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Hedging and optimization of energy asset portfolios

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Dedication

To my wife Luz María and my children. To the memory of my parents.

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Hedging and optimization of energy asset portfolios

By

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Abstract

This thesis includes three papers on hedging and optimization of energy asset portfolios. The regulatory scheme for natural gas (NG) prices in Mexico is described and the behavior of international and domestic gas prices and the peso-dollar exchange rate from January 2012 to June 2017 is analyzed. Statistical analysis reveals that volatility in the daily growth rate of international NG prices exceeds daily fluctuations in the exchange rate. Based on this knowledge, the behavior of First-Hand Sales prices is modeled, and two price hedging strategies are proposed, one through futures and the other through swaps. Given how First-Hand Sales prices are calculated, the optimal futures hedge should consider the acquisition of gas futures one and two months prior as well as contemporary exchange rate futures.

Based on a hedging strategy that includes NG futures and using an MGARCH VCC (MGARCH stands for Multivariate Generalized Autoregressive Conditional Heteroskedasticity and VCC for Variable Conditional Correlation) model, conditional variances were estimated with lags of 20 and 40 days between the prices of NG Futures. Dynamic hedges of NG were calculated assuming theoretical futures prices of the US dollar in Mexican pesos. By applying backtesting, it was found that the forecasts of optimal hedge ratios improve with short prediction periods and proximate observed data. The dynamic hedging model proposed can be extended to other fuel markets. The importance of hedging NG prices derives from the size of the market and the extent of the risks to which the market participants are exposed.

Using the share price data of six energy companies of Latin America and other regions and two crude oil futures, this thesis proposes the integration of hedging portfolios and the calculation of efficient frontiers under different risk measures. The original financial series are transformed into new ones to improve the risk measurement. With the new series obtained through simulation with the support of the Extreme Value Theory and t-copulas, different conditional risk measures are calculated. These conditional risk measures are used to solve the hedging and optimization problems. Non-linear integer programming techniques are used to obtain these solutions. The programming codes used to generate the new series and solve the hedging and optimization problems are presented in the annexes. Due to the economic value and the volatility of energy markets, hedging strategies and portfolio optimization are useful tools to reduce non-desired levels of risk or to avoid unnecessary costs.

Contents

Abstract	vi
Introduction	13
1.1. Motivation	13
1.2. Problem Statement and Context	13
1.3. Research Questions	14
1.4. Solution Overview	15
Strategies for Hedging First-Hand Natural Gas Prices in Mexico	17
2.1. Introduction	17
2.2. Conceptual Framework	17
2.2.1. Background	17
2.2.2. Regulation of Natural Gas Prices	19
2.3. Methodology	19
2.4. Analysis	20
2.4.1. Statistical overview of the price series	20
2.4.2. Characteristics of the First Hand-Selling Price in Reynosa	26
2.5. Hedging strategy with futures	28
2.5.1. The use of futures contracts for hedging	28
2.5.2. Other Hedges	35
2.6. Conclusions	36
2.7. Post Data	37
2.7.1. Effectiveness of the hedging model using backtesting	37
2.7.2. Operational suitability for the hedging of the Futures with one and two-month lags	39
Dynamic hedging of prices of Natural Gas in Mexico	41
Abstract	41
3.1. Introduction	41
3.2. The Mexican NG regulation.	43
3.3. State of the Art	43
3.4. Methodology	46
3.4.1. Use of future contracts as hedging	46
3.4.2. Multivariate GARCH VCC model	48
3.5. Data and Results	49
3.5.1. Data	49
3.5.2. The simple hedging strategy	53
3.5.3. Hedging under the MGARCH VCC model	55
3.5.4. Backtesting in the VCC model	60
3.6. Conclusions and final considerations	62

3.7	Post Data	63
3.7.1	Effectiveness of the hedging results with the backtesting	63
Hedging and optimization of energy asset portfolios		65
	Abstract	65
4.1.	Introduction	65
4.1.1.	Organization of the study	67
4.2.	State of the art	67
4.3.	Methodology	69
4.3.1.	Data adjustments and simulation	69
4.3.1.1	Nyström & Skoglund (2002) basic theoretical assumptions	69
4.3.1.2	Data adjustments and simulation process	70
4.3.2.	Risk measures	72
4.3.2.1.	Standard deviation, CVaR and MAD	73
4.3.3.	Optimization of functions	75
4.3.4.	Portfolio hedging	76
4.3.5.	Portfolio optimization	77
4.4.	Data and Results	78
4.4.1.	Data	78
4.4.2.	Results	79
4.5	A computational alternative to estimate CVaR based on MAD results	84
4.6.	Conclusions and final considerations	85
Conclusions		87
5.1.	Contributions	87
5.2.	Conclusions	87
5.3.	Future work	88
References		91
Appendixes		97
A.1.	'Stepzero'. Code in MATLAB® that generates a series of simulated returns from the historic price data of the portfolio assets.	97
A.2.	'Stephedge'. Code in MATLAB® that calculates the Efficient Frontier values, including the one with minimum risk for the hedging portfolio. It also calculates the values of alternative risk measures for optimal hedge solutions.	101
A.3.	'Stepone'. Code in MATLAB® that calculates the Efficient Frontier values for a securities portfolio under different risk measures.	105
Curriculum Vitae		107

List of tables

Table 2.1.	Statistics of the series of daily PVPM in Reynosa (US\$/MMBtu) during the period of analysis.	22
Table 2.2.	Statistics of the daily continuous growth rate of PVPM in Reynosa during the period of analysis.	22
Table 2.3.	Statistics of the daily spot price series of NG at NYMEX (USD/MMBtu) during the period of analysis.	24
Table 2.4.	Statistics of the daily continuous growth rate of NG spot prices at NYMEX during the period of analysis.	24
Table 2.5.	Statistics of the daily continuous growth rate of the peso – dollar Exchange rate during the period of analysis.	26
Table 2.6.	Statistics of the simple linear regression between the daily differences of the PVPM in dollars in Reynosa and the differences of the NG Future #1 prices at Nymex with a lag of one day during the period of analysis.	27
Table 2.7.	Correlation between the monthly growth of different international prices of NG during the period of analysis.	30
Table 2.8.	Optimal hedge ratios of the PVPM in dollars with regards to the one-month futures at the CME with lags during the period of analysis.	32
Table 2.9.	Optimal hedge ratios of the PVPM in dollars with regards to two combined Henry Hub futures during the period of analysis.	32
Table 2.10.	Optimal hedge ratios of the PVPM in pesos in Reynosa with futures of the peso-dollar exchange rate with one-month lag during the period of analysis.	34
Table 2.11.	Hedge models of the PVPM in pesos, Henry Hub futures and futures of the peso-dollar exchange rate during the period of analysis.	34
Table 2.12.	Hedge Model 1 of the PVPM in pesos, Henry Hub futures and futures of the peso-dollar exchange rate during the in-sample subperiod of analysis.	37
Table 2.13.	Hedge Model 1 of the PVPM in pesos, Henry Hub futures and futures of the peso-dollar exchange rate during the out-of-sample subperiod of analysis.	38
Table 2.14.	Comparative results of the hedge Model 1 of the PVPM in pesos, Henry Hub futures and futures of the peso-dollar exchange rate during the in-sample and out-of-sample subperiods.	39
Table 3.1.	Statistics of selective series in the study period.	51
Table 3.2.	Standard deviations observed in sub periods during the study period	52

Table 3.3.	Optimal hedge ratios of PVPM of Reynosa with three-month futures with delay.	54
Table 3.4.	PVPM hedging in dollars with two Henry Hub futures instruments with one- and two-month lag.	55
Table 3.5.	PVPM hedging models in pesos in the study period with Henry Hub futures and Mex Ps-USD exchange rate futures in the study period.	55
Table 3.6.	MGARCH VCC model of the daily variations of the PVPM Reynosa in dollars and the Henry Hub two-month futures, with lags of 20 and 40 days in the study period.	56
Table 3.7.	Relationship of conditional covariances and optimal hedge ratios h_j^* predicted for 10 days with the two-month Henry Hub futures, with lags of 20 and 40 days.	59
Table 3.8.	Statistics of the optimal h_{20}^* and h_{40}^* in-sample ratios and out-of-sample forecasts in the entire period and the estimation period in the backtesting.	61
Table 3.9.	Statistics of the optimal h_{20}^* and h_{40}^* in-sample ratios and out-of-sample forecasts for the period of the last 252 days in the backtesting.	61
Table 3.10.	Effectiveness of the optimal hedge ratios h_{20}^* and h_{40}^* forecasts for the 132-day out-of-sample period in the backtesting.	63
Table 3.11.	Effectiveness of the optimal hedge ratios h_{20}^* and h_{40}^* forecasts for the 10-day out-of-sample period in the backtesting.	64
Table 4.1.	Statistics of the daily log-returns of the energy assets during the period of analysis.	79
Table 4.2.	Correlation matrix of the original daily log-returns of the energy assets during the period of analysis.	79
Table 4.3.	Statistics of the daily log returns generated by the model for the period of analysis.	80
Table 4.4.	Correlation matrix of the daily log returns generated by the model for the period of analysis.	80
Table 4.5.	Composition and risk levels of the hedged portfolios by risk model for the period of analysis.	82
Table 4.6.	Weights, Returns, and Risk levels of Portfolios lying on the Efficient Frontiers for the period of analysis.	83

List of figures

Figure 2.1. Daily and monthly PVPM in Reynosa during the period of analysis.	21
Figure 2.2. Spot prices of NG at NYMEX in US\$/MMBtu during the period of analysis.	23
Figure 2.3. Mex Peso -- US Dollar exchange rate during the period of analysis.	25
Figure 2.4. Monthly PVPM in dollars in Reynosa vs contract prices of the Future #1 in the NYMEX during the period of analysis.	27
Figure 2.5. Diagram of the use of hedges with NG futures and dollars.	29
Figure 2.6. Correlation between the growth of the Henry Hub one-month futures and the growth of the monthly PVPM in dollars in Reynosa during the period of analysis.	31
Figure 2.7. Cross correlations between the monthly growth of the PVPM in pesos in Reynosa and the monthly growth of the peso-dollar exchange rate during the period of analysis.	33
Figure 2.8. Diagram of the exchange of flows in a swap.	35
Figure 2.9. Actual and fitted values of the growth of the monthly PVPM in pesos using Model 1 during the first months of the period of analysis.	37
Figure 3.1. Employment diagram of NG and dollar futures hedge.	47
Figure 3.2. Daily and monthly PVPM in Reynosa in the period of study.	50
Figure 3.3. Daily exchange peso-dollar rate during the study period.	51
Figure 3.4. Daily PVPM of Reynosa in dollars vs. prices of NYMEX Futures Contract # 1 in study period.	52
Figure 3.5. Correlation between the growth of the Henry Hub three-month futures and those of the PVPM in Reynosa in dollars during the study period.	53
Figure 3.6. Estimated conditional covariances between the daily growth of the PVPM in Reynosa in dollars and the daily growth of the two-month Henry Hub futures with lags of 20 and 40 days in the study period.	57
Figure 3.7. Estimated conditional covariances between the daily growth of the PVPM in Reynosa in dollars and the daily growths of the two-month Henry Hub futures, with lags of 20 and 40 days in the period of the last 90 days of the historical series and the first 10 days forecast.	58
Figure 3.8. Optimal hedge ratios h_j^* between the daily growth of the PVPM in Reynosa in dollars and the daily growth of the two-month Henry Hub futures, with lags of 20 and 40 days in the study period.	59
Figure 3.9. Optimal hedge ratios h_j^* in-sample and forecasted out-of-sample in the last 10% period of the data observed through backtesting.	60

Figure 4.1. VaR95% and CVaR95% of a random variable standard normally distributed.	75
Figure 4.2. Mean-Standard Deviation Efficient Frontier for the Equally Weighted Portfolio during the period of analysis.	81
Figure 4.3. Efficient frontiers for the portfolios under different risk measures for the period of analysis.	82
Figure 4.4. The efficient frontier of the Mean-Std Dev portfolio with and without constraints for the period of analysis.	84

Chapter 1

Introduction

1.1. Motivation

This thesis deals with energy risk management and how tools are combined to obtain practical solutions to problems: whether it be how to minimize risk in a Natural Gas (NG) position, how to update that solution over time, or how to forecast solutions and, for international energy asset portfolios, how to obtain combinations that give the smallest possible risk and how to combine portfolio components in order to minimize risk that is subject to certain performance standards. All the preceding without bypassing the issue of how to measure the risk.

According to the Energy Information Administration (EIA), in 2017 total energy demand around the world was 14,034,897 ktoe¹ (kilo tonnes of oil equivalent). Of this demand, 40.46% was for petroleum products, 18.45% for electricity and 14.99% for NG. Part of the demand for NG was used in electricity generation. The energy market is volatile: in the 12 months leading up to March 2020, daily volatility in the US dollar price for West Texas Intermediate (WTI) was US \$9.28 and volatility of the daily log returns for the same period was 5.14%. The energy market, and in particular the hydrocarbon market, is large (economically valuable) and volatile.

