Chapter 4 we will address the construction of optimal portfolios with some instruments of oil mixes and indexes of BRIC economies, now we will focus on the calculation of VaR and CVaR risk measures considering NIG distribution and their comparative values assuming normality.

3.3. Methodology

For the study conducted in this chapter, we made use of the daily data collected from Bloomberg of the following equities' crude oil producing companies of BRIC economies:

Petróleo Brasileiro SA in Brazil; PJSC Rosneft Oil Company in Russia; Oil & Natural Gas Corporation Limited, Oil India Limited, Gail India Limited and Cairn India Limited, in India; and Kunlun Energy Company Limited, China Petroleum & Chemical Corporation and China National Offshore Oil Corporation in China.

The period selected for the data series was from 2004 to 2017, where several events of instability were presented, such as:

- The Price bubble due to the growth in oil demand from China and India and fixing oil level supply by producing countries (2004 - 2007)
- The financial crisis (2007 - 2008)
- The rise in prices due to the reactivation of economies and the increase in demand by emerging economies (2009 - 2013)
- The drop in oil prices since 2014 due to oversupply and weakening demand in oil markets, as well as Saudi Arabia's decision to maintain its oil production levels.

Due to data availability, we only considered certain events as in the case of Rosneft, where we obtained data from 2006 and Oil India, with information since 2009 (Baffes, Kose, Ohnsorge and Stocker, 2015).

For every equity series, we computed the logarithmic return for daily data.

3.3.1. Descriptive Statistics

We calculated skewness and kurtosis for each equity series in order to validate distributions with higher values of these moments and inferring that data series fit to a non-normal distribution class.

The following table presents descriptive statistics for the series considered. Note that Oil India's kurtosis differs in proportion from the rest of the equities, being less than 3 in the selected period that includes shocks and instability. Therefore, as discussed in previous chapter, about the case of kurtosis obtained for Indian and Chinese market indexes, we also suggest a future study on this equity.

Country	Company	Equity	Mean	Variance	Skewness	Kurtosis
Brazil	Petrobras	PETR3 BZ	0.0001	0.0010	-0.017	3.925
		PETR4 BZ	0.0001	0.0010	-0.2081	4.7732
Russia	Rosneft	ROSN RX	-0.0001	0.0008	1.3606	36.2001
India	ONGC	ONGC IN	0.0001	0.0005	0.1444	4.8558
		ONGC IB	0.0001	0.0005	-0.1443	4.9206
		ONGC IS	0.0001	0.0005	-0.1517	4.8777
	Oil India	OINL IN	0.00004	0.0003	-0.0039	2.2466
		OINL IB	0.00004	0.0003	0.0895	2.5768
		OINL IS	0.00004	0.0003	-0.0039	2.2466
	GAIL	GAIL IN	0.0003	0.0006	-0.1290	13.3321
		GAIL IB	0.0003	0.0006	-0.1205	14.0466
		GAIL IS	0.0003	0.0006	-0.1813	13.2171
	CAIRN	CAIRN IN	0.0002	0.0007	-0.3793	4.7279
		CAIRN IS	0.0002	0.0007	-0.3793	4.7279
	China	CNPC	135 HK	0.0005	0.0006	-0.0446
SINOPEC		386 HK	0.0002	0.0006	0.1606	5.2435
CNOOC		883 HK	0.0004	0.0006	0.1702	5.1632

Table 3.1. Descriptive statistics of BRIC oil equity returns. * High kurtosis values appeared in all data series distributions, except for Oil India equities. Source: own elaboration in R Software with data from Bloomberg.

3.3.2. Goodness of Fit Test

In the same way as in the previous chapter, we used the Anderson-Darling and Shapiro-Francia goodness-of-fit tests, expecting to reject the null hypothesis of normality and the non-rejection of statistical similarity between empirical data series and the simulated series using a NIG distribution class. In the first case, we established normality tests as follows:

H_0 : We confirm a sample resulting from Normal distribution.

H_a : We discard a sample that follows normality.

Regarding the goodness-of-fit test of the simulated vector with NIG distribution and the data series, the proposed hypotheses were the following:

H₀: Sample that comes from Normal Inverse Gaussian distribution is confirmed.

H_a: We discard a sample derived from a Normal Inverse Gaussian distribution.

For the purpose of testing both null hypothesis, the p-value for each data series was calculated using Anderson-Darling (1954) and Shapiro-Francia (1972) proofs when testing normality, and Kolmogorov-Smirnov (Massey,1951) and Anderson-Darling for NIG distribution goodness-of-fit test, with a significance level of 0.05, so, if $p - value \geq 0.05$ the null hypothesis is not rejected.

3.3.3. VaR and CVaR estimates

From definition 1 we can obtain the VaR knowing the cumulative distribution function of the asset's returns:

$$-VaR_{1-q}^X = F_X^{-1}(q)$$

where F_X^{-1} is the inverse of the cumulative distribution function of the returns of the asset in a period. To be precise, VaR is the quantile q of F_X . Therefore, calculation of the VaR fundamentally consists of the estimate of the lower quantiles of the cumulative distribution function of the assets' returns.

Assuming that the distribution of the returns of the assets is normal, described by the mean parameters μ and the standard deviation σ , the VaR calculation consists

of finding the q^{th} percentile of the standard normal distribution z_q :

$$q = \int_{-\infty}^{X^*} g(x)dx = \int_{-\infty}^{z_q} \Phi(z)dz = N(z_q)$$

where $X^* = z_q\sigma + \mu$, and $\Phi(z)$ is the standard normal density function, $N(z)$ is the cumulative normal distribution function, X is the asset performance, $g(x)$ is the normal distribution function of the returns with mean μ and standard deviation σ , and X^* is the minimum return at confidence level $1 - q$.

The CVaR of X at level $1 - q$ is defined as the expected loss, given that this loss exceeds the VaR, that is:

$$CVaR_{1-q}^X = E [X: X > VaR_{1-q}^X]$$

The CVaR complements the information provided by the VaR. As mentioned before, it is very useful when we have asymmetric distributions. Besides, it is an excellent risk management tool, applicable in distributions with peaks.

3.4. Results

Analyzing the values of calculated descriptive statistics, we noted that all series have high kurtosis values, except in Oil India Limited, with a value less than 3. Additionally, and as we expressed before, we applied the Anderson-Darling and Shapiro-Francia Normality tests to confirm that the series were not Normal. In both tests, we assumed a significance level of 0.05 and we obtained a p-value less than 2.2×10^{-16} in all cases, meaning the null hypothesis of normality was rejected. Then we sought to fit the series using a member of the GH family: the NIG distribution class.

Again, we applied Maximum Likelihood Estimation (MLE) for NIG parameters

estimate. Table 3.2 shows NIG parameters using R software.

Country	Company	Equity	α	β	δ	μ	n
Brazil	Petrobras	PETR3 BZ	27.7696	-2.5228	0.0259	0.0025	3463
		PETR4 BZ	27.8003	-1.2370	0.0265	0.0013	3463
Russia	Rosneft	ROSN RX	27.4717	1.1942	0.0185	-0.0009	2870
India	ONGC	ONGC IN	45.2371	-0.2926	0.0237	0.0003	3480
		ONGC IB	44.6404	-0.0489	0.0228	0.0002	3480
		ONGC IS	45.3151	-0.2969	0.0238	0.0003	3480
	Oil India	OINL IN	62.0062	-0.9141	0.0200	0.0003	2049
		OINL IB	58.4773	0.5684	0.0183	-0.0001	2049
		OINL IS	62.0062	-0.9140	0.0200	0.0003	2049
	GAIL	GAIL IN	39.9015	0.7049	0.0228	-0.0001	3480
		GAIL IB	39.6728	0.9236	0.0221	-0.0002	3480
		GAIL IS	40.0952	0.8189	0.0228	-0.0002	3480
	CAIRN	CAIRN IN	34.5280	-1.5625	0.0238	0.0012	2549
		CAIRN IS	34.5280	-1.5625	0.0238	0.0012	2549
	China	CNPC	135 HK	29.6789	1.8679	0.0191	-0.0007
SINOPEC		386 HK	35.2436	0.0443	0.0194	0.0002	3454
CNOOC		883 HK	35.2196	0.0726	0.0210	0.0003	3454

Table 3.2. NIG Parameters obtained for each oil equity series. Source: own elaboration in R Software with data from Bloomberg

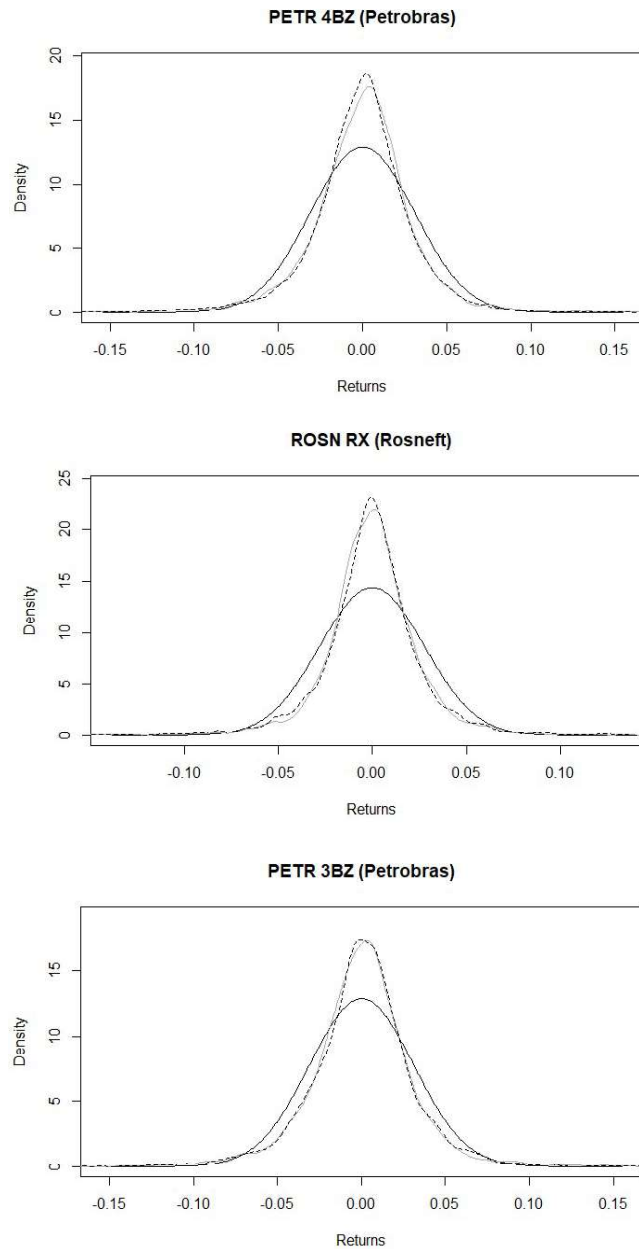
Applying the methodology outlined in previous chapter, series with a particular NIG distribution were simulated in order to compare the similarity of the empirical and simulated data series. We used Kolmogorov-Smirnov and Anderson-Darling tests for this purpose.