These two characteristics of the energy market are an invitation to manage risk. For this purpose, there are financial derivatives such as futures, options, and others that, together with analysis and optimization tools, make it possible to obtain solutions that limit exposure to risk, transferring it to third parties. All this for the benefit of whoever owns the energy assets.

Despite the development of risk management tools in the North American, Asian, and European energy markets, their use has not been widespread in countries such as Mexico. This thesis looks at certain analyses carried out for Latin American markets.

1.2. Problem Statement and Context

Some energy markets, such as that for NG distribution, are natural monopolies; consequently, their activities should be regulated to avoid predatory practices. One of the most frequently used regulatory instruments is price capping. Until June 2017, the maximum price for NG in Mexico was established by the country's Energy Regulatory Commission (CRE for the acronym in Spanish). This maximum price is known as the First-Hand Sales Price (PVPM for the acronym in Spanish), and the mechanism to establish it will be explained later. Since participants in the NG market are usually exposed to fluctuations in the raw material price (i.e., the price of the NG

¹ A ton of oil equivalent corresponds to 41,868,000,000 J (joules) or 11,630 kWh (kilo-watt hours).

molecule), and possibly exchange rate fluctuations for the commodity's price, strategies should be developed and specific solutions found that reduce exposure to price and exchange rate risk. In this work, hedging strategies are proposed and optimal solutions to minimize risk are obtained for a given period.

Optimal solutions to NG hedging problems are changeable over time and depend on collecting new data. These changes can be explained through models whose results serve to produce forecasts for the immediate future, which in turn can be used to determine solutions that can be applied even before new events occur. These forecasts and solutions should be evaluated to see how accurate they are and, if necessary, to make adjustments. In this thesis, the problem of dynamic NG hedging is also raised, and a model is proposed to obtain conditional variances and correlations. Forecasted optimal hedging solutions are obtained and evaluated by backtesting, and measures to improve the effectiveness of the forecast are proposed.

The NG hedging problem can be extended to a portfolio comprised of energy company shares and crude oil futures, with a different methodology - that of portfolio optimization - and extending it to other risk measures. Portfolio hedging, which is understood to mean a given solution that minimizes risk, is a specific example of a solution for the risk-return model in which the risk measure may be different from the portfolio returns variance. If other risk measures are adopted and simulations are used, it is advisable to generate more data than those observed, using interpolation and extrapolation techniques. Using a portfolio of emerging international energy companies who receive less coverage from industry analysts, hedging and optimization models are developed employing three different risk measures. These risk measures are evaluated, and optimal solutions are obtained through simulations.

1.3. Research Questions

In approaching, developing and solving the NG hedging model, it is necessary to verify whether gas futures are desirable instruments to incorporate into a hedging portfolio, the periods during which they best correlate to the NG position to be hedged, and whether the returns on the futures in those time periods are statistically independent from each other. Additionally, it is essential to determine the optimal hedge ratios, i.e., the proportions between the hedged portfolio's long and short positions that minimize its variance. In a portfolio of optimal hedges, the volatilities of the short positions closely replicate the volatilities of the long positions; therefore, the route to hedging solutions is the one used to create synthetic instruments. The solutions so obtained are applicable to the periods analyzed, although it would be important to find out if they can continue to be effective in the future.

Changes in the prices of assets in a hedged position can be explained through statistical models that feed back to the results of previous estimates. If these models

attain high likelihood values and their estimates have high levels of confidence, they can effectively forecast future results based on the latest actual observations. This thesis proposes a model that determines forecasts, which are then tested by backtesting to ascertain whether the results are effective, i.e., whether they are close to future values. To increase the effectiveness of the forecasts, a set of data closer to the forecast period should be tested as well as determining the horizon of the period for which the forecasts are closest to the observed values from backtesting.

In a portfolio of energy assets, it is likely that asset returns are correlated. For this reason, obtaining optimal hedging ratios with changing information becomes complex. In addition, since the historical data are discrete, the calculation of some risk measures yield "step patterns". Therefore, it is desirable that, without altering the most important statistical characteristics of the series, the number of data is increased by means of interpolation and extrapolation techniques. On the other hand, some risk measures are not coherent, and it is therefore necessary to measure the risk differently, which leads to a much more complex calculation.

This thesis proposes a portfolio of shares in energy companies and oil futures whose historical data are transformed into new series to improve the measurement of risk under different measures and different scenarios. The first problem that arises is that of hedging. For its solution, a reference portfolio must be defined. The second, more general problem is obtaining efficient frontiers with different risk measures. Risk minimization as an objective, subject to certain performance levels, requires the solution of non-linear integer programming models.

1.4. Solution Overview

Since Henry Hub NG prices in the United States play a role in determining PVPMs, it should not be surprising that Henry Hub NG futures are a good hedge for PVPMs and that peso-dollar futures can also reduce currency risk exposure in peso NG positions. Given the statistical independence between dollar NG futures with one- and two-month lags and that the NG position to be hedged, the optimal hedging ratios can be obtained through closed formulas or regression coefficients using ordinary least squares (OLS). The optimal hedge is the one that is best for the whole period analyzed but not necessarily for sub-periods or for the immediate future. The objective of the hedge is to minimize risk without considering return, since the aim is to replicate the volatility of the position to be hedged; however, the hedge may also have a return objective.

The conditional variances and correlations of the data from the analysis period are calculated, as has already been mentioned, with a model whose estimates are significant. The model allows for predicting values for the immediate future through known data. The forecasts are more accurate to the extent that the data used are closer to the period to be predicted, provided that the solutions with high degrees of likelihood produce convergent solutions. The effectiveness of forecasts decreases

as the prediction horizon increases. The use of backtesting allows the model to be calibrated, both in terms of coefficients and the extent of the actual data and the horizon to be predicted. The process of dynamic hedges is iterative, as new information is received.

In the transformation of the data from the analysis period by interpolation and extrapolation, one type of distribution function is used for the central part of the original data series and another for the extreme tails. The application of the proposed method adds new internal and extreme elements to the original data, without losing its original statistical characteristics, so that it is possible to estimate risk measures with greater precision. In order to determine the hedging solutions and efficient frontiers under different risk measures, optimization algorithms are executed so that, in the case of hedging, the minimum of minimums is reached for each risk measure and, for the efficient frontiers, the minimum risk portfolios are obtained for a range of returns. This way the investor can select the risk-return combination that best matches their preferences.

In the following chapters, three essays on hedging and optimization of energy assets are presented: *Strategies for Hedging First-Hand Natural Gas Prices in Mexico*: Barrera-Rivera, R. & Valencia-Herrera, H. (2019); *Dynamic Hedging of Prices of Natural Gas in Mexico*: Barrera-Rivera, R. & Valencia-Herrera, H. (2020); and *Hedging and Optimization of Energy Asset Portfolios*: Barrera-Rivera, R. & Valencia-Herrera, H. (2021). The final chapter presents the conclusions of the thesis. Additionally, references and annexes are included.

After the review of the thesis' examiners, they recommended to include some elements of analysis at the end of chapter 2 and chapter 3 which were not included in the original versions of the articles.

Chapter 2

Strategies for Hedging First-Hand Natural Gas Prices in Mexico

2.1. Introduction

The structure of energy markets sometimes requires price regulation. One example is the Natural Gas (NG) market, where natural monopolies are created based on activity or geographic location. In such cases, governments decide to regulate prices by imposing prices caps as a defensive measure to support market participants.

Until June 2017, Mexico's Energy Regulatory Commission (CRE for the acronym in Spanish) restricted the NG prices that Petróleos Mexicanos (PEMEX), the state oil consortium, used for the first sale of gas to other participants in the distribution chain. Such prices are known in Spanish as the *precio de venta de primera mano* (PVPM), or first-hand sales price. These were fixed for a period, usually one month, were denominated in pesos, and initially referred to two strategic geographic points: the main gas import site (Reynosa) and the main production hub (Ciudad Pemex).

Other prices in the distribution chain were determined based on PVPM, considering other factors such as transportation costs, taxes, margins, and return on investment. Since PVPM remained fixed for one month and was denominated in pesos, and NG prices in the U.S. market changed frequently and were denominated in dollars, there was a possibility that the importer, distributor, or consumer of gas would use a hedge. This was to mitigate risks from selling or consuming at a set price in one currency (Mexican pesos) and eventually having to acquire the commodity in the future at another price, which was fixed according to floating prices (prices in the southern U.S.) in another currency (U.S. dollar).

In the following sections we describe the literature on oil and gas hedging and the mechanism of PVPM in Mexico, analyze the statistical series of prices, indexes and quotes, propose hedging strategies with futures, outline work that could be developed in the future, and finally, draw conclusions.

2.2. Conceptual Framework

2.2.1. Background

Rosellón (2008) highlights the CRE's work since 1996 to adopt a methodology for NG prices that linked them to prices in the southern United States through the netback rule and the Little-Mirrlees approach. He points out that, when setting a reference price, it is advisable to use two geographical points - the import point and the domestic production point - and to adopt an intermediate point that presumably reduces the arbitrage between the options to import or to buy gas from domestic production.

Literature that analyses oil and gas hedging is extensive. Pindyck (2003) examines the behavior of volatility in natural gas and crude oil futures prices since 1990. He finds that there is a short-term trend in volatility due to shocks and that, during these, the interrelationship between the volatilities of both hydrocarbons increases. Jin & Jorion (2006) study the hedging activities of 109 U.S. oil and gas producers and analyze the effects that these activities have on the value of the companies. They find that hedging reduces the companies' stock price sensitivity to fluctuations in hydrocarbon prices. Using a partial regression model, Woo, Olson & Horowitz (2006) prove that California NG users can exploit cross-hedging opportunities with the Henry Hub index and even forecast the index's behavior, thereby improving risk management through futures or swaps. The authors point out that in 2000, the price of NG in that state increased tenfold.

Regarding pricing forecasts, Wong-Parodi, Dale & Lekov (2006) compare the forecasts for NG prices published by the U.S. Energy Information Administration (EIA) with those from the New York Mercantile Exchange (NYMEX). They found that prices on the futures market are a better indicator than the forecasts released by the government agency, which consistently overestimated prices during the 1982-2005 period. Brown & Yücel (2008) study the independent movement of NG and crude oil prices. They develop an error-correction model to show that crude oil prices influence NG prices; consequently, both products can be regarded as substitutes. Kaufmann & Ullman (2009) study the effect of innovation on hydrocarbon prices and how these effects spread to other prices in the spot and futures markets. They also analyze the long-term relationship between spot and futures prices. Nomikos & Andiosopoulos (2012) investigate the behavior of the prices of eight energy products listed on the NYMEX, both in the spot and futures markets, and conclude that there is a leverage effect for West Texas Intermediate (WTI) and heating oil while in the remaining markets the effect is inverse.

Other authors have analyzed the volatility of oil prices. Suenaga, Smith & Williams (2008) examine the volatility of NYMEX NG futures prices and conclude that prices exhibit seasonality in winter, in addition to the fact that the effect of price shocks is persistent; therefore, hedging strategies that do not consider these factors are sub-optimal. Agnolucci (2009) compares the predictive ability of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models and implied volatility models to estimate volatility in the prices of WTI futures contracts on the NYMEX based on statistical and regression results. Wei, Wang & Huang (2010) use different GARCH models to estimate the volatility of Brent and WTI crude oil prices and find that non-linear models are better at capturing long-term effects and asymmetric volatility.

More recently, Scholtens & van Goor (2014) analyze the volatility in NG prices in the United Kingdom and conclude that GARCH models based on supply and demand and supported by theoretical assumptions of an economic nature are good predictors.

Asche, Oglend & Osmundsen (2017) find that when NG prices are decoupled from oil prices due to short-term effects, models such as Error Vector Correction can lead to erroneous conclusions about the nature of the co-integration relationship. Ghodussi & Emamzadehfard (2017) experiment with hedging alternatives in the U.S. NG market. They contrast the use of a single futures contract with the use of futures contracts that exceed the time-to-maturity for hedging six different physical price positions. They conclude that using longer maturity contracts can increase the effectiveness of the hedge.

2.2.2. Regulation of Natural Gas Prices

Since NG transportation and distribution are natural monopolies, the government regulates prices to protect the consumer. Investment in transmission and distribution networks in certain areas is so costly that it would be problematic to have overlapping networks belonging to different suppliers; it would simply ensure that some investments lay idle, the costs of which would eventually be passed on to the consumer. For this reason, regulators usually fix maximum selling prices that allow regulated companies to recover their investments and costs at a reasonable capitalization rate.

This asymmetrical regulation is applicable to other parts of the production, transportation and distribution chains whenever monopolistic situations arise. For example, when there is a single producer or a dominant warehouse, or a carrier that owns the only pipeline in a region, or a distributor in an urban area.

In addition to limits on prices and tariffs, regulators use other measures, which could include allowing use of a sole operator's facilities and equipment, mandatory unbundling, and restrictions on monopolies. Recently, for example, regional distributors in Mexico have had to open their networks to sellers with marketing and administration capabilities but no infrastructure, who offer NG to consumers and handle the final sale.