The results confirmed the statistical similarity of empirical data and NIG distributions in all cases. Therefore, according to statistical criteria, NIG distributions can fit the equity return distributions.

Country	Company	Equity	Kolmogorov-Smirnov	Anderson-Darling
Brazil	Petrobras	PETR3 BZ	0.4789	0.4103
		PETR4 BZ	0.5555	0.5466
Russia	Rosneft	ROSN RX	0.281	0.533
India	ONGC	ONGC IN	0.2888	0.3001
		ONGC IB	0.8652	0.7755
		ONGC IS	0.07921	0.08834
	Oil India	OINL IN	0.3053	0.3152
		OINL IB	0.5496	0.5372
		OINL IS	0.964	0.9378
	GAIL	GAIL IN	0.1756	0.307
		GAIL IB	0.07921	0.06038
		GAIL IS	0.7584	0.7136
	CAIRN	CAIRN IN	0.4588	0.5876
CAIRN IS		0.6752	0.6754	
China	CNPC	135 HK	0.2346	0.4428
	SINOPEC	386 HK	0.3269	0.414
	CNOOC	883 HK	0.8926	0.8357

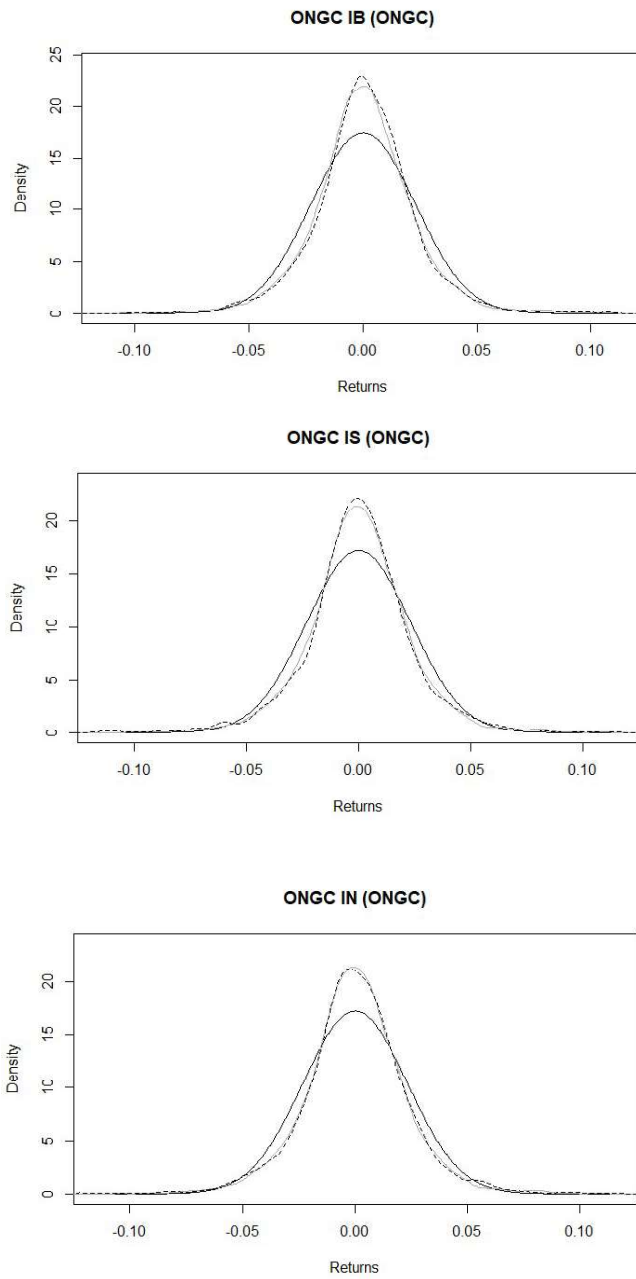
Table 3.3. Likelihood Test (p-value) for BRIC oil equities. Both tests assume a significance level of 0.05, that means that if p-value is greater than or equal to 0.05 we do not reject the null hypothesis, otherwise we confirm the alternative hypothesis. Source: own elaboration in R Software with data from Bloomberg

The following figures show graphically the qualitative comparison of Normal Distribution (solid black line), empirical data distribution (dashed gray line) and simulated NIG distribution (dashed black line), in the figures bellow.



Distribution graphics show Normal Distribution (—), Empirical Data Distribution (---) and NIG simulated Distribution (....), for interpretation purposes. We generated these graphics in R software.

Figure 3.1. Normal, Empirical and NIG data series distribution for equities in Petrobras and Rosneft companies. Source: own elaboration in R Software with data from Bloomberg.



Distribution graphics show Normal Distribution (—), Empirical Data Distribution (---) and NIG simulated Distribution (....), for interpretation purposes. We generated these graphics in R software

Figure 3.2. Normal, Empirical and NIG data series distribution for equities in Oil & Natural Gas Corporation Limited (ONGC). Source: own elaboration in R Software with data from Bloomberg.

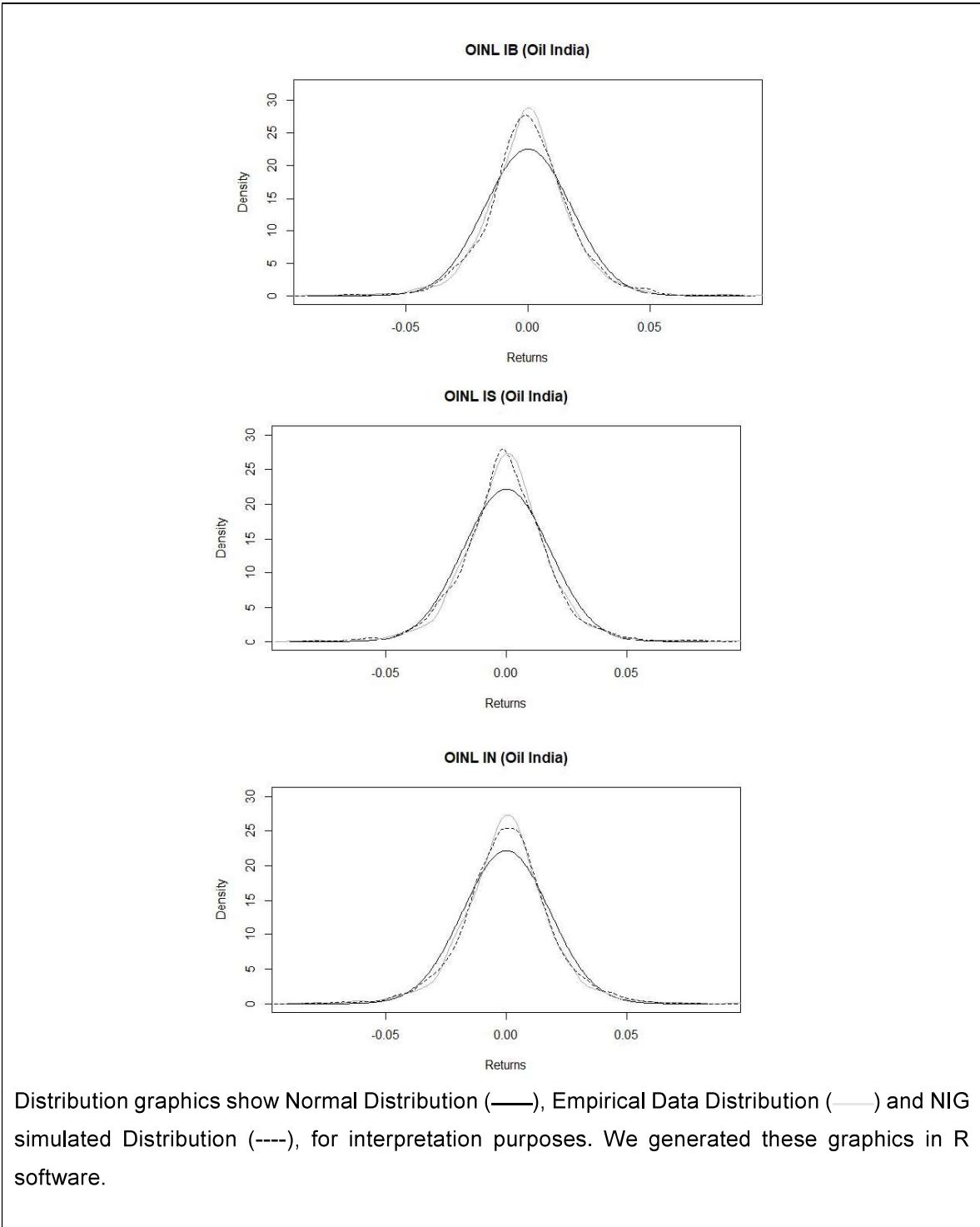


Figure 3.3. Normal, Empirical and NIG data series distribution for equities in Oil India Limited (OINL). Source: own elaboration in R Software with data from Bloomberg.

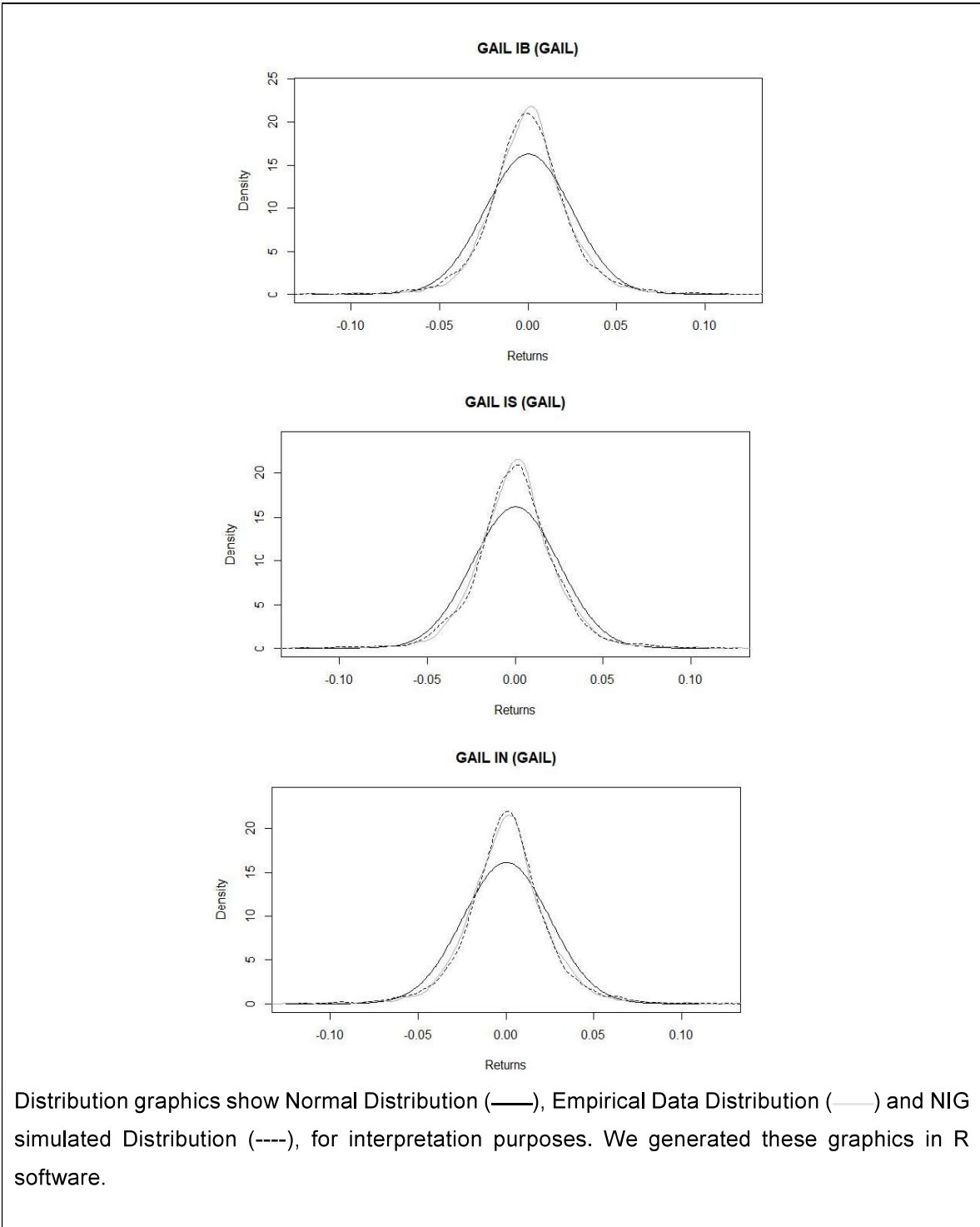


Figure 3.4. Normal, Empirical and NIG data series distribution for equities in Gail India Limited. Source: own elaboration in R Software with data from Bloomberg.

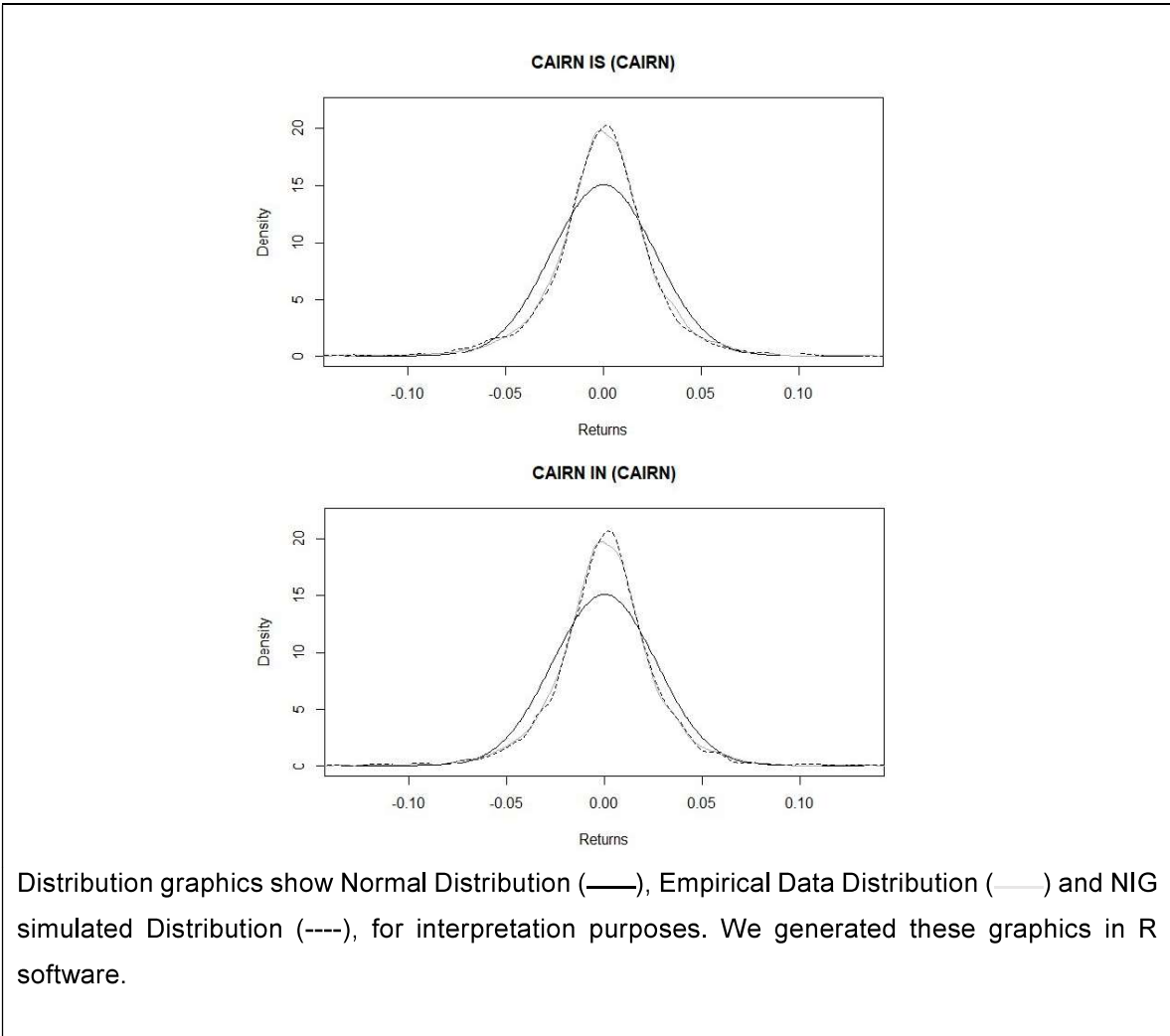


Figure 3.5. Normal, Empirical and NIG data series distribution for equities in Cairn India Limited. Source: own elaboration in R Software with data from Bloomberg.