The NG price regulation model is widely used in market economies. The methodology for regulating NG prices has evolved since its first appearance in Mexico 22 years ago. In February 2016, the CRE published its most recent approach, establishing the method for calculating the PVPM in two hubs: Reynosa and Ciudad Pemex, with two different frequencies: daily and monthly.

2.3. Methodology

To develop this work, price series, indexes, and quotations from the beginning of 2013 to September 30, 2016 were used. The NG price series in the U.S. were taken from the EIA website. Spot and futures market prices correspond to those from the NYMEX; the PVPM in Reynosa and Ciudad Pemex are those published by the CRE. The Henry Hub and Houston Ship Channel indexes are from Platts and the peso-dollar exchange

rate are those published by the Bank of Mexico (BANXICO for the acronym in Spanish).

The following factors were taken into account for each of these prices: (1) the estimated price of NG in South Texas; (2) whether there is a net import or export of NG in the country; (3) the cost of transportation between Reynosa and South Texas; (4) the transportation rate of the Sintragás system from Reynosa to Ciudad Pemex, and (5) the peso-dollar exchange rate.

To estimate the price of NG in South Texas, the indexes used were: (1) Henry Hub, (2) Houston Ship Channel published by Platts, and (3) other local Texas indexes.

Transportation rates in the U.S. were estimated using the following systems: (1) Tennessee Gas Pipeline Company, L.L.C.; (2) El Paso Natural Gas Company, L.L.C., and (3) Texas Eastern Transmission, LP, published by the United States Federal Energy Regulatory Commission (FERC).

The parameters obtained from an Ordinary Least Squares model using an Engle-Granger procedure, in which the dependent variable is the PVPM and the factors are the independent variables, were applied to the above-mentioned factors. Such parameters are public and are updated from time to time.

There is a time lag for data. For example, the exchange rate is based on an average taken from the 15 days prior to the month corresponding to the PVPM. In the model, values of factors from previous periods (months or days) are introduced.

Using the values of the PVPM in Reynosa or Ciudad Pemex, PVPMs are calculated for each of the Gas Processing Terminals and other geographic points where PEMEX delivers fuel to be transported and distributed. The PVPMs are transmitted along the whole chain and become the component that most affects the price to the end-consumer, whether industrial or domestic. In most cases, prices to distributors or end-users are sustained for one month and are quoted in Mexican pesos per gigajoule or by volume.

Consequently, the price of fuel remains fixed in pesos during the period while the dollar price of imported fuel frequently varies.

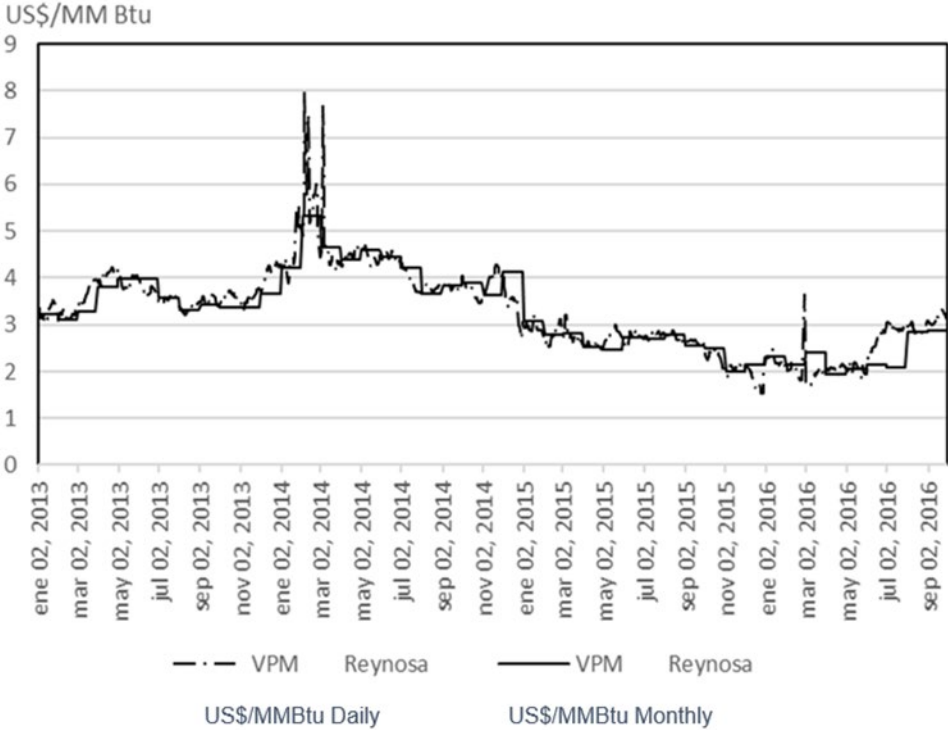
2.4. Analysis

2.4.1. Statistical overview of the price series

As has been mentioned, several factors influence setting the PVPM. In this essay, we will concentrate on two: (1) the price of NG in dollars in South Texas; and (2) the peso-dollar exchange rate. The other factors have less relevance in determining PVPM and are less volatile.

Figure 2.1 shows the daily and monthly PVPM in Reynosa for the analysis period. Note that the monthly value does not correspond to the average of the daily values. This is because the monthly PVPM was determined one day before the beginning of the month and was sustained throughout the period, while the daily prices, although also calculated one day in advance, were modified daily, allowing them to reflect more updated international price information.

Figure 2.1. Daily and monthly PVPM in Reynosa during the period of analysis.



Source: Own elaboration with data of the CRE.

Statistics of the series of daily PVPMs in Reynosa are shown in Table 2.1. Table 2.2 includes statistics of the natural log series of PVPM variations, to eliminate the effect of the PVPM's first value.

Table 2.1. Statistics of the series of daily PVPM in Reynosa (US\$/MMBtu) during the period of analysis.

Mean	3.27564
Typical error	0.028812
Median	3.2662
Mode	3.776
Standard deviation	0.894104
Variance	0.799423
Kurtosis	2.021911
Skewness	0.753176
Range	6.4395
Minimum	1.5458
Maximum	7.9853
Observations	963

Source: Own elaboration with data of the CRE.

Table 2.2. Statistics of the daily continuous growth rate of PVPM in Reynosa during the period of analysis.

Mean	-6.376E-05
Typical error	0.00153
Median	0
Mode	0
Standard deviation	0.047719
Variance	0.002277
Kurtosis	59.72235
Skewness	-0.789370
Range	1.095689
Minimum	-0.662498
Maximum	0.4331
Observations	962

Source: Own elaboration with data of the CRE.

The dispersions are of special interest to this work: the standard deviation for daily PVPM in Reynosa, in dollars, for the period analyzed was 0.894104 and that of the logarithmic variation was 0.047719.

Similarly, the Henry Hub and Houston Ship Channel index series were analyzed, as well as the spot and futures prices for 1-month and 2-month NG contracts on the NYMEX and the peso-dollar exchange rate quotations. Since a high correlation was found between the daily PVPM and the spot price of NG on the NYMEX, we will focus on the graphs and statistics of NG on the NYMEX and the peso-dollar exchange rate.

Figure 2.2 shows the NYMEX NG spot price chart for the period analyzed. At a glance, you can see the similarity of the profile with Figure 2.1 that shows the daily PVPM in dollars in Reynosa.

Figure 2.2. Spot prices of NG at NYMEX in US\$/MMBtu during the period of analysis.



Source: Own elaboration with data of the NYMEX.

As indicated in Tables 2.3 and 2.4, the standard deviation of the spot price series from the NYMEX was 0.961053 and the continuous daily growth rate was 0.040707. The standard deviation of the continuous daily growth rate of NG on the NYMEX was less than that of the daily growth rate of PVPMs in dollars in Reynosa for the same period (0.047719). This means that, during the period analyzed, the PVPM in Reynosa was more volatile than the spot price of NG on the NYMEX. Finally, the behavior of the peso-dollar exchange rate for the period is shown in Figure 2.3.

Table 2.3. Statistics of the daily spot price series of NG at NYMEX (USD/MMBtu) during the period of analysis.

Mean	3.323406
Typical error	0.030969
Median	3.32
Mode	2.88
Standard deviation	0.961053
Variance	0.923623
Kurtosis	1.820140
Skewness	0.700156
Range	6.66
Minimum	1.49
Maximum	8.15
Observations	963

Source: Own elaboration with data of the NYMEX.

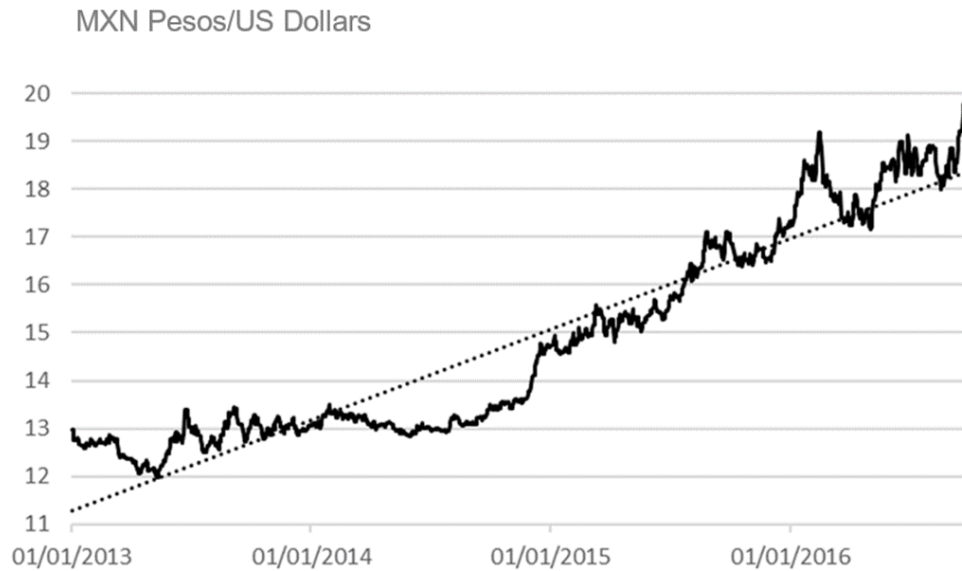
Table 2.4. Statistics of the daily continuous growth rate of NG spot prices at NYMEX during the period of analysis.

Mean	0.000297
Typical error	0.000147
Median	0
Mode	0
Standard deviation	0.005437
Variance	2.96E-05
Kurtosis	5.254254
Skewness	0.269750
Range	0.070360
Minimum	-0.029854
Maximum	0.04050
Observations	1368

Source: Own elaboration with data of the NYMEX.

Finally, the following shows the behavior of the peso-dollar exchange rate during the period:

Figure 2.3. Mex Peso -- US Dollar exchange rate during the period of analysis.



Source: Own elaboration with data of the BANXICO.

As can be seen in Figure 2.3, the exchange rate underwent greater volatility starting in 2015, which continued through to the last date in the period analyzed.

Table 2.5 shows statistics of the daily continuous growth rate (logarithmic differences) for the exchange rate. During the period of analysis, the standard deviation of the daily continuous growth rate was 0.005437. From 2015 onwards, this standard deviation increased to 0.006380 (not shown in the Table). Accordingly, and during the period considered, the volatility of NG prices in dollars (NYMEX), measured through the standard deviation of the logarithmic variation, was eight times the volatility of the peso-dollar exchange rate.

Table 2.5. Statistics of the daily continuous growth rate of the peso – dollar Exchange rate during the period of analysis.

Mean	0.000297
Typical error	0.000147
Median	0
Mode	0
Standard deviation	0.005437
Variance	2.956E-05
Kurtosis	5.254254
Skewness	0.269750
Range	0.070360
Minimum	-0.02985
Maximum	0.04050
Observations	1368

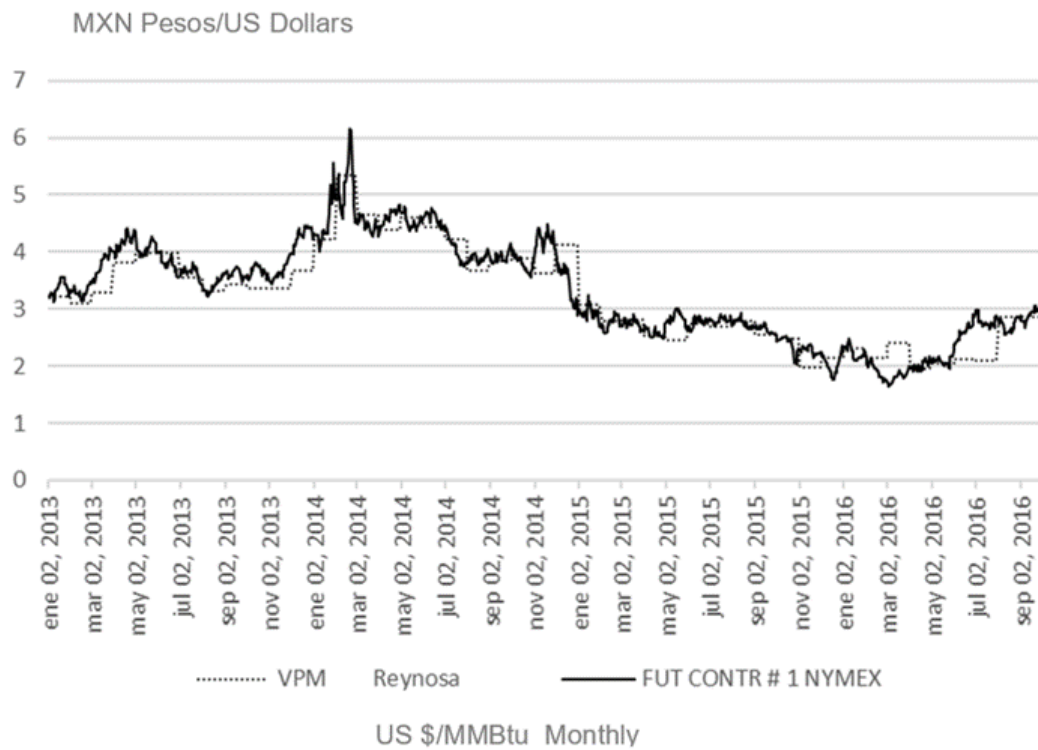
Source: Own elaboration with data of the BANXICO.