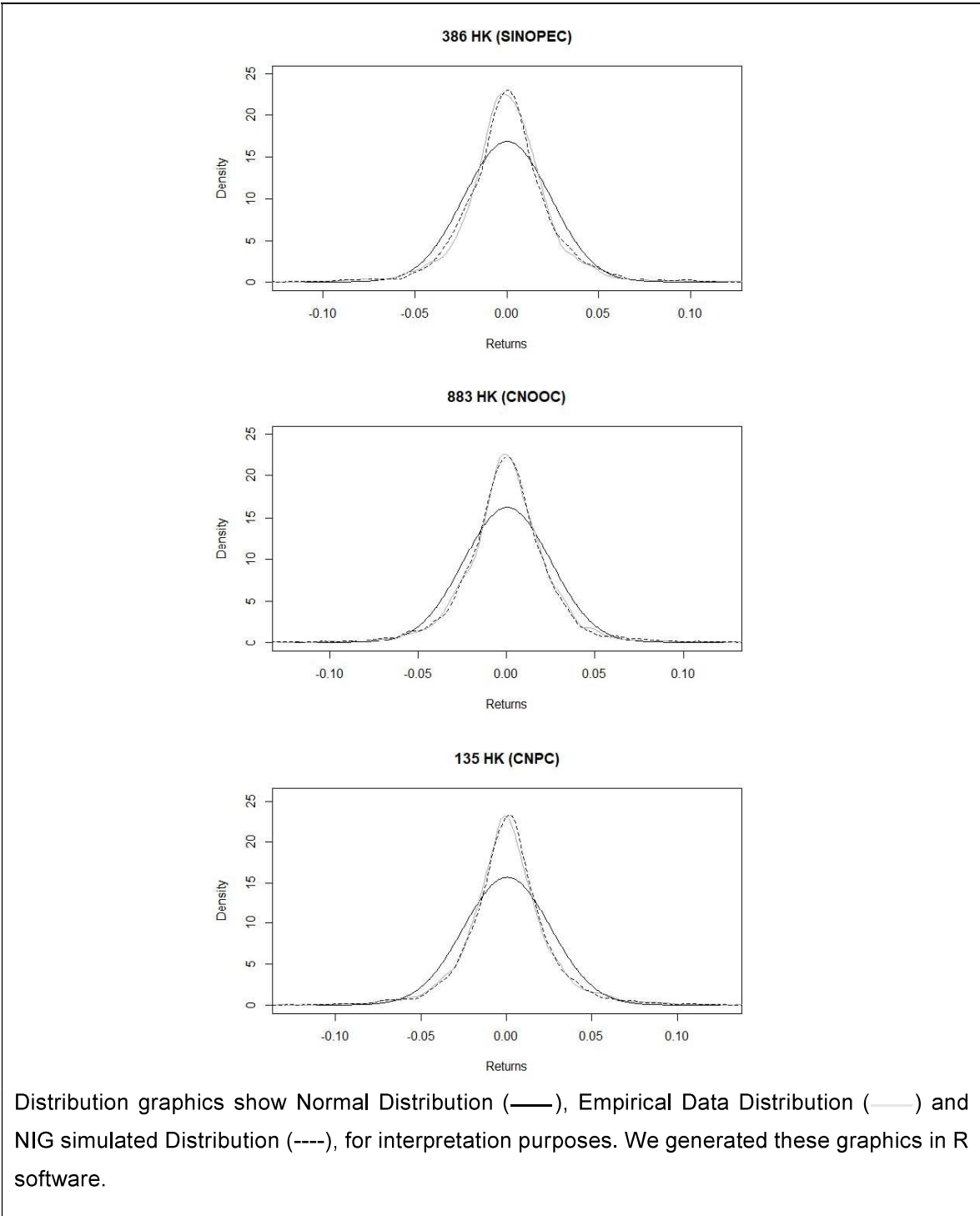


Figure 3.6. Normal, Empirical and NIG data series distribution for equities in Kunlun Energy Company Limited (CNPC), China Petroleum & Chemical Corporation (SINOPEC) and China National Offshore Oil Corporation (CNOOC). Source: own elaboration in R Software with data from Bloomberg.

In addition, we calculated Value at Risk and Conditional Value at Risk measures using R software, first considering a normal distribution and establishing mean (μ) and standard deviation (σ) parameters indicated in Table 3.1. Then we used NIG distributions with flatness (α), skewness (β), scale (δ) and shift (μ) parameters specified in Table 3.2.

Table 3.4 shows calculated values for both risk measures, where in all cases we observed that VaR obtained using normal distribution were less than VaR calculated through the NIG distribution type; and we found the same case for CVaR, where the values obtained with normality assumption were less than those obtained when supposing NIG distribution, i.e.:

$$\begin{aligned} VaR_{Normal\ distribution} &< VaR_{NIG\ distribution} \\ CVaR_{Normal\ distribution} &< CVaR_{NIG\ distribution} \end{aligned}$$

Consequently, in VaR and CVaR estimates, we observed that normality assumption represents an underestimation of risk. In other words, the NIG distribution model provided estimates of the largest potential losses.

Country	Company	Equity	Normal distribution		NIG distribution	
			VaR	CVaR	VaR	CVaR
Brazil	Petrobras	PETR3 BZ	-0.0718	-0.0822	-0.0896	-0.1178
		PETR4 BZ	-0.0720	-0.0825	-0.0877	-0.1146
Russia	Rosneft	ROSN RX	-0.0645	-0.0739	-0.0724	-0.0962
India	ONGC	ONGC IN	-0.0537	-0.0615	-0.0616	-0.0784
		ONGC IB	-0.0530	-0.0608	-0.0609	-0.0777
		ONGC IS	-0.0537	-0.0616	-0.0616	-0.0784
	Oil India	OINL IN	-0.0418	-0.0479	-0.0481	-0.0607
		OINL IB	-0.0412	-0.0472	-0.0473	-0.0601
		OINL IS	-0.0418	-0.0479	-0.0481	-0.0607
	GAIL	GAIL IN	-0.0571	-0.0655	-0.0641	-0.0823
		GAIL IB	-0.0566	-0.0649	-0.0633	-0.0815
		GAIL IS	-0.0571	-0.0654	-0.0639	-0.0820
		CAIRN	CAIRN IN	-0.0614	-0.0703	-0.0738
		CAIRN IS	-0.0614	-0.0703	-0.0738	-0.0958
China	CNPC	135 HK	-0.0586	-0.0672	-0.0687	-0.0907
	SINOPEC	386 HK	-0.0548	-0.0628	-0.0652	-0.0853
	CNOOC	883 HK	-0.0568	-0.0651	-0.0672	-0.0875

Table 3.4. Values at Risk (VaR) and Conditional Values at Risk (CVaR). All both measures obtained, VaR and CVaR, are higher respectively, when a NIG distribution is considered, than the values calculated when normality is assumed. Source: own elaboration in R Software with data from Bloomberg.

3.5 Conclusions

The study provided in this chapter evidence that in equities of BRIC oil companies, the CVaR model built using NIG distribution provides the biggest estimate of potential losses, compared to the CVaR estimate assuming normality or a VaR considering either NIG or Normal distribution. The results were obtained by computing both VaR and CVaR for the assets of oil producing companies in Brazil, Russia, India and China in periods of volatility between 2004 and 2017. For VaR and CVaR estimates based on NIG distribution, we obtained the parameters describing this function and we confirmed that it adjusted the empirical data of the equity returns reasonably.

This represents a benefit by calculating, with less underestimation, the risks that respond to the extreme price fluctuations associated with the oil sector, under a given level of confidence.

The analysis of VaR and CVaR performance assuming heavier tails distributions, such as Normal Inverse Gaussian distribution is required to explain the stronger movements in volatility of oil equities in times of uncertainty. In the next chapter, we explore the statistical relationship between oil variables and stock market variables of BRIC countries, in order to find some diversification opportunities in a portfolio optimization context.

Chapter 4. Diversification opportunities in oil and stock indexes of BRIC countries

4.1. Introduction

There are presently many studies dealing with the connection between the oil price and economic variables, including: Akram, 2004; Chen and Chen, 2007; Cunado and Perez de Gracia, 2005; Davis and Haltiwanger, 2001; Hamilton, 1983,2003; Hamilton and Herrera, 2004; Hooker, 2002; Huang & Guo, 2007; Lee, Lee and Ratti, 2001; Lee and Ni, 2002; Nandha and Hammoudeh, 2007, among others.

Due to the presence and development of oil financial instruments such as futures and derivatives in the last decades, the commodity adopted financial characteristics that led to the analysis of its relations with the financial market (Alquist and Gervais, 2013; Ma, Ji and Pan, 2019; Zhang and Wang, 2015).

The Hamilton's seminal work in 1983, where it is argued that most financial crises in the United States are preceded by a significant increase in the price of oil, establishes a line of research that continues to this day to be very fruitful.

Even though authors such as Fang and You (2014) pointed out that there are fewer studies which have studied the relationship between the price of oil and financial markets, there is a significant literature that shows us the interest and relevance of the link between these variables. As Lin and Su (2020) point out, the interaction between oil and stock markets has become one of the most important issues in the academic field of finance. In their study Lin and Su counted 1342 papers in English, published since 1980 and divided the production of research into two stages: Phase 1 or flourishing, from 1985 to 2007, with less than 25 publications per year, and with a superficial level of analysis in the oil-stocks issue. Phase 2 or phase of development from 2007 to 2019, which presents the studies with various

fundamental theories and research methods, and represents more than 200 publications in 2019 and with a continuous increasing tendency.

There are several works and with various methodologies that have studied, and continue to do so, the interaction of oil and financial markets. Lin and Su pointed some subdivisions in this topic, including the spillover effect of crude oil price and stock market (Du and He, 2015; Ji, Liu, Zhao and Fan, 2018; Wen, Wang, Ma and Wang, 2019); the relations between oil price and stocks from the industry level or national level (Badeeb and Lean, 2018; Pal and Mitra, 2019); and the impact of structural oil shocks on stock markets (Apergis and Miller, 2009; Bastianin, Conti and Manera 2016; Kang, Ratti and Yoon, 2015).

There is also some work on the subject of oil-stock oriented to BRIC emerging economies. Naming just some of them, Tiwari, et al, 2019; Fang and You, 2014; Aloui, Nguyen, and Njeh, 2013.

The original term BRIC composed by the first letter of the 4 countries: Brazil, Russia, India and China, was created by the English economist Jim O'Neill to designate emerging economies that, according to O'Neill's forecasts, would exceed in 2050 the World's main economies. The power of these countries lies both on the size of its GDP, population and territory, as well as in the wealth of its natural and energy resources, its continued growth and in the volume of its foreign trade (O'Neill, Jim, 2001)

According to the International Monetary Fund, the Nominal Gross Domestic Product in 2018 of the four countries represented in the BRIC, was near to 24% of world GDP, 95.7% of the GDP of the United States of America; and according to the World Bank, the block has more than 40% of the world's population and together more than 25% of the planet's surface, according to data from the Central Intelligence Agency (CIA) of the United States.

Therefore, the objective of this chapter focuses on the study of the relationship between the economies of BRIC block, specifically on the movement of stock indexes and crude oil mixes in those countries, in order to find some investment opportunities through the Markowitz theory (Markowitz, 1952), making a portfolio with main indexes and crude oil mixes of the BRIC countries.

Let us not forget that in previous chapters we represented empirical data series as random vectors using NIG distribution parameters which were derived from the same empirical series in a univariate approach. In this chapter, we departed from a univariate approach and then moved to the multivariate case, excluding the independence assumption and allowing the construction of portfolios with related variables.

The remainder of this chapter is structured as follows: In section 4.2 we discuss the Multivariate Normal Inverse Gaussian distribution in order to find a portfolio that maximizes the relation return–risk with the variables mentioned before using that distribution. In sections 4.3 and 4.4 we explain the methodology applied to the data and results obtained. We present the conclusions in section 4.5.