2.4.2. Characteristics of the First Hand-Selling Price in Reynosa

Ever since an open market for NG imports has existed in Mexico, the PVPM in Reynosa has been the reference for prices in other markets, including the PVPM in Ciudad Pemex (the production center). PVPM in Ciudad Pemex was not calculated based on production costs but as a price that balanced the import with the hypothetical export of NG. For this reason, we will focus on the PVPM in Reynosa and on its monthly rate, which more clearly demonstrates the option of adopting a hedging strategy.

During the period studied, the monthly PVPM in dollars in Reynosa was highly correlated with the daily prices of the NYMEX's NG Future Contract 1. Figure 2.4 shows the similarity of monthly PVPMs to the daily prices of the futures contract.

Figure 2.4. Monthly PVPM in dollars in Reynosa vs contract prices of the Future #1 in the NYMEX during the period of analysis.



Source: Own elaboration with data of the CRE and NYMEX.

We can get a closer look at the behavior of these prices by analyzing their differences or changes, i.e., whether the rise or fall in daily NG prices in Reynosa has any relation to, for example, the change in futures prices on prior days. Table 2.6 shows the results of simple linear regression between daily differences of the PVPM in dollars in Reynosa and daily differences of the NG Futures Contract 1 prices on the NYMEX with a one-day lag.

Table 2.6. Statistics of the simple linear regression between the daily differences of the PVPM in dollars in Reynosa and the differences of the NG Future #1 prices at Nymex with a lag of one day during the period of analysis.

Variable	Coefficient	Standard error	T statistic	Prob.
Future # 1 (-1)	0.980709	0.010414	94.16891	0.0000
C	0.043812	0.035484	1.234683	0.2173

$R^2 = 0.902318$; R^2 adjusted = 0.902216

Source: Own elaboration with data of the CRE and NYMEX.

We can see in Table 2.6 that 90.2216% of the variances of the adjusted daily differences of PVPM in dollars in Reynosa can be explained by the variations of the daily differences in the NG Futures Contract 1 prices on the NYMEX with a one-day lag, during the period analyzed.

The peso-dollar exchange rate comes into play when converting the PVPM in dollars to pesos. However, there is no significant correlation between NG prices in dollars (NYMEX) and the peso-dollar exchange rate. The correlation between the logarithmic variations of both factors was -0.0545 for the analysis period.

2.5. Hedging strategy with futures

Among the multiple hedging strategies that participants in the NG production or distribution chain in Mexico can adopt is one using futures contracts. To demonstrate, we will take the case of an urban NG distributor and an industrial consumer.

2.5.1. The use of futures contracts for hedging

The NG distributor in an urban zone acquires from PEMEX, or from an importer, fuel that will later be sold to domestic or industrial consumers. The distributor acquires the NG at a fixed price at the city gate once the gas has been transported from the point of importation or from the processing terminal. For one month, the gas purchase price will be fixed in pesos and the distributor will, in turn, have to sell it at a fixed price to their consumers. The following month, the distributor will purchase the NG at a different price that will depend on fuel prices in South Texas and the peso-dollar exchange rate, among other factors. To manage the risk represented by fluctuations in both the price of NG in dollars and the exchange rate, the distributor may enter into futures contracts for gas as well as for the exchange rate.

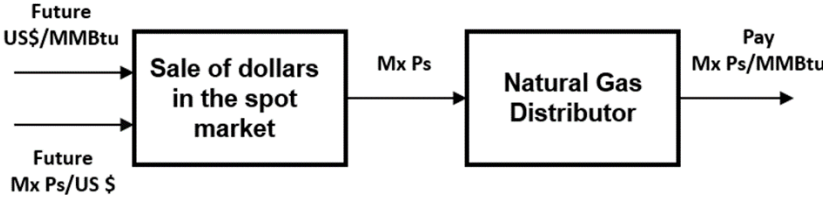
It should be recalled that, as noted in Figure 2.4 above, the volatility in the price of NG in dollars is generally higher than the volatility of the exchange rate, so the two hedging strategies - for the price of NG in dollars and for the peso - could be independent and intermittent.

The NYMEX market offers NG futures contracts with monthly expirations that span a decade ahead. For example, the last trading day for the December 2016 contract was the prior November 28 for delivery by December 31, 2016. Each contract size was for 10,000 MMBtu and the tick size is USD \$0.001.

The Chicago Mercantile Exchange (CME) peso-dollar futures contract is for 500,000 MXN, with a minimum dollar-to-peso exchange rate fluctuation of 0.00001, equivalent to USD \$5.00 per contract. The contracts have monthly expirations and cover an 18-month period.

A distributor who speculates that NG prices in dollars will rise and that the peso will depreciate in the next weeks or months can buy NG futures on the NYMEX and buy dollar futures on the CME. To do so, they will need to open contracts and deposit collaterals. Before their expiration date, the distributor should reverse their position unless they wish to take "physical delivery" of the goods. In the event of a reverse trade, the distributor will take their profit or loss to the spot exchange market to convert the dollars into pesos. With the potential profit, the distributor will be able to acquire NG at the new PVPM. If the hedging strategy was successful, the distributor will have the ability to acquire the same or a higher volume of NG as a result of good risk management. Figure 2.5 diagrams the use of futures contracts as a hedging tool.

Figure 2.5. Diagram of the use of hedges with NG futures and dollars.



Source: Own elaboration.

Evidently, it would be the inverse operation if expectations were for prices to decrease. Futures would be sold on the NYMEX and, if necessary, dollar futures would be sold on the CME. In any case, the result in dollars would be expected to be positive.

The classic theory of hedging with futures - see for example Hull (2007) and Ghoddusi and Emamzadehfard (2017) - consists of reducing or nullifying the price volatility of a spot position by including a certain number of futures contracts in the portfolio. If you have a portfolio P with n_S long positions in assets and n_F short positions in futures, the hedge ratio would be the number of future positions to hedge one unit of the spot position, i.e., $h = \frac{n_F}{n_S}$.

The value of the portfolio, considering n_S units of assets to be hedged and n_F units of futures, would result from the equation (2.1).

$$P = n_S S - n_F F \tag{2.1}$$

Consequently, the changes in the hedged portfolio result from the equation (2.2).

$$\Delta P = n_S \Delta S - n_F \Delta F \tag{2.2}$$

The minimum variance hedge ratio is estimated by selecting the number of futures contracts that minimize the conditional variance of changes in portfolio value. The optimal hedge ratio is the equation (2.3).

$$h^* = \frac{n_F}{n_S} = \frac{cov(\Delta S, \Delta F|I)}{var(\Delta F|I)} \quad (2.3)$$

where I is the set of information in time t and h^* is the optimal hedge ratio.

As mentioned, the PVPM in Reynosa is a maximum price for NG in Mexico. The formula for determining the monthly price according to the CRE considers the previous monthly international NG prices from Henry Hub, Texas Eastern STX, Tennessee Zone 0, and Houston Ship Channel, adjusted for other factors such as transportation costs. In particular, the CME quotes NG futures contracts using the Henry Hub numbers as a reference.

These futures contracts are closely related to their underlying. Also, gas prices at Henry Hub are closely related to those of Texas Eastern STX, Tennessee Zone 0 and Houston Ship Channel, as can be seen in Table 2.7. Consequently, hedge ratios could be estimated using the PVPM in Reynosa and CME's Henry Hub NG futures as spot prices.

Table 2.7. Correlation between the monthly growth of different international prices of NG during the period of analysis.

	Futures # 1	Henry Hub	Houston SC	Tennessee	Texas STX
Futures # 1	1.000000	0.832366	0.881157	0.853803	0.846315
Henry Hub	0.832366	1.000000	0.975052	0.992991	0.990873
Houston SC	0.881157	0.975052	1.000000	0.980367	0.985568
Tennessee	0.853803	0.992991	0.980367	1.000000	0.994432
Texas STX	0.846315	0.990873	0.985568	0.994432	1.000000

Source: Own elaboration with data of the NYMEX and Platts.

To estimate the hedge ratios, equation (2.4) is used, where ΔP_t is the monthly growth of the PVPM in Reynosa with respect to its price in month t and $\Delta F_{1,T}$ is the growth of Henry Hub one-month gas futures vis-à-vis its price in period T on the NYMEX of the previous month.

$$\Delta P_t = a_0 + a_1 \Delta F_{1,T} + \varepsilon_t \quad (2.4)$$

Given that the estimate for PVPM takes into account a lag in international NG prices, futures from previous periods could be useful for hedging PVPMS.

Figure 2.6 shows the growth in one-month futures prices and the growth of PVPM in Reynosa in dollars. This shows that the Reynosa one-month and two-month lead PVPMs have a statistically significant correlation with one-month futures.

Figure 2.6. Correlation between the growth of the Henry Hub one-month futures and the growth of the monthly PVPM in dollars in Reynosa during the period of analysis.

Future #1 Reynosa (-i)	Future #1 Reynosa (+i)	i	Lag	Lead
		0	0.0708	0.0708
		1	-0.0815	0.4599
		2	-0.1456	0.3893
		3	0.1038	0.0160
		4	0.0987	0.0818
		5	-0.1157	-0.0614
		6	0.0480	-0.0934
		7	-0.0768	-0.1173
		8	0.0262	-0.0491
		9	0.2075	0.0872
		10	-0.1022	0.0515
		11	-0.1614	0.2728
		12	0.0634	-0.2436
		13	0.0606	-0.0824
		14	-0.1625	0.0979
		15	-0.0550	-0.1906
		16	-0.0095	0.2215
		17	-0.0718	-0.1811
		18	0.1737	0.0095
		19	-0.0537	0.0344
		20	-0.1879	-0.2024

Source: Own elaboration with data of the NYMEX and CRE.

Table 2.8 shows the hedge ratios of CME NG futures during the analysis period and prior periods. We can see that only futures at a one- or two-month lag behind the PVPM offer hedging possibilities, since only in these cases is h^* statistically significant. The optimal hedge for a seller of natural gas in the Mexican market could be structured with a one-month lag, taking a short position in futures for 51.4961% of the value of the position to be covered one month before the natural gas is sold at PVPM. The R^2 is an indicator of the potential risk reduction using the hedge, in this case 20.9224%. Since the optimal hedge ratio with two-month futures is statistically significant, one could hedge, for example, the purchase at PVPM by buying futures for 44.3225% of the value of the position to be covered two months before the purchase is made.

Table 2.8. Optimal hedge ratios of the PVPM in dollars with regards to the one-month futures at the CME with lags during the period of analysis.

Months of lags of the Future #1	h^*	Standard error	R^2
0	0.079718	0.162166	0.005010
1	0.514961	0.281373*	0.209224
2	0.443225	0.179516**	0.155223
3	0.030040	0.076684	0.000718

* and **, statistically significant at 90% and 95%, respectively.

Source: Own elaboration with data of the NYMEX and CRE.

Hedging can be structured by purchasing multiple futures over several prior periods. More specifically, given that the autocorrelation in future growth is very small and not statistically significant, futures with a lag of one month and two months can be considered independent instruments. Therefore, the coefficients obtained by applying linear regression to the growth in PVPMs with respect to growth in futures with one-month and two-month lags can be considered as optimal hedge ratios with each instrument, in a multiple hedge. From Table 2.9, a position in gas subject to PVPM could be hedged using one-month futures with one- and two-month lags, acquiring futures worth 49.8119% of what will be paid a month earlier and worth 41.8258% of what will be paid two months earlier, for a risk reduction of 35.0306% (R^2).

Table 2.9. Optimal hedge ratios of the PVPM in dollars with regards to two combined Henry Hub futures during the period of analysis.

Months of lags of the Future #1	h^*	Standard error
1	0.498119	0.315104
2	0.418258	0.191188**

* and **, statistically significant at 90% and 95%, respectively.

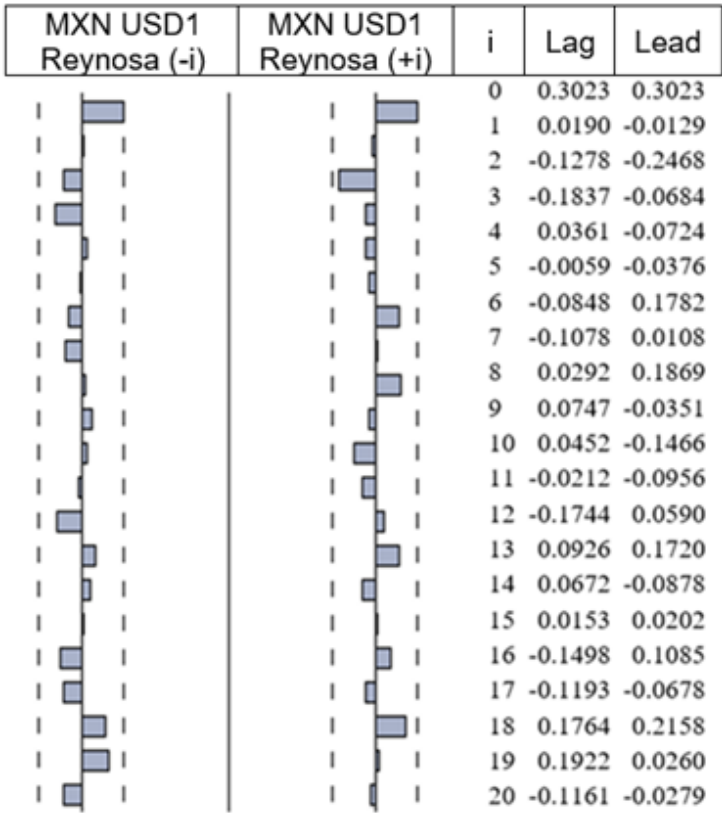
$$R^2 = 0.350306$$

Source: Own elaboration with data of the NYMEX and CRE.

To analyze the hedge with exchange rate futures, for analysis purposes synthetic futures were estimated using the hedged interest rate parity $F_t = \frac{S_t(1+r_d)}{(1+r_f)}$, where F_t is the directly quoted future price in period t, S_t is the directly quoted spot exchange rate, and r_d and r_f are the effective domestic and foreign forward rates in period t, respectively. In this case, rates for 91-day Cetes (Mexican Federal Treasury Certificates) and 90-day Treasury Bills, adjusted to a one-month term, were used.

Figure 2.7 shows cross correlations between the monthly growth of the PVPM in pesos in Reynosa with the growth of MXN-USD exchange rate futures, with different lag and lead periods. From the correlogram, it can be seen that the only statistically significant correlation is when there are no lags and leads between these instruments, and possibly when there are two lags. This is confirmed by the results of Table 2.10, in which the optimal hedge ratio without lags is 1.211913 and with two lags is -1.129440. This is statistically significant in both cases: the first at 95%, the second at 99%, although paradoxically these relationships have opposite signs.

Figure 2.7. Cross correlations between the monthly growth of the PVPM in pesos in Reynosa and the monthly growth of the peso-dollar exchange rate during the period of analysis.



Source: Own elaboration with data of the CRE, BANXICO and Bloomberg.

Table 2.10. Optimal hedge ratios of the PVPM in pesos in Reynosa with futures of the peso-dollar exchange rate with one-month lag during the period of analysis.

Months of lags of the future	h^*	Standard error	R^2
0	1.211913	0.521351**	0.091359
1	-0.055272	0.727505	0.000174
2	-1.129440	0.409645***	0.069684
3	-0.314282	0.810577	0.005377

*, ** and ***, statistically significant at 90%, 95% and 99%, respectively.*

Source: Own elaboration with data of the CRE, BANXICO and Bloomberg.

Finally, Table 2.11 shows the results of hedging with a model that uses a hedge combining gas futures and MXN-USD exchange rate futures for the PVPM in Reynosa.

Table 2.11. Hedge models of the PVPM in pesos, Henry Hub futures and futures of the peso-dollar exchange rate during the period of analysis.

Model 1

Instrument	h^*	Standard error
MXNUSD1	1.032752	0.259077***
Future #1(-2)	0.419196	0.286295
Future #1(-1)	0.495714	0.480097

$R^2 = 0.407014$

Model 2

Instrument	h^*	Standard error
MXNUSD1	1.360383	0.507579**
Future #1(-2)	0.452500	0.175684**

$R^2 = 0.237207$

*** and ***, statistically significant at 95% and 99%, respectively.*

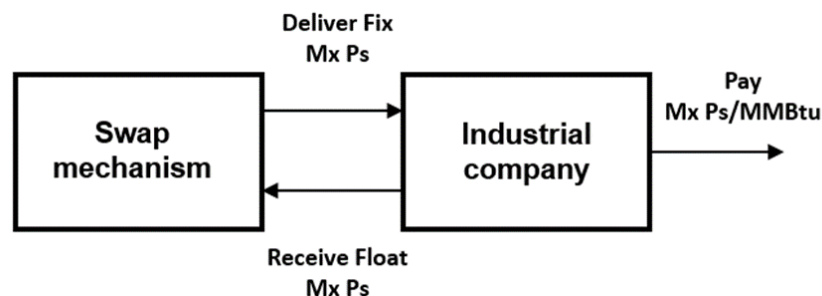
Future #1, one-month Henry Hub futures of the CME, MXNUSD1, one-month futures of the peso-dollar exchange rate, in parenthesis the lag of the instrument.

Source: Own elaboration with data of the CRE, NYMEX, BANXICO and Bloomberg.

2.5.2. Other Hedges

Another possibility to hedge for an unfavorable shift in the NG price in dollars and in the peso-dollar exchange rate is the use of swaps. Swaps are private agreements between two parties, one of which is usually a financial institution, to exchange cash flows. For example, suppose an industrial company decides to pay a fixed monthly amount in pesos during a given quarter in exchange for receiving, in pesos each month, the equivalent value in a certain volume of NG. In this way, they would be fixing the price at which they would like to buy gas during the quarter, regardless of the price fluctuations for NG in South Texas or the exchange rate. Each month, for three months, the company would pay the middleman a fixed amount in pesos and, in return, would receive a varying amount in pesos to purchase the same volume of NG, regardless of price fluctuations. This is a fixed vs. floating swap, with the arrangement that the floating part represents the value of the commodity in pesos. Figure 2.8 shows graphically the swap mechanism.

Figure 2.8. Diagram of the exchange of flows in a swap.



Source: Own elaboration.

Swaps are flexible and are agreed to based on the needs of the clients and the possibilities of the intermediaries.

There are other hedge possibilities, such as forwards, options, and exotic options, among others. Each vary as to costs and risk patterns that in certain cases may offer better hedging possibilities.

In addition, the behavior of PVPMs changes during different periods of time, so an analysis of the PVPm in sub-periods should provide an interesting perspective. Volatility in the price of hydrocarbons seems to allow some breathing room at times, while at others, the ups and downs are pronounced. Forecasting parameters change during these intervals.

Hedging strategies can be evaluated by backtesting. On principle, we know that it is not advantageous to completely hedge all the time, so it is important to learn when

and how much it would be worthwhile to do so, and what kind of strategy would be the most advisable.

A possible hedging and investment tool would be the creation of a hydrocarbon fund, a commodity fund whose value would change based on international prices and the exchange rate, in addition to offering returns. The possibility of buying or selling shares in the fund would allow hedging against fluctuations and could also be an attractive investment, especially for those who do not have easy access to international markets.

The hydrocarbon market and the NG market invite forecasting. Short- and medium-term forecasts would complement the use of hedges once the uncertainty about the direction and volatility of future prices is limited. Work related to NG can be extended to other fossil fuels: LPG, gasoline and diesel are logical next steps, although the markets may not be regulated for the latter two.

NG hedging strategies provide lessons that can be followed in electricity. Both markets are regulated and even have a forthcoming connection, as NG is a fuel widely used in electricity generation.

Finally, the findings can be extended to other markets and international environments. The NG market is regulated in many countries and many of them import fuels that are listed on global exchanges. The proposal made here could well be adopted internationally.

2.6. Conclusions

A major finding of this essay confirms what international energy traders know: that price volatility for NG and other hydrocarbon exceeds exchange rate volatilities. For this case and during the analysis period, NG volatility on the NYMEX was eight times that of the peso-dollar exchange. Even during critical periods for the peso (2015-2016), the volatility in the commodity price was six times that of the peso-dollar. The correlation between NG price and exchange rate variations is close to zero.

It would be worth examining the autocorrelation that can appear in the Linear Regression of PVPM and the price of NG on the NYMEX, focusing the model on the relationship between variations in both prices.

Optimal hedging of gas First-Hand-Sales prices considers the acquisition of futures months before the hedge date, which may allow for arbitrage. In view of the opening of the gas market in Mexico, it is recommended that the schemes for determining the prices of First-Hand gas sales in Mexico be revised to reflect the current international gas market.

The analysis and proposed hedging strategy could be extended to other fuels and other international markets with little effort. NG price regulation is an international

regulatory practice and many countries are net importers of hydrocarbons. The generation of electric energy from NG heightens the importance of this effort.

Finally, the field appears promising. Setting prices for a period implies costs and risks that someone must assume: the final consumer, the distributor, the importer, or the government consortium. The hedging strategy allows this risk and cost to be distributed among other participants with different capital structures and market perspectives.

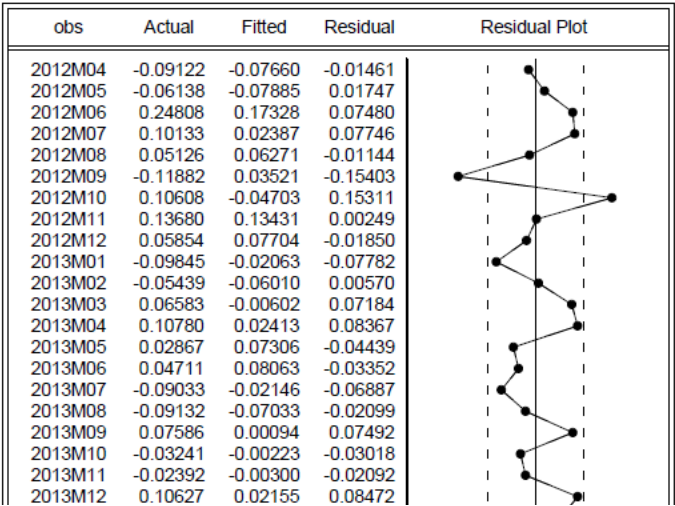
2.7 Post Data

This section includes materials which were not part of the original essay, however, in the view of the thesis' examiners and my own, we consider they are pertinent to include:

2.7.1 Effectiveness of the hedging model using backtesting

One question that might arise with the proposed hedging model is how stable it is. Can it be used in subperiods? To address these questions, first, we show Figure 2.9 which includes the actual and fitted values for the first months of the data series using Model 1, and then we made a backtesting analysis for two subperiods of the period of analysis; the first subperiod from January 2012 to December 2014 (in-sample) and the second from January 2015 to September 2016 (out-of-sample). Based on the Model 1 whose results appear on Table 2.11, we show in Table 2.12 the results for the in-sample subperiod and in Table 2.13 those for the out-of-sample.

Figure 2.9. Actual and fitted values of the growth of the monthly PVPM in pesos using Model 1 during the first months of the period of analysis.



Source: Own elaboration with data of the CRE, NYMEX, BANXICO and Bloomberg

Table 2.12. Hedge Model 1 of the PVPM in pesos, Henry Hub futures and futures of the peso-dollar exchange rate during the in-sample subperiod of analysis.

Instrument	h^*	Std. Error	t-Statistic	Prob.
FUTURES_1(-1)	0.733938	0.149983	4.89349	0.0000
FUTURES_1(-2)	0.047851	0.153338	0.31206	0.7572
FUT_TC	1.132081	0.542226	2.08784	0.0454

R^2 0.475321

Source: Own elaboration with data of the CRE, NYMEX, BANXICO and Bloomberg.

Table 2.13. Hedge Model 1 of the PVPM in pesos, Henry Hub futures and futures of the peso-dollar exchange rate during the out-of-sample subperiod of analysis.

Instrument	h^*	Std. Error	t-Statistic	Prob.
FUTURES_1(-1)	0.097874	0.276319	0.354205	0.7278
FUTURES_1(-2)	0.589016	0.24842	2.371049	0.0306
FUT_TC	1.863486	0.801361	2.3254	0.0335

R^2 0.464808

Source: Own elaboration with data of the CRE, NYMEX, BANXICO and Bloomberg.

The first thing we can say from the analysis of Tables 2.12 and 2.13 is that the explanatory power of the tests is high and similar. In the in-sample subperiod we have an R^2 of 0.4753 and for the out-of-sample the R^2 is 0.4648. Meaning that the logarithmic variations of the regressors can explain (and eliminate) such fractions (the R^2) of the logarithmic variations of the PVPM in pesos. In the in-sample subperiod, the statistical significance of the hedge ratio h^* associated to the Future#1 with a two-month lag is low, while in the out-of-sample subperiod the hedge ratio with the low statistical significance is the one related to the Future#1 with a lag of one month. In Model 1 of the original essay (Table 2.11), both hedge ratios have a significance above 95%. The hedge ratios for the peso-dollar exchange rate futures in both subperiods are statistically significant at the 95% level.