4.2. Literature review

Taking up some previous concepts, the Generalized Hyperbolic (GH) family distributions, specifically the Normal Inverse Gaussian Distribution (NIG) has been a notable option to model the distribution of return of numerous financial instruments, such as commodities, equities, options and so on. Nevertheless, empirical studies frequently focus on univariate cases. In this paper we have extended previous studies (Núñez et al, 2018; Sánchez-Ruenes et al, 2020) to model returns of BRIC Indexes and Crude Oil prices using the Multivariate NIG distribution.

In the paper of 1977, Barndorff-Nielsen proposed the family of the Generalized Hyperbolic, which is considered in the literature as a seminal paper. In

particular, one of the main characteristics of the NIG is its ability to adjust heavy tails, and a second property is that it is closed under convolution.

On the other hand, the Multivariate Normal Inverse Gaussian (MNIG) is a mean-variance mixture combination of a Gaussian distribution with an Inverse Gaussian distribution (Oigard and Hanssen, 2002).

The Multivariate Normal distribution is defined as $X = (X_1, \dots, X_d)^t$ if $X = \mu + AZ$, where $Z = (Z_1, \dots, Z_k)^t$ is an independent identically distributed vector of random variables with univariate Standard Normal distribution, $A \in \mathbb{R}^{d \times k}$ and $\mu \in \mathbb{R}^d$. With this specification, we can express the multivariate distribution with the following density (Mardia et al, 1979):

$$f(x) = \frac{1}{(2\pi)^{\frac{d}{2}} |\Sigma|^{\frac{1}{2}}} \exp\left(-\frac{1}{2}(x - \mu)^t \Sigma^{-1}(x - \mu)\right)$$

where Σ is a semidefinite positive and non-singular matrix representing the covariance. Once the parameters are determined, Press et al. (1992) proposed an algorithm to simulate random vectors that follows that same distribution by employing the Cholesky decomposition of Σ .

If we consider a random vector, expressed as $X = \mu + \sqrt{W}AZ$ where $W \geq 0$ is a random variable scalar. Assuming W has a Generalize Inverse Gaussian distribution with density

$$f(x) = \frac{1}{k_\lambda(\chi, \psi)} x^{\lambda-1} \exp\left[-\frac{1}{2}(\chi x^{-1} + \psi x)\right]$$

where k_λ is the Bessel function of third kind defined as:

$$k_\lambda = \int_0^\infty x^{\lambda-1} \exp\left[-\frac{1}{2}(\chi x^{-1} + \psi x)\right] dx$$

then, the variance mixture of distributions has the joint density of:

$$f(x) = \frac{(\sqrt{\chi\psi})^{-\lambda} \psi^{\frac{d}{2}} K_{\lambda-\frac{d}{2}}(\sqrt{\chi + (x - \mu)^t \Sigma^{-1}(x - \mu)})}{(2\pi)^{\frac{d}{2}} |\Sigma|^{\frac{1}{2}} k_{\lambda}(\sqrt{\chi\psi}) \left(\sqrt{\chi + (x - \mu)^t \Sigma^{-1}(x - \mu)}\right)^{\frac{d}{2}-\lambda}}$$

which is a special case of the Generalized Hyperbolic case of:

$$f(x) = \frac{(\sqrt{\chi\psi})^{-\lambda} \psi^{\lambda} (\psi + \gamma^t \Sigma^{-1} \gamma)^{\frac{d}{2}-\lambda} K_{\lambda-\frac{d}{2}}(\sqrt{\chi + (x - \mu)^t \Sigma^{-1}(x - \mu)}) \exp((x - \mu)^t \Sigma^{-1} \gamma)}{(2\pi)^{\frac{d}{2}} |\Sigma|^{\frac{1}{2}} k_{\lambda}(\sqrt{\chi\psi}) \left(\sqrt{\chi + (x - \mu)^t \Sigma^{-1}(x - \mu)}\right)^{\frac{d}{2}-\lambda}}$$

With this expression, particular cases may be extended, for the purpose of this paper, the Multivariate NIG distribution is obtained when $\lambda = -0.5$ which becomes close to the Hyperbolic distribution but with heavier tails (McNeil, Frey, and Embrechts, 2005).

It should be recalled that the applications and the potential of the family of the Generalized Hyperbolic are vast. Andresen, A., Koekebakker, S., & Westgaard, S. (2010) presented a discrete random-field model for forward prices driven by the Multivariate Normal Inverse Gaussian distribution. Likewise, O'Hagan, A., Murphy, T. B., Gormley, I. C., McNicholas, P. D., & Karlis, D. (2016) establish that a combination of MNIG distributions is a great alternative to a mixture of Gaussian distributions as a tool to develop a model of clustering.

On the other hand, Eberlein, E. & Prause, K. (2012) confirm that the Generalized Hyperbolic family allows a better description of the returns of financial assets. In particular, these authors derive a formula for pricing options for models that include the Generalized Hyperbolic using the Esscher transformation as a possibility to determine prices in an incomplete market.

4.3. Methodology

In this chapter we worked with daily data obtained from Bloomberg of market indexes of BRIC economies: IBOV from Brazil, RTSI from Russia, NIFTY from India and SHCOMP from China. We also studied the main crude oil mixes in those countries, except in India, owing to the unavailability of data. The oil mixes included in this study were: Marlim from Brazil, Urals from Russia and Daqing from China, and the data were obtained from Platts.

The period of time selected for the stock and oil prices series was from 2004 to 2019, owing to include structural oil price breakpoints and important events of oil market in the current millennium: such as the beginning of oil price surge until financial crisis in 2005 and the oil price rebound in 2012 until 2014. The bubble periods identified by Perifanis (2019) are included in the studying interval as well. Finally, the decline in oil prices since 2014 was considered due to oversupply and demand decreasing in oil markets, besides Saudi Arabia's decision to maintain its oil production levels.

In order to prove the stationarity, we applied the Augmented Dickey-Fuller test with Schwarz information criterion and the Phillips-Perron test with Bartlett kernel spectral estimation method, where:

H_0 : The series has a unit root.

H_1 : The series is stationary.

The p-values were the following:

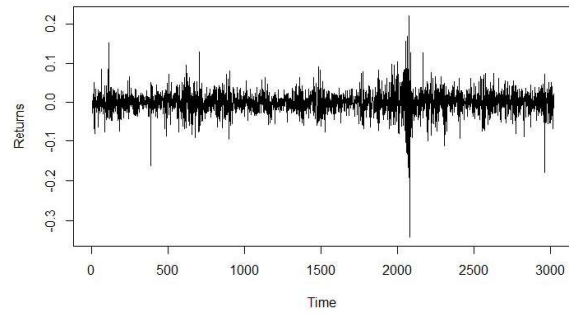
Indexes and prices	p-value	
	DF	PP
IBOV	0.5144	0.6076
RTSI	0.4908	0.4929
NIFTY	0.5531	0.5248
SHCOMP	0.4987	0.5245
Marlim	0.5321	0.5037
Urals	0.6493	0.6082
Daqing	0.6622	0.5663

Table 4.1. p-values for BRIC indexes and oil prices obtained through Augmented Dickey-Fuller test with Schwarz information criterion (DF) and Phillips-Perron with Bartlett kernel spectral estimation method (PP) for indexes and prices.

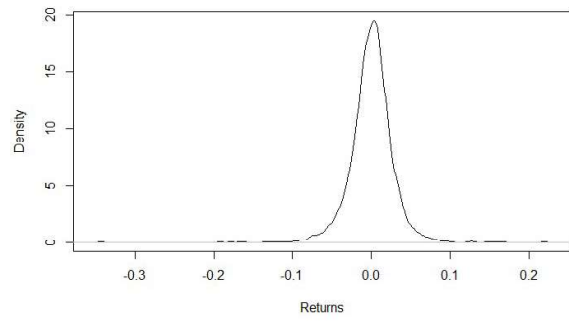
In all cases p-values were greater than .01, i.e. we did not reject the null hypothesis, therefore, the series were not stationary. Subsequently, we calculated the logarithmic returns for each index and price series.

The following graphics represent the logarithmic returns of the series, where we can observe mean reversion:

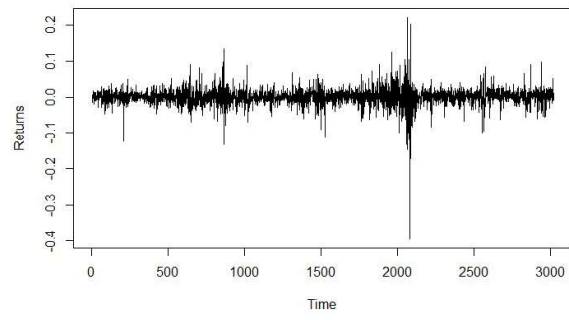
IBOV



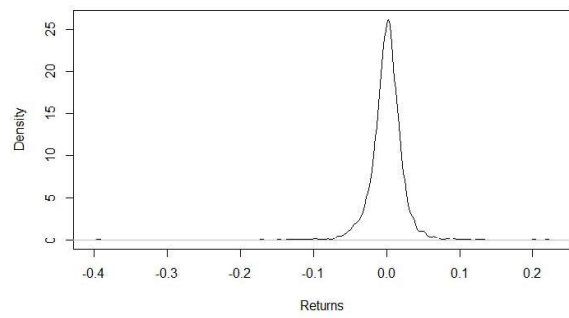
IBOV



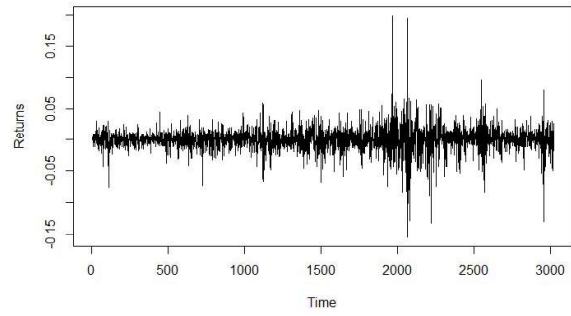
RTSI



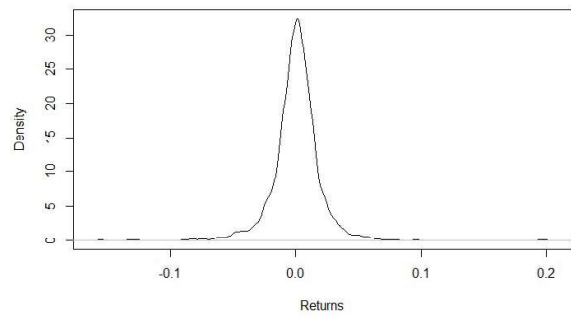
RTSI



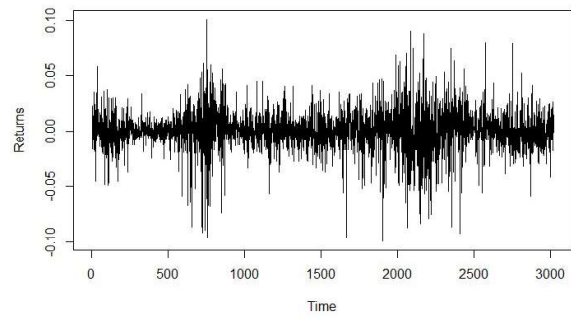
NIFTY



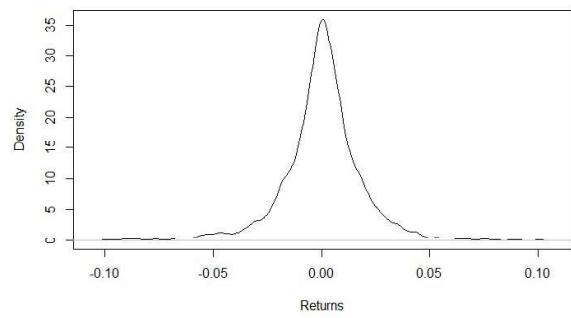
NIFTY



SHCOMP



SHCOMP



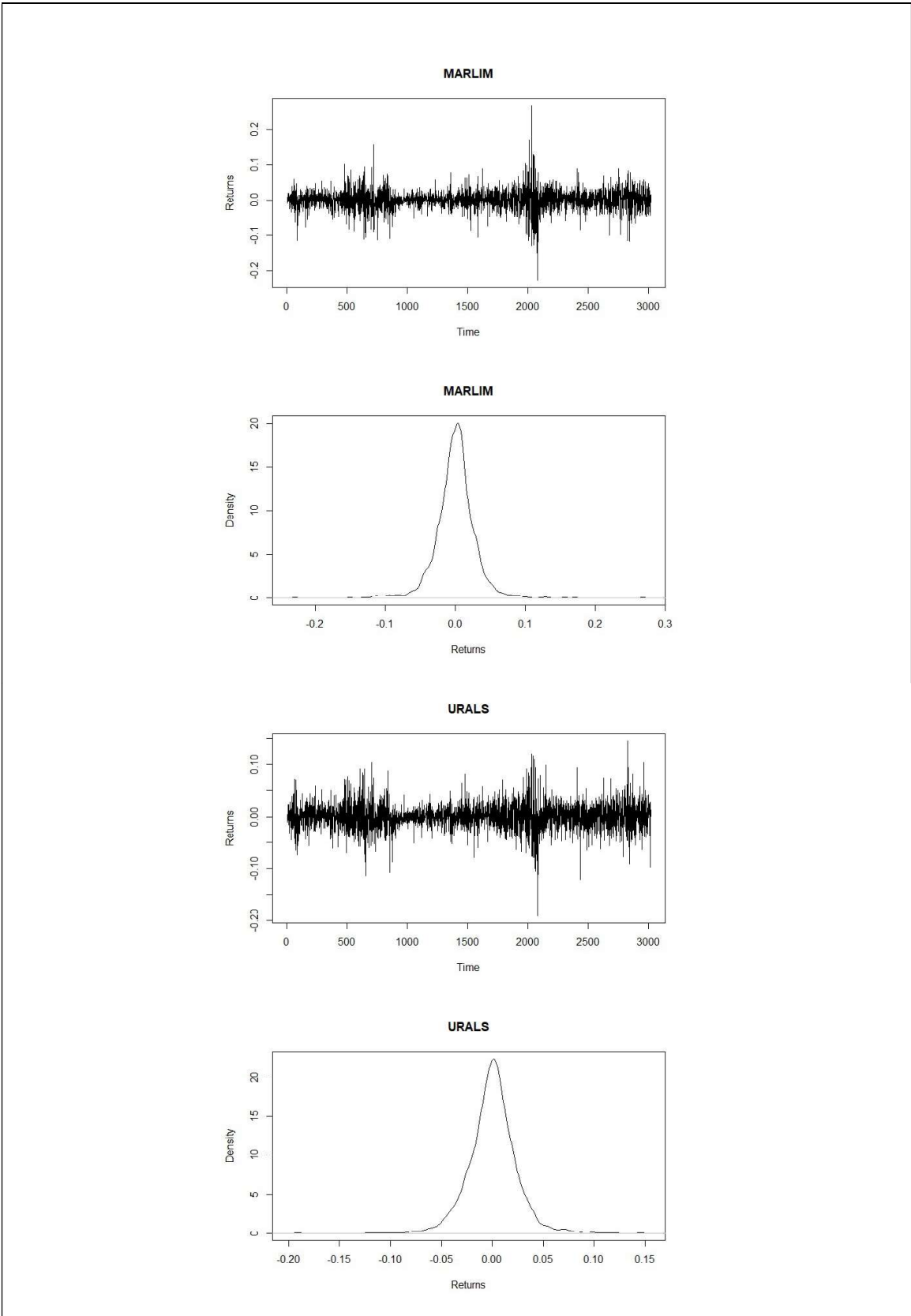


Figure 4.1. Logarithmic returns for BRIC indexes and crude oil mixes and their respective probability density functions. Source: own elaboration in R Software with data from Bloomberg.

Most of the graphs in figure 1 show high volatility in observations close to 500 and 2000, which correspond to the last months of 2014 and 2008, respectively, and represent the fall in prices due to excess supply and weakening demand, as well as the establishment of a level of production by Saudi Arabia, in 2014, and the financial crisis derived from the depreciation of the subprime mortgage market in the United States, in 2008.

We re-evaluated the stationarity using the logarithmic returns of the series and we obtained for every series a p-value less than 0.0001, that means the null hypothesis is rejected, that is, the series are stationary. Table 4.3 presents the descriptive statistics for those series.

	n	mean	variance	skewness	kurtosis
IBOV	3022	0.000353703	0.000773655	-0.6705952	13.157355
RTSI	3022	0.000253878	0.00058478	-1.38471987	31.182532
NIFTY	3022	0.000454259	0.000359029	-0.02677346	13.064744
SHCOMP	3022	0.000310674	0.000342458	-0.40594055	4.142914
MARLIM	3022	0.000304422	0.000780304	-0.0322947	7.423756
URALS	3022	0.000294209	0.000588745	-0.02802895	4.068729
DAQING	3022	0.000252584	0.000624716	-0.0513526	5.094316

Table 4.2. Descriptive statistics for logarithmic returns of BRIC indexes and oil mixes. Source: own elaboration with data from Bloomberg.

Taking into account the stylized facts, mainly the presence of high levels of kurtosis, this could lead intuitively to the empirical adjustment of a distribution with semi-heavy tails. In this case, the proposal is the Normal Inverse Gaussian distribution, as it has praiseworthy properties for financial analysis (Barndorff-Nielsen, 1977).

In a previous research (Núñez et al, 2018) and as detailed in chapter 2, the returns of BRIC indexes: IBOV, RTSI, NIFTY and SHCOMP, adjusted significantly in a univariate approach, to NIG distribution, even during periods of economic crisis. In this chapter, we first verified if NIG distribution also fits the series of crude oil returns: MARLIM, URALS and DAQING. The results confirm that NIG distribution reasonably adjusted to the empirical data in a univariate case for those crude oil mixes. However, the world is connected through different variables, including the financial variables, so it makes sense to look for a multivariate distribution.

For this research, we created some vectors with returns of BRIC indexes and crude oil mixes, and then we confirmed their adjustment to a Multivariate Normal Inverse Gaussian distribution using goodness-of-fit test through Cramer criteria with significance levels of 0.95 and 0.99 (Baringhaus and Franz, 2004).

For every proposed vector, we did not reject the null hypothesis for NIG distribution – H_0 : the empirical data follow a Multivariate NIG distribution–. We used R software to obtain these results, as well as the later ones.

Vector	Dimension	MNIG Distribution	
		Cramer Test significance 0.95	Cramer Test significance 0.99
Daqing, Urals, Marlim, SHCOMP, IBOV, NIFTY	6	0.7052947	0.7112887
Daqing, Urals, Marlim, SHCOMP, RTSI, IBOV	6	0.1138861	0.1038961
Daqing, Marlim, IBOV, NIFTY, SHCOMP	5	0.7102897	0.692307
Daqing, Marlim, IBOV, SHCOMP	4	0.4215784	0.4155844
Daqing, Urals, RTSI, SHCOMP	4	0.4035964	0.4115884
Marlim, Urals, IBOV, RTSI	4	0.3416583	0.3606394

Table 4.3. p-values for every proposed vector using the Cramer test considering a NIG distribution. Elaborated by authors with data from Bloomberg.

4.4. Results

The results from the previous tests indicate that the logarithmic returns from the oil prices and indexes altogether adjust statistically, to a Multivariate NIG distribution.

We observed greater correlations to the oil mix and stock index of Russia. We could intuit, excluding Russia, that it is possible to create a diversified portfolio and we proved it through an empirical study.

In this regard, we created a portfolio for every vector, according to the theory of Markowitz (1952), finding the optimal weights (ω 's), as seen in table 4.4, for each investment instrument, in our case market indexes and commodities, in order to obtain a higher return with a lower risk.