Table 2.14 include the comparative results of the hedge ratios and the R^2 for the two subperiods.

Table 2.14. Comparative results of the hedge Model 1 of the PVPM in pesos, Henry Hub futures and futures of the peso-dollar exchange rate during the *in-sample* and *out-of-sample* subperiods.

Instrument	In-sample h^*	Out-of-sample h^*	% Abs Diff
MXNUSD1	0.733938	0.097874	86.66%
Future #1(-1)	0.047851	0.589016	1130.94%
Future #1(-2)	1.132081	1.863486	64.61%
R^2	0.475321	0.464808	

Source: Own elaboration with data of the CRE, NYMEX, BANXICO and Bloomberg.

It is clear from the percentage absolute differences (% Abs Diff) in Table 2.14 that the hedge ratios for the in-sample subperiod are not good predictors of the hedge ratios for the out-of-sample subperiod. However, the hedge model for the PVPM in pesos based on the Henry Hub Future#1 and the peso-dollar exchange rate futures stands, not with the same hedge ratios but with the same structure of long position in PVPM in pesos and shorts in Henry Hub Future#1 and peso-dollar exchange rate futures.

A lesson from the backtesting in the two subperiods is that it is necessary to calibrate the model and to choose those regressors which are statistically significant and contribute to improve the explanatory power of the test.

2.7.2 Operational suitability for the hedging of the Futures with one and two-month lags

Someone trying to implement the hedging strategy and model developed in the essay, might have some concerns about the tradability of the Henry Hub Futures#1 with one and two-month lags.

On July 21, 2020, at the opening of the trades at Globex (Global Exchange -- an electronic trading system managed by the Chicago Mercantile Exchange), there were 452,086 contracts of Henry Hub NG pending to be traded. At the end of the day, there was an open interest of 1,298,974 contracts. As it was pointed out in the essay, each contract size is for 10,000 MMBtu. During the same trading day, 19,848 contracts were traded with futures with maturities in the coming two months (August and September), with an economic value of USD \$335.2 million.

If the NG position to be hedged is large compared to the daily trade volume of the selected futures, then a basket of futures with different maturities should be considered. However, this may have not been the case for most of the local players

who would have been interested in hedging a PVPM position during the period of analysis.

Chapter 3

Dynamic hedging of prices of Natural Gas in Mexico

Abstract

The first-hand sale prices of Natural Gas (NG) in Mexico had a dynamic lagged relationship with international NG futures prices during the period of January 2012 to June 2017. Based on a hedging strategy which includes NG futures and using an MGARCH VCC (MGARCH stands for Multivariate Generalized Autoregressive Conditional Heteroskedasticity and VCC for Variable Conditional Correlation) model, conditional variances were estimated with 20 and 40 days of lag between the prices of NG Futures. Dynamic hedges of NG were calculated assuming theoretical futures prices of the US dollar in Mexican pesos. With the use of backtesting, it was found that the forecasts of optimal hedge ratios improve with short prediction periods and proximate observed data. The dynamic hedging model proposed can be extended to other fuel markets. The importance of hedging NG prices derives from the size of the market and the extent of the risks to which the market participants are exposed.

Keywords: Natural gas prices, first-hand sale prices, dynamic hedging, backtesting.

JEL Classification: G13, G15, Q41, Q48

3.1. Introduction

The structure of energy markets usually requires price regulation as in the Natural Gas (NG) markets in which there are natural monopolies. In these cases, governments regulate prices by imposing limits on them as a defense measure in favor of the other market participants.

In Mexico, the Energy Regulatory Commission (CRE) is the regulatory body that, until June 2017, limited the prices of the NG that Petróleos Mexicanos (PEMEX), the state oil and gas company, used in its first sales to the other participants in the distribution chain. These prices are known as First-hand Sale Prices (PVPM, for its initials in Spanish). These prices typically set for a one-month period were denominated in pesos, and initially referred to two strategic geographical points: the main gas import gate (Reynosa) and the main production point (Pemex city) in the country. See CRE (2016).

The other prices in the distribution chain were determined from the PVPM, considering transportation costs, taxes, and investment recovery, among others. The PVPM remained fixed for a month and was denominated in Mexican pesos. The NG prices in the US market changed frequently and were quoted in dollars. Hence, there was a possibility that the importer, the distributor, or the consumer would use hedges to manage the risk that was assumed when selling or consuming at a

constant price in one currency (Mexican pesos) and eventually buying the product in the future at another price, which was set according to floating prices (prices in the South of the United States) in another currency (US dollar).

After June 2017, the CRE stopped the releasing of PVPM. The risk management problem was transformed because the NG distributor or consumer continued to face an environment of fixed prices in pesos for sale to the public versus permanently changing dollar prices of the commodity. The problem of NG price hedging becomes increasingly important internationally due to the growing demand for hydrocarbons, which is driven by the also greater generation of electricity using NG, and the gap between NG exporting and importing countries. In countries that import NG with a weak regulatory scheme, the wholesale prices of the NG are set by independent contracts in which the international price component is the most critical factor.

Since the transport and distribution of NG are natural monopolies, the authorities regulate prices in such a way that the consumer is not deprecated. Given the necessary investment in transportation and distribution networks by a provider to serve an area, the overlapping of networks of different providers will result in significant additional costs. For this reason, regulators usually set maximum selling prices that allow the regulated parties to recover their investments and costs at a reasonable capital rate. This asymmetric regulation applies to other elements of the production chain, for example, a single producer or a preponderant storage facility.

In addition to the limits on prices and tariffs, regulators employ other measures, such as allowing the use of facilities and equipment of the monopolist, ordering disintegrations, and limiting concentrations. It is important to notice that, in the absence of an appropriate regulatory system, price fluctuations and risks are (at least in part) transferred to the final consumer. The NG price regulation model is widely used, even in market economies. The following section includes a revision of some relevant work.

The main objectives of this investigation are the following two:

(1) To introduce a dynamic hedging approach based on a MGARCH-VCC model to predict the values of the best hedges for an immediate future period, and

(2) To evaluate the predictions obtained through backtesting and make recommendations to improve these predictions.

The importance of the study is based on the considerable size of the NG import market in Mexico, the possibility of a resurgence of NG regulated prices in Mexico, and the existence of regulation in the energy sector in many countries of the world, for which the Mexican experience in first-hand NG prices can be relevant.

The organization of the paper is as follows. In this section, we include the introduction. The following section briefly discusses relevant Mexican NG price regulation. The third section is a bibliographical review. The fourth section presents the methodology. The fifth section analyzes the data and the results. Finally, we make concluding remarks in the last section.

3.2. The Mexican NG regulation.

In Mexico, the NG price regulation methodology evolved since its first publication in 1996. In February 2016, CRE (2016) published its latest methodology, which explains the calculation of the PVPM at two points: Reynosa and Pemex city, with two different frequencies: daily and monthly. The considered factors are: (1) the estimation of the price of NG in South Texas; (2) the existence of a net import or export of NG in the country; (3) the cost of transportation between Reynosa and South Texas; (4) the Sintragás system transportation fee from Reynosa to Pemex city, and (5) the peso-dollar exchange rate. In turn, the estimate of the price of NG in South Texas considers the following price indexes: (1) Henry Hub; (2) Houston Ship Channel published by Platts, and (3) other local Texas indexes. For the estimation of the tariffs of transport in the United States, the rates of the systems were used: (1) Tennessee Gas Pipeline Company, L.L.C.; (2) El Paso Natural Gas Company, L.L.C., and (3) Texas Eastern Transmission, LP, published by the Federal Energy Regulatory Commission of the United States.

3.3. State of the Art

Tse & Tsui (2002) propose a GARCH model for multiple variables (MGARCH) in which the correlations vary over time; the conditional variance follows a single variable GARCH formulation, and the conditional correlation matrix adopts a self-regressive average behavior. Tolmasky & Hindanov (2002) present a family of term structure models to evaluate contingent obligations of contingency goods and seasonal markets, in particular the oil market. Pindyck (2003) examines the behavior of volatility in the prices of natural gas and crude oil futures since 1990 and finds that there is a short-term trend of volatility due to shocks and that, during these, the interrelation between volatilities of both hydrocarbons increases. Jin & Jorion (2006) study the hedging activities in 109 oil and gas producing companies in the United States and analyze the effects that these activities have on the value of the companies. They found that the hedges reduce the price sensitivity of the companies' shares to variations in the prices of hydrocarbons.

Woo, Olson & Horowitz (2006) prove, through a partial regression model, that California NG users in the United States can take advantage of the opportunity to cross-cover with the Henry Hub index and can even predict the behavior of the index in the future and, thereby, improve risk management through futures contracts or swaps. Wong-Parodi, Dale & Lekov (2006) compare the NG price forecasts

published by the Energy Information Administration (EIA) with those of the New York Mercantile Exchange (NYMEX) market and found that the futures market prices are a better forecast than the forecasts released by the government agency.

Rosellón (2008) highlights the work of the CRE since 1996 for adopting an NG pricing methodology that links them to the prices of this same substance in the South of the United States. The author points out that, in setting a reference price, it is appropriate to use two geographical points: the one of importation and the one of domestic production, and to adopt an intermediate point that presumably reduces the arbitrage between the option of importing and buying gas from domestic production. Brown & Yücel (2008) study the separation between the prices of NG and those of crude oil. They develop a vector error correction (VEC) model, with which they demonstrate that the prices of the crude affect those of the NG. So, both goods can be considered substitutes. Suenaga, Smith & Williams (2008) examine the volatility of the prices of the NG futures in NYMEX and conclude that the prices show seasonality in the winter. Besides, the effect of price shocks is persistent. Therefore, hedging strategies that do not consider these factors are sub-optimal.

Agnolucci (2009) compares the predictive capacity of GARCH models and that of implied volatility to estimate the volatility in the prices of West Texas intermediate (WTI) futures contracts in the NYMEX based on statistical and regression results. Kaufmann & Ullmann (2009) study the effect of innovation on hydrocarbon prices and how these effects are propagated to other prices in the spot and futures markets, they also analyze the long-term relationship between spot and futures prices. Wei, Wang & Huang (2010) use different models of the GARCH type to estimate the price volatility of the Brent and WTI crude markers. They found that non-linear models are better for capturing long-term effects and asymmetric volatility.

Laurent, Rombouts & Violante (2012) investigate the selection of different MGARCH models in large-scale portfolios and find that the models are inaccurate in periods of instability. Nomikos & Andriosopoulos (2012) investigate the behavior of the prices of eight energy products listed on the NYMEX, both in the spot market and in the futures market, and conclude that there is a leverage effect on the WTI and heating oil, while in the rest of the markets the effect is inverse. Wang & Wu (2012) forecast energy market volatility using uni and multivariate GARCH models. They propose hedging strategies based on multivariate models. Lv & Shan (2013) model the volatility of the NG market using GARCH models with long memory distributions and fat tails. Gannon & Liu (2013) propose a dynamic method of rebalancing asset hedges extending the GARCH-BEKK (BEKK are the initials of the authors Baba, Engle, Kraft and Kroner) approach to one MGARCH DCC (DCC stands for Dynamic Conditional Correlation). Scholtens & Van Goor (2014) analyze the volatility in NG prices in the United Kingdom and conclude that GARCH models based on supply and demand and theoretical assumptions of an economic nature are good predictors.

Blazsek & Villatoro (2015) compare GARCH and EGARCH (Exponential GARCH) Beta-t models and conclude that the EGARCH Beta-t models had a higher forecasting capacity in the period after the 2008 financial crisis in the United States. Asche, Oglend & Osmundsen (2017) find that when NG prices are decoupled from crude oil prices due to short-term effects. So, models such as VEC ones lead to erroneous conclusions about the nature of the cointegration relationship. Ghodussi & Emamzadehfard (2017) experiment with hedging alternatives in the US NG market. They contrast the use of a single type of futures contract with the use of futures contracts that exceed the maturities of the obligations to cover six different physical positions. They found that extending the term of future contracts can increase the effectiveness of the hedging. Gulay & Emec (2018) compare the variance normalization and stabilization method (NoVaS) with different GARCH methods in forecasting the volatility of different financial series and find that the NoVaS method has a higher forecasting capacity for values that are out of the sample.

Few studies analyze energy hedging in Mexico or even Latin America. For example, Barrera-Rivera & Valencia-Herrera (2019) describe a regulatory price model for NG in Mexico, propose an NG price hedging model, estimate optimal hedge ratios, and evaluate positions with some suitable future contracts. They propose two price hedging strategies: the first one through futures contracts and the other one using swaps. Based on the methodology of the PVPM, the optimal hedging with futures considers NG futures contracted one and two months earlier, plus contemporary exchange rate futures. Another study is Gutiérrez (2016) that focuses on cross hedging in the Mexican oil market with a multivariate GARCH model. Also, Díaz Contreras et al. (2014) analyze hedging strategies for the Colombian energy market. Related literature analyzes the use of international agricultural derivatives for hedging agricultural commodities in Latin America, see, for example, Troncoso-Sepúlveda & Cabas-Monje (2019), Ortiz Arango & Montiel Guzmán (2017), Ortiz Alvarado & Girón (2015), Guízar Mateos, et al. (2012), and Godínez Placencia (2007).

The present study proposes a dynamic hedging approach that considers conditional variances and covariances within an MGARCH VCC model and a larger sample than in Barrera-Rivera & Valencia-Herrera (2019). With this approach, we can predict optimal hedge ratios that can be used in immediate periods beyond the sample. In the following section, we give an overview of the methodology followed in this paper, the single hedging strategy and the main proposal in this paper, a dynamic hedging approach based on an MGARCH VCC model.

3.4. Methodology

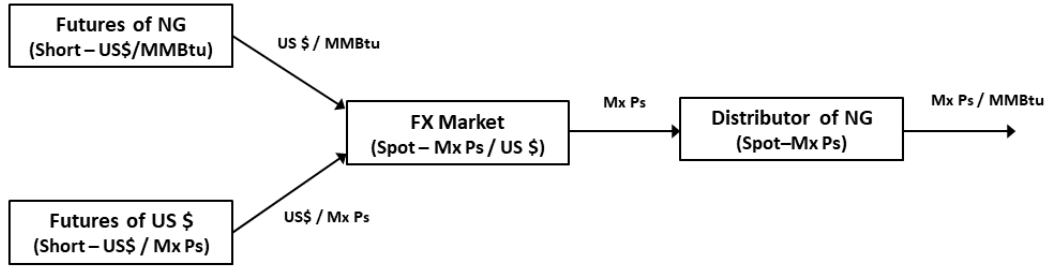
3.4.1. Use of future contracts as hedging

For purpose of explanation, let's introduce the following case: A NG distributor of an urban area acquires the fuel that it will subsequently sell to domestic or industrial users from PEMEX or an importer. The price of the NG is acquired at a fixed price at the entrance of the urban area (city gate) once the gas has been transported from the point of importation or from a processing terminal. For a month, the purchase price of the gas will be fixed in pesos and the distributor, in turn, must sell it at a fixed price to its users. The next month, the distributor will buy the NG at another price, which will depend on fuel prices in South Texas and the peso-dollar exchange rate, among others. To manage the risk represented by the variation of the NG in dollars and the exchange rate, the distributor may take positions of futures contracts for the gas and for the exchange rate. As was stated, the price volatility of NG in dollars may be higher than the volatility of the exchange rate so that the two hedging strategies, one for the price of NG in dollars and the other for the exchange rate in pesos, could be independent and intermittent.

The NYMEX market offers NG futures contracts with monthly maturities that span a decade ahead. For example, the December 2016 contract was last listed on November 28, had physical delivery on December 31, 2016 and each contract covers 10,000 MMBtu (ten billion Btu). The pulse (tick) of quotation prices is 0.001 US dollars. On the other hand, the contract of future peso-dollars in the Chicago Mercantile Exchange (CME) covers Mx Ps 500,000, with a minimum fluctuation in the price of USD 0.00001 per peso, equivalent to 5 dollars per contract. The contracts have monthly maturities and cover a period of 18 months.

A distributor that estimates that the prices of NG in dollars will be on the rise and that the peso will depreciate in the coming weeks or months can buy NG futures in the NYMEX and buy dollar futures in the CME. To allow these operations, the distributor will need to open contracts and provide guarantees, and before the expiration of the contracts, he or she must revert them, unless the distributor wishes to reach the "physical delivery" of the goods. In case of the reversal, the distributor will take his or her profit or loss, and with it he or she will go to the spot exchange market to convert the dollars to pesos. With the possible benefit, the distributor can acquire NG from the new PVPM. If the hedging strategy was successful, the distributor will have the ability to acquire the same or a higher volume of NG as a result of good risk management. The operation would be contrary if the price expectation were down: Futures would be sold in the NYMEX and, if necessary, peso futures would be bought in the CME. In any case, the resulting dollar would be expected to be positive. Figure 3.1 shows the use of futures contracts as a hedging tool.

Figure 3.1. Employment diagram of NG and dollar futures hedge.



Source: Own elaboration

The classic theory of hedging with futures, see for example Hull (2009) and Ghoddsi & Emamzadehfard (2017), consists of reducing or nullifying the price volatility of a spot position with the inclusion of a certain number of futures contracts in the portfolio. If we have a P portfolio with n_S long asset positions and n_F short futures positions, the hedge ratio is defined as the number of futures positions that are occupied to cover a unit of the spot position, that is, $h = n_F/n_S$.

The value of the portfolio, considering n_S units of assets to be covered and n_F units of futures, would be given by equation (3.1),

$$P = n_S S - n_F F \quad (3.1)$$

Therefore, changes in the covered portfolio are given by equation (3.2),

$$\Delta P = n_S \Delta S - n_F \Delta F \quad (3.2)$$

The minimum variance hedge ratio is estimated by selecting the number of futures contracts that minimizes the conditional variance of changes in the value of the portfolio. The optimal hedging ratio is given by equation (3.3),

$$h^* = \frac{n_F}{n_S} = \frac{Cov(\Delta S, \Delta F|I)}{Var(\Delta F|I)} \quad (3.3)$$

where I is the set of information in time t and h^* and is the optimal hedging ratio. The hedge ratio h^* can be easily estimated using ordinary least squares (OLS), as in equation (3.4), where ΔP_t is the PVPM monthly growth rate and $\Delta F_{l,t}$ is the monthly growth rate of an NG Futures Contract with l lags,

$$\Delta P_t = a_0 + a_1 \Delta F_{l,t} + \varepsilon_t \quad (3.4)$$

and thus, the optimal hedge ratio h_j^* would be applicable to the hedging instrument F_j , as shown in equation (3.5),

$$h_j^* = \frac{n_F}{n_S} = \frac{\text{Cov}(\Delta S, \Delta F_j | I)}{\text{Var}(\Delta F_j | I)} \quad (3.5)$$

The historical data can not only serve to determine an optimal hedging up to the last date of the data, it can also contribute to estimating the hedge that must be taken to face a risk that is expected in the immediate future through the prediction of conditional variances. Additionally, the initial strategy may change as new data is known that makes it necessary to rebalance the portfolio. In summary, in cases where there is a certain seasonality, historical data can be used to estimate future parameters, and it is convenient to update the information with newly available data that, in turn, will result in new estimates. Let us introduce the VCC multivariate GARCH model proposed to replicate the volatility of the underlying and suitable hedging instruments.

3.4.2. Multivariate GARCH VCC model

GARCH models are those in which the conditional variance of the errors can be explained through the variance of the previous errors and, usually, they are used together with the ARCH (Autoregressive Conditional Heteroscedasticity) models in which the conditional variance of the errors is explained through the behavior of the errors of the past periods. See Engle (1982) and Bollerslev (1986).

Different authors have used and evaluated the use of GARCH models as predictive tools to estimate price volatility, particularly in energy. See Agnolucci (2009), Wei, Wang & Huang (2010), Wang & Wu (2012), Lv & Shan (2013), Gannon & Liu (2013), Scholtens & Van Goor (2014), and Blazsek & Villatoro (2015).

The multivariate GARCH models (MGARCH), following the notation of Orskaug (2009), are defined as:

$$r_t = \mu_t + a_t \quad , \quad (3.6)$$

$$a_t = H_t^{1/2} z_t \quad , \quad (3.7)$$

where:

r_t : $n \times 1$ vector of the logarithmic returns of n assets in time t ,

a_t : $n \times 1$ vector of mean-corrected returns of n assets in time t , so that $E[a_t] = 0$,

$\text{Cov}[a_t] = H_t$.

μ_t : $n \times 1$ vector of the expected conditional values of r_t .

H_t : $n \times n$ matrix of conditional variances - covariances of a_t in time t .

$H_t^{1/2}$: any $n \times n$ matrix in time t as H_t is the matrix of conditional variances of a_t . $H_t^{1/2}$ may be obtained by a Cholesky factorization of H_t .

z_t : $n \times 1$ vector of errors iid such that $E[z_t] = 0$ and $E[z_t z_t^T] = I$.

μ_t in equation (3.6) can be modeled as a constant vector or as a time series; it is not correlated in time, which does not mean that it does not have a serial dependency, but that the dependency can be non-linear. On the other hand, H_t in equation (3.7) is a matrix of conditional variances, which needs to be inverted every period t . Besides, for Cholesky factorization to be possible, H_t must be positive and defined.

In the VCC multivariate GARCH model, conditional variances are modeled as univariate GARCH models and conditional covariances are modeled as non-linear functions of conditional variances. The parameters of the quasi-correlations involved in the non-linear functions of the conditional variances follow a GARCH model specified by Engel (2002). In the MGARCH VCC there is a revolving estimator of the covariance matrix of standardized residues, following the development of Tse & Tsui (2002).

The optimal hedge ratios h_j^* of equation (3.5) can be calculated with the conditional variances and covariances obtained through the MGARCH VCC model. These optimal hedge ratios can correspond to the whole period of data or they can be estimated for subperiods, even on a daily basis, as conditional variances and covariances can be obtained dynamically, that is, the newest estimates considers the last historical data available, as new information arrives, a new set of conditional variances and covariances can be calculated, and thus, new optimal hedge ratios. This can be performed with in-sample or out-of- sample data.

In the following section, we discuss the data, the results from a single hedging strategy, the optimal hedging strategy from a MGARCH VCC model, and the suitability of the dynamic hedging proposal with the use of a backtesting tool.

3.5. Data and Results

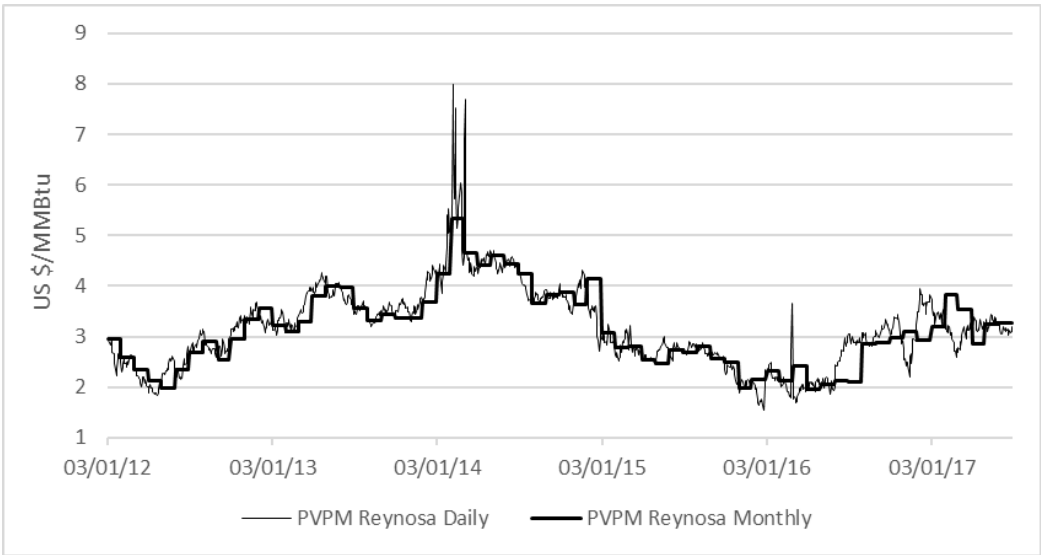
3.5.1. Data

The data sample is from the beginning of 2012, until June 30, 2017 when CRE ended the publication of the PVPM. The NG price series in the United States are from the US Energy Information Administration (EIA) website; spot and futures market prices correspond to those of NYMEX; the PVPMs in Reynosa and Pemex City are those published by the CRE, and the exchange rates of the peso-dollar are those published

by Banco de México (BANXICO). In its first part, as was stated, this study follows the methodology of Barrera-Rivera & Valencia-Herrera (2019) for an extended study period.

Figure 3.2 shows the graph of the daily and monthly PVPM in Reynosa during the study period. Notice that the monthly values do not correspond to the average of the daily values. The reason is that monthly PVPMs were determined one day before the beginning of the month and sustained throughout the period. However, daily prices were also calculated one day in advance and modified daily, which allowed them to reflect information more up to date on international prices. It should be noted that the PVPMs correspond only to business days, they exclude weekends and holidays.

Figure 3.2. Daily and monthly PVPM in Reynosa in the period of study.



Source: Own elaboration with data of the CRE

Table 3.1 shows the statistics of the continuous growth rates of the Reynosa's daily PVPM in dollars (USD PVPM), of the NG Spot Price at NYMEX (Spot NYMEX) and the Mexico United States exchange rate (Mx Ps – USD XR). The value of the skewness in the PVPM (-0.63395) indicates that the distribution is moderately biased and the value of kurtosis (57.51959) shows that the distribution is sharply leptokurtic; therefore, it is not normal. However, the elements of greatest interest for this work are those of volatilities: the standard deviation for the logarithmic variation of the daily PVPM was 0.04421 and the standard deviation of the logarithmic variations of the NG Spot Price in NYMEX was 0.03920, which means that, during the study period, the PVPM in Reynosa was more volatile than the NG spot price in NYMEX.