No.	PORTFOLIO	ω_{Daqing}	ω_{Urals}	ω_{Marlim}	ω_{SHCOMP}	ω_{RTSI}	ω_{IBOV}	ω_{NIFTY}
1	Daqing, Urals, Marlim, SHCOMP, IBOV, NIFTY	0.08657	0.04096	0.05774	0.25627	-	0.05879	0.49968
2	Daqing, Urals, Marlim, SHCOMP, RTSI, IBOV	0.13106	0.08274	0.08798	0.47673	0.04694	0.17455	-
3	Daqing, Marlim, SHCOMP, IBOV, NIFTY	0.09874	-	0.07767	0.25804	-	0.06038	0.50517
4	Daqing, Marlim, SHCOMP, IBOV	0.16757	-	0.13729	0.50737	-	0.18777	-
5	Daqing, Urals, SHCOMP, RTSI	0.11166	0.18530	-	0.55792	0.14512	-	-
6	Urals, Marlim, RTSI, IBOV	-	0.27662	0.11611	-	0.24402	0.36325	-

Table 4.4. Asset weights obtained for suggested portfolios 1 to 6. Elaborated by authors with data from Bloomberg.

Therefore, we employed the relation return – standard deviation to show the efficiency of every portfolio, obtaining the following ratios:

No.	PORTFOLIO	μ/σ
1	Daqing, Urals, Marlim, SHCOMP, IBOV, NIFTY	0.02912
2	Daqing, Urals, Marlim, SHCOMP, RTSI, IBOV	0.02212
3	Daqing, Marlim, SHCOMP, IBOV, NIFTY	0.02908
4	Daqing, Marlim, SHCOMP, IBOV	0.02195
5	Daqing, Urals, SHCOMP, RTSI	0.02056
6	Urals, Marlim, RTSI, IBOV	0.01660

Table 4.5. Return – standard deviation relationship for portfolios 1 to 6. Elaborated by authors with data from Bloomberg.

It should be noted that the variance–covariance matrix plays a fundamental role in portfolio optimization theory (Markowitz, 1952) and in the case of the family of generalized hyperbolic distributions (GH), it is calculated with the skewness (γ) and dispersion (Σ) parameters, through the variance equation according to McNeil, Frey, and Embrechts (2005). In Appendix A, we present the variance–covariance matrixes obtained through the maximum likelihood criterion with a NIG distribution for each portfolio.

We also obtained the following curves as a result of the optimization of the return – standard deviation relationship for those portfolios.

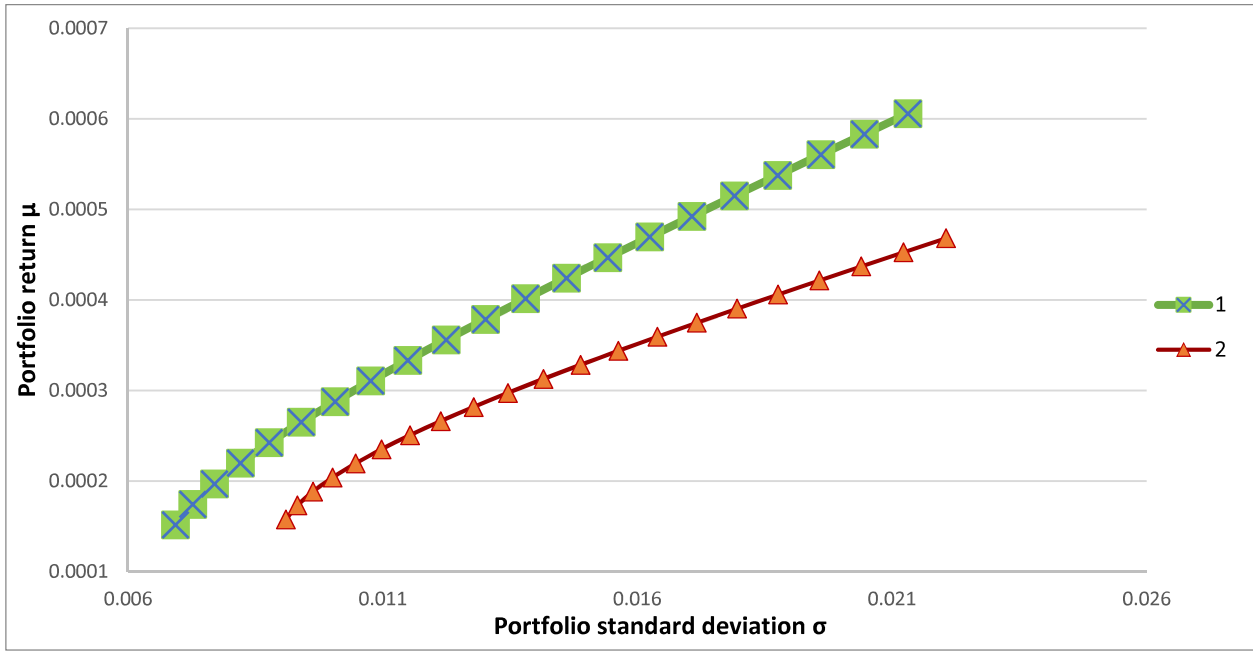


Figure 4.2. Efficient frontier curves for portfolios 1 and 2 described in table 4.4. Elaborated by authors with data from Bloomberg.

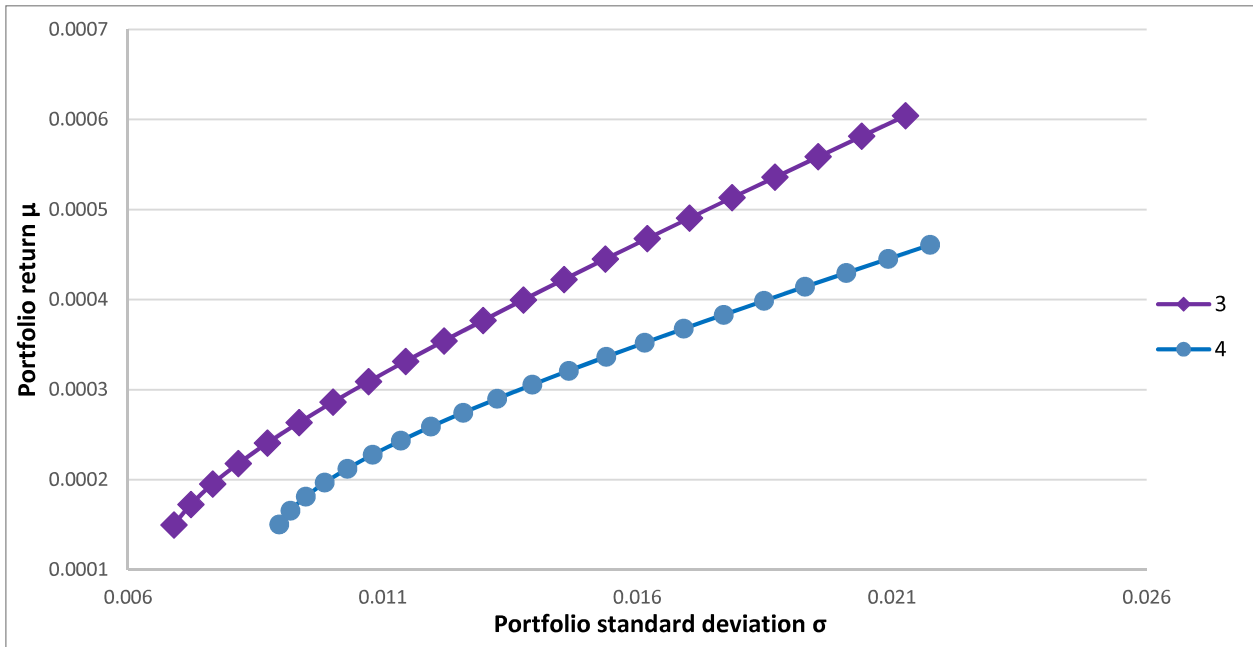


Figure 4.3. Efficient frontier curves for portfolios 3 and 4 described in table 4.4. Elaborated by authors with data from Bloomberg.

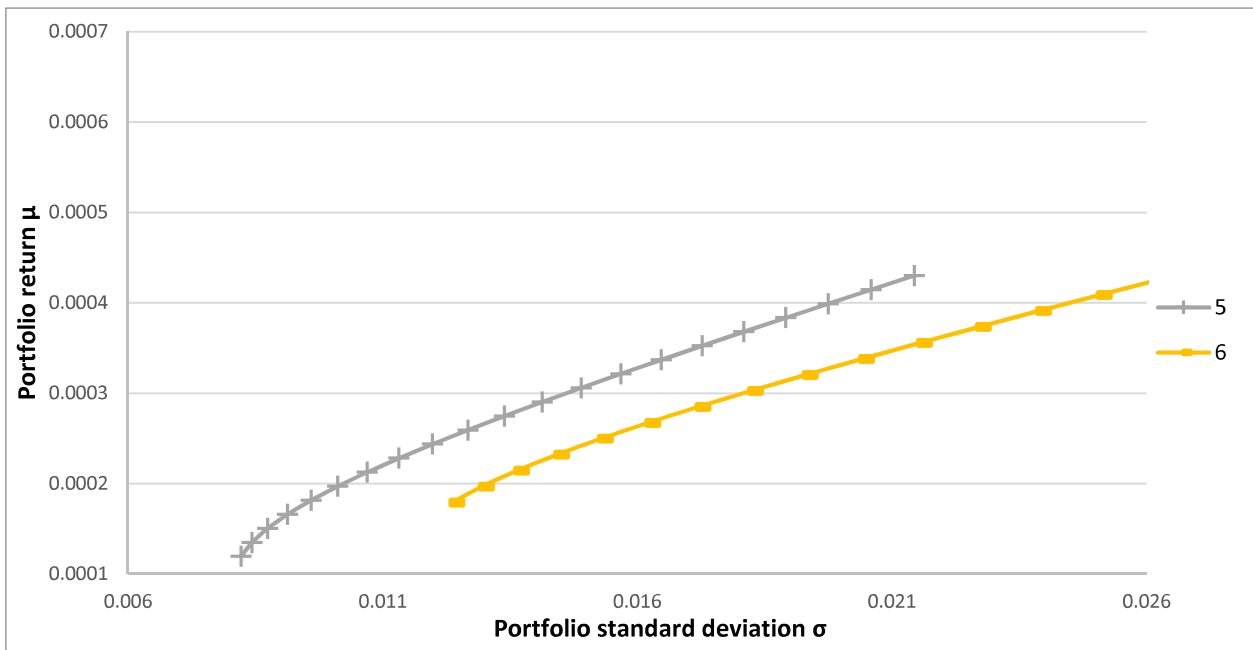


Figure 4.4. Efficient frontier curves for portfolios 5 and 6 described in table 4.4. Elaborated by authors with data from Bloomberg.

The curves are presented in three different graphs, since the curves of portfolio 1 (oil mixes from China, Russia and Brazil; and indexes from China, Brazil and India) and portfolio 3 (oil mixes from China and Brazil and indexes of China,

Brazil and India); as well as portfolio 2 (oil mixes and indexes from China, Russia and Brazil) and portfolio 4 (oil mixes and indexes from China and Brazil) would be almost overlapping, with a difference of less than 0.14% and 0.8%, respectively. Comparing the curves of portfolios 1 and 3, we observe that the inclusion of the Russian mix does not represent a significant increase in the efficiency of the portfolio. Similarly, if we compare curves 2 and 4, we also find that, incorporating the Russian mix and index, it does not have a significant impact on portfolio efficiency.

On the other hand, the curves of portfolios 1 and 3 are above the pair of curves 2 and 4, revealing that the incorporation of the Hindu index contributes more than 32.6% to the efficiency of the portfolio. However, the increase greater than 6.7% in the curves of portfolios 2 and 4 with respect to curve 5, responds to the inclusion of the Brazilian mix and index. Finally, the growth of more than 23.8% in the efficiency of portfolio 5 compared to 6, is due to the exchange of instruments from Brazil for those from China.

In this sense, the instruments that had the greatest impact on portfolio efficiency (highest return with the lowest risk) are, in this order: NIFTY index (India); Daqing mix and the SHCOMP index (China); and Marlim mix and IBOV index (Brazil). On the opposite side, the contribution of Urals mix and RTSI index (Russia) represents less than 1% in the return-standard deviation ratio of the portfolio.

4.5 Conclusions

In this chapter, a multivariate approach for portfolios construction is employed using the market indexes IBOV, RTSI, NIFTY and SHCOMP, and crude oil mixes of Marlim, Urals and Daqing. Employing a Multivariate Normal Inverse Gaussian test for the logarithmic returns it was possible to determine that Normal Inverse Gaussian assumptions hold even in the multivariate case fitting the empirical data appropriately. In order to make efficient portfolios using Multivariate Normal Inverse

Gaussian distributions, the Markowitz theory was employed for some combinations of the oil mixes and market indexes of BRIC. The results show a no-significant diversification effect of including Russia's index and/or oil mix in the Portfolio. From an investment optimization perspective, the results of this work show the possibility to diversify with market indexes and oil mixes of India, China and Brazil, even in periods including extreme and crisis events, showing that the greatest impact on portfolio efficiency is due to the Hindu index and the Chinese mix and index.

Chapter 5. Conclusions

Summing up the study presented in each chapter, it was possible to model, in the framework of BRIC economies, the market index returns, as well as the returns of crude oil mixes, adjusting the empirical data series to a distribution of the Generalized Hyperbolic family: NIG distribution.

Fitting financial data to a semi-heavy tailed parametric distribution is highly convenient, since it is possible to capture extreme values in periods of instability or crisis, without forgetting the qualities of the NIG distribution class such as the feasibility to compute the moments and parameters with closed solutions and being closed under convolution.

In this way and proving the precision of the adjusted series, we compared the risk measures Value at Risk and Conditional Value at Risk (Expected Shortfall) to verify the estimates of these values assuming normality and using a distribution that better represents the study variables.

The results were consistent in all cases: the risk measures of considered assets were underestimated under the assumption of normality, resulting in differences that ranged from 11 % to 43 %; that is, the values obtained when using NIG distribution to calculate the VaR and CVaR were higher in the proportions indicated with respect to the use of Normal distribution.

Finally, returning to the *purpose* research question, we oriented the relationship of the interest variables in this study, as well as risk management, to the construction of efficient portfolios through the Markowitz optimization theory, made

up of intuitively related variables that form part of the so-called BRIC block in periods of volatility.

The results showed the possibility of creating optimal investment portfolios obtaining for this purpose the returns and variances considering NIG distribution. Furthermore, it was proved that the efficiency of the portfolios increases with the inclusion of the Hindu (NIFTY) and Chinese (SHCOMP) indexes, as well as with the Chinese crude oil mix (Daqing). However, the inclusion of the Russian index (RTSI) and / or Russian mix (Urals) does not have a significant effect on the efficiency of the portfolio.

Appendixes

Appendix A. Variance–covariance matrixes calculated using R software through the maximum likelihood criterion of a NIG distribution for every portfolio of table 4.4.

A.1 Portfolio 1: Daqing, Urals, Marlim, SHCOMP, IBOV, NIFTY

Variance - Covariance Matrix Portfolio 1	Daqing	Urals	Marlim	SHCOMP	IBOV	NIFTY
Daqing	5.90E-04	2.73E-04	1.33E-04	5.62E-05	7.05E-05	4.87E-05
Urals	2.73E-04	5.97E-04	4.31E-04	4.95E-05	1.58E-04	7.31E-05
Marlim	1.33E-04	4.31E-04	7.13E-04	5.26E-05	2.28E-04	7.77E-05
SHCOMP	5.62E-05	4.95E-05	5.26E-05	3.33E-04	7.94E-05	7.78E-05
IBOV	7.05E-05	1.58E-04	2.28E-04	7.94E-05	7.43E-04	1.37E-04
NIFTY	4.87E-05	7.31E-05	7.77E-05	7.78E-05	1.37E-04	3.27E-04

Table A.1. Maximum Likelihood Variance -Covariance Matrices for portfolio 1. Elaborated by authors with data from Bloomberg

A.2 Portfolio 2: Daqing, Urals, Marlim, SHCOMP, RTSI, IBOV

Variance - Covariance Matrix Portfolio 2	Daqing	Urals	Marlim	SHCOMP	RTSI	IBOV
Daqing	5.92E-04	2.74E-04	1.36E-04	5.59E-05	1.54E-04	6.92E-05
Urals	2.74E-04	5.97E-04	4.30E-04	4.83E-05	2.12E-04	1.59E-04
Marlim	1.36E-04	4.30E-04	7.15E-04	5.20E-05	1.95E-04	2.30E-04
SHCOMP	5.59E-05	4.83E-05	5.20E-05	3.37E-04	8.38E-05	7.82E-05
RTSI	1.54E-04	2.12E-04	1.95E-04	8.38E-05	4.89E-04	2.36E-04
IBOV	6.92E-05	1.59E-04	2.30E-04	7.82E-05	2.36E-04	7.45E-04

Table A.2. Maximum Likelihood Variance -Covariance Matrices for portfolio 2. Elaborated by authors with data from Bloomberg

A.3 Portfolio 3: Daqing, Marlim, SHCOMP, IBOV, NIFTY

Variance - Covariance Matrix Portfolio 3	Daqing	Marlim	SHCOMP	IBOV	NIFTY
Daqing	6.10E-04	1.33E-04	5.57E-05	7.02E-05	4.83E-05
Marlim	1.33E-04	7.35E-04	5.26E-05	2.30E-04	7.74E-05
SHCOMP	5.57E-05	5.26E-05	3.32E-04	7.89E-05	7.82E-05
IBOV	7.02E-05	2.30E-04	7.89E-05	7.45E-04	1.36E-04
NIFTY	4.83E-05	7.74E-05	7.82E-05	1.36E-04	3.26E-04

Table A.3. Maximum Likelihood Variance -Covariance Matrices for portfolio 3. Elaborated by authors with data from Bloomberg

A.4 Portfolio 4: Daqing, Marlim, IBOV, SHCOMP

Variance - Covariance Matrix Portfolio 4	Daqing	Marlim	IBOV	SHCOMP
Daqing	6.06E-04	1.36E-04	7.14E-05	5.52E-05
Marlim	1.36E-04	7.34E-04	2.37E-04	5.28E-05
IBOV	7.14E-05	2.37E-04	7.55E-04	7.84E-05
SHCOMP	5.52E-05	5.28E-05	7.84E-05	3.30E-04

Table A.4. Maximum Likelihood Variance -Covariance Matrices for portfolio 4. Elaborated by authors with data from Bloomberg

A.5 Portfolio 5: Daqing, Urals, SHCOMP, RTSI

Variance - Covariance Matrix Portfolio 5	Daqing	Urals	SHCOMP	RTSI
Daqing	6.07E-04	2.83E-04	5.79E-05	1.57E-04
Urals	2.83E-04	6.15E-04	4.84E-05	2.16E-04
SHCOMP	5.79E-05	4.84E-05	3.37E-04	8.38E-05
RTSI	1.57E-04	2.16E-04	8.38E-05	4.96E-04

Table A.5. Maximum Likelihood Variance -Covariance Matrices for portfolio 5. Elaborated by authors with data from Bloomberg

A.6 Portfolio 6: Urals, Marlim, RTSI, IBOV

Variance - Covariance Matrix Portfolio 6	Urals	Marlim	RTSI	IBOV
Urals	6.03E-04	4.40E-04	2.12E-04	1.60E-04
Marlim	4.40E-04	7.30E-04	2.00E-04	2.31E-04
RTSI	2.12E-04	2.00E-04	4.77E-04	2.34E-04
IBOV	1.60E-04	2.31E-04	2.34E-04	7.32E-04

Table A.6. Maximum Likelihood Variance -Covariance Matrices for portfolio 6. Elaborated by authors with data from Bloomberg

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Eduardo Sánchez Ruenes holds a Bachelor's degree in Mechanical Engineering and Administration from Tecnológico de Monterrey. He received from the same institution a M.Sc. degree with major in Quality and Productivity.

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