Table 3.1 also shows the statistics of the logarithmic exchange rate variations (Mex Ps - USD). During the study period, the standard deviation of the continuous daily growth rate was 0.0058. It should be noted that, although the prices and quotations of this study refer to the same period of analysis, the observations of the exchange rate include dates of weekends and others in addition to those of NG prices. During the period considered, the volatility of NG dollar spot prices (NYMEX), measured through the standard deviation of the logarithmic variation, was 6.7 times the volatility of the peso-dollar exchange rate.

Table 3.1. Statistics of selective series in the study period.

	Mean	Standard Deviation	Kurtosis	Skewness	Observations
USD PVPM	5.05045E-05	0.04421	57.51959	-0.63395	1,364
Spot NYMEX	2.46433E-06	0.03920	20.55434	1.03573	1,364
Mx Ps – USD XR	0.00012	0.00588	15.82514	1.12651	2,007

Source: Own elaboration with data of the CRE

Figure 3.3 shows graphically the peso-dollar exchange rate in the study period. As can be seen in the figure, the exchange rate experienced significant volatility from 2015 until the last date of the analyzed period.

Figure 3.3. Daily exchange peso-dollar rate during the study period.



Source: Own elaboration with BANXICO data

Table 3.2 depicts numerically the differences in the standard deviations of the prices during two subperiods: from January 2012 to December 2014 and from January 2015 to the end of the sample period. Standard deviations of the second subperiod

are larger than those of the first subperiod in the domestic markets. The opposite happens with the NG spot prices of NYMEX where the volatility is greater in the first subperiod. This shows that the local market had its own sources of variations. Notice that in both subperiods, the volatility of the NG spot price in the NYMEX was several times greater than the volatility of the peso-dollar exchange rate.

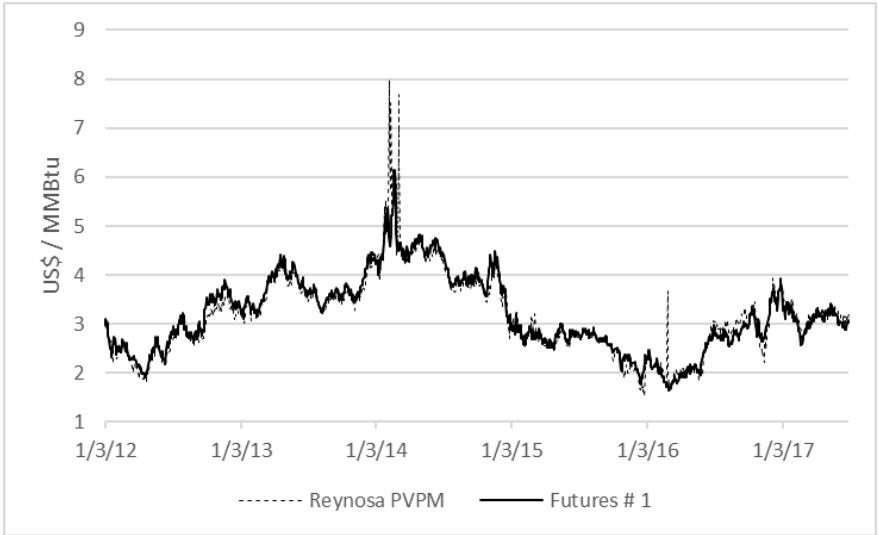
Table 3.2. Standard deviations observed in sub periods during the study period

	Standard Deviation Whole Period	Standard Deviation 2012-2014	Standard Deviation 2015-Jun 2017
USD PVPM	0.04421	0.04006	0.04879
Spot NYMEX	0.03920	0.04139	0.03641
Mx Ps – USD XR	0.00588	0.00499	0.00779

Source: Own elaboration with data of the CRE and BANXICO

Since there is an open market to import NG to Mexico, the PVPM in Reynosa was the reference for the other local market prices, even for the PVPM in Pemex city, the main production center. Therefore, we will focus on the PVPM of Reynosa and, first, on its monthly version. During the study period, the monthly PVPM in Reynosa in dollars is highly correlated with the daily one-month future prices 'Future # 1' of the NYMEX NG. Figure 3.4 shows graphically the proximity of the monthly PVPM and the daily prices of the future contract.

Figure 3.4. Daily PVPM of Reynosa in dollars vs. prices of NYMEX Futures Contract # 1 in study period.



Source: Own elaboration with data from CRE and NYMEX.

3.5.2. The simple hedging strategy

In the case of an urban NG distributor and that of an industrial user of the product. We consider the CME lists NG futures contracts that take Henry Hub index prices as a reference. These futures contracts have a very close relationship with their underlying. Also, gas prices in Henry Hub have a very close relationship with those of Texas Eastern STX, Tennessee Zone 0 and Houston Ship Channel, as can be seen in Barrera-Rivera & Valencia-Herrera (2019). Therefore, the hedge ratios consider PVPMs in Reynosa as spot prices and the CME NG Henry Hub futures.

In order to estimate the hedge ratios, we use equation (3.4), where ΔP_t the monthly growth of the PVPM in Reynosa at month t and $\Delta F_{1,t}$ is the one-month growth of the one-month Henry Hub gas future price at month t. Note that, due to the solution of the OLS method, the coefficient a_1 in equation (3.4) is the same as the optimal hedge ratio h^* in equation (3.3).

Since the estimation of PVPMs considers international NG previous prices, futures from previous periods can be useful for making PVPM hedges. Figure 3.5 shows the growth in the prices of the three-month futures and the growth of the PVPM prices in Reynosa in dollars. Notice that PVPM of Reynosa with one and two months of advance and delay have a statistically significant relationship with the futures at three months.

Figure 3.5. Correlation between the growth of the Henry Hub three-month futures and those of the PVPM in Reynosa in dollars during the study period.

Future #3, Reynosa (-i)	Future #3, Reynosa (+i)	I	Delay	Advance
		0	0.1632	0.1632
		1	0.4306	0.1037
		2	0.2483	0.0158
		3	-0.0919	0.0164
		4	-0.0212	0.0694
		5	0.1360	-0.0795
		6	-0.1308	-0.1402
		7	0.0585	-0.0645
		8	-0.0576	-0.0568
		9	0.0398	0.2435
		10	0.1552	0.0495
		11	-0.0744	-0.1489
		12	-0.1433	-0.1192
		13	-0.0845	-0.0745
		14	-0.0625	0.0405
		15	0.0605	-0.0214
		16	0.2066	0.1056
		17	-0.0679	-0.0985
		18	0.0645	0.1463
		19	-0.0366	-0.0155
		20	-0.0868	-0.1127

Source: Own elaboration with data of the NYMEX and CRE

From Figure 3.5, it could also be stated that there may be more than one hedging instrument, for example, the Future # 3 with zero and one month of offset, so that equation (3.3) could be extended to more than one hedging instrument.

Table 3.3 shows the hedge ratios with NG futures of the CME for the period and previous periods. From the table, only the futures of one and two delayed periods to the PVPM offer hedging possibilities, since only in these cases h_j^* are statistically significant. The optimal hedging of a natural gas seller in the Mexican market could be structured with the instrument lagged one month by taking a short futures position for 54.7082% of the value of the position to be filled a month before the natural gas is sold to PVPM. The R^2 is an indicator of the potential risk reduction using hedging, here 14.0259 %. Since the optimal hedging ratio for futures with two months of delay is statistically significant, it could be hedged, for example, the purchase of PVPM buying futures for 36.4429% of the value of the position to be filled two months before it was made the purchase.

Table 3.3. Optimal hedge ratios of PVPM of Reynosa with three-month futures with delay.

Delayed months of Future #3	h_j^*	Standard Error	T Statistics	R^2
0	0.211115	0.179270	1.177642	0.018822
1	0.547082	0.167063	3.274710**	0.140259
2	0.364429	0.174800	2.084838**	0.057112

** , statistically significant at 95%.

Source: Own elaboration with data of the NYMEX and CRE.

The hedging can be structured by acquiring multiple futures during several previous periods. Because the autocorrelation in the growth of futures with months of lag is very small and not statistically significant, it is possible to consider futures with arrears of one and two months as independent instruments. Therefore, the coefficients that are obtained when making a linear regression of the growth in PVPM with respect to the growth of futures with one and two months of lag can be considered as optimal hedge ratios with each instrument, in a multiple hedging. From Table 3.4, a position of gas subject to PVPM could be filled with futures of different maturities with one and two months of lag, acquiring futures at two months, with a value of 56.0542% of the position, one month before and 31.11712% of the value to cover two months before, for a risk reduction of 23.3381% (R^2).

Table 3.4. PVPM hedging in dollars with two Henry Hub futures instruments with one- and two-month lag.

	h_j^*	Standard Error	T Statistics
Future # 2 (-1)	0.560542	0.149673	3.745105**
Future # 3 (-2)	0.311712	0.159525	1.95399**

and, statistically significant at 99% and 94%, respectively.*

$$R^2 = 0.233381$$

Source: Own elaboration with data of the CRE and NYMEX.

To analyze the hedge with exchange rate futures for the purpose of analysis, synthetic futures prices were estimated using the interest rate parity $F_t = S_t(1 + r_d)^t / (1 + r_f)^t$, where F_t the price of the future quoted at period t , S_t is the exchange rate spot in direct quotation and r_d y r_f are the effective domestic and foreign rates at the future term in period t , respectively. In this case, the rates of the 91-day Cetes and the 90-day Treasury Bills were considered, adjusted for a period of one month. In a similar exercise, the PVPM in Reynosa can be covered with two- and three-month NG futures with one and two lags and one-month MXN-USD exchange rate futures, see Table 3.5.

Table 3.5. PVPM hedging models in pesos in the study period with Henry Hub futures and Mex Ps-USD exchange rate futures in the study period.

Instrument	h_j^*	Standard Error	T Statistics
Mex Ps-USD (1)	1.420590	0.392058	3.623420**
Future # 2 (-1)	0.609386	0.137362	4.436364**
Future # 3 (-2)	0.278413	0.145986	1.907120**

$$R^2 = 0.371015$$

Source: Own elaboration with data of the CRE, NYMEX, BANXICO and Bloomberg.

3.5.3. Hedging under the MGARCH VCC model

In Barrera-Rivera & Valencia-Herrera (2019), optimal hedging ratios h_j^* are estimated for a PVPM spot position with the exchange rate and one month and two months lagged NG futures. Once the hedge is determined, it is can be necessary to rebalance the hedge based on estimates of conditional variance forecasts and correlations, both variations in the PVPM in Reynosa and of the NG futures used, since these elements concentrate the risk.

Table 3.5 shows the results of the MGARCH VCC for the daily series of the variations of the PVPM in dollars ('reynosavpm') and the two-month futures of the NG Henry Hub, with 20 and 40 days of lag ('lag20' and 'lag40', respectively). The 20 and 40 days of lag are equivalent, in the daily series of prices, to 1 and 2 months of lag in the monthly series used in section 3.5.1. For the model, 1,324 daily observations were used, distributed in a t-student manner and a Newton-Raphson optimization method. From the results of Table 3.6 it follows that the ARCH and GARCH coefficients are statistically significant at more than 99%; in the estimation of correlations, an acceptable statistical security was not achieved.

Table 3.6. MGARCH VCC model of the daily variations of the PVPM Reynosa in dollars and the Henry Hub two-month futures, with lags of 20 and 40 days in the study period.

	Coefficient	Standard Error	Z	
ARCH_reynosavpm				
Arch L1.	.172906	.026373	6.56**	
Garch L1.	.7638289	.0314717	24.27**	
_cons	.0000642	.0000165	3.89**	
ARCH_lag20				
Arch L1.	.0391612	.0107685	3.64**	
Garch LI.	.9315916	.0188782	49.35**	
_cons	.0000201	8.47e-06	2.37**	
ARCH_lag40				
Arch L1.	.0508818	.0120538	4.22**	
Garch L1.	.9248127	.0177219	52.18**	
_cons	.0000178	7.52e-06	2.36*	
corr(reynosavpm,lag20)	.0060658	.0308313	0.20	
corr(reynoaavpm,lag40)	.0408179	.0307444	1.33	
corr(lag20,lag40)	-.0016276	.0310237	-0.05	
Adjustment	lambda1	.0130937	.039509	0.33
	lambda2	.7153054	1.327244	0.54
Degree of Freedom	_cons	9.49200	1.00104	8.77**

**,* statistically significant at 99% and 95%, respectively.

Source: Own elaboration with data of CRE and NYMEX.

In the case of daily variations in Reynosa PVPM in dollars, the two-month futures with 20 days lag, and the two-month futures with 40 days lag, the conditional variance is estimated as in equations (3.8) to (3.10), respectively,

$$\sigma^2_{1,t} = 0.0000642 + 0.172906 \varepsilon^2_{1,t-1} + 0.7638289 \sigma^2_{1,t-1} \quad (3.8)$$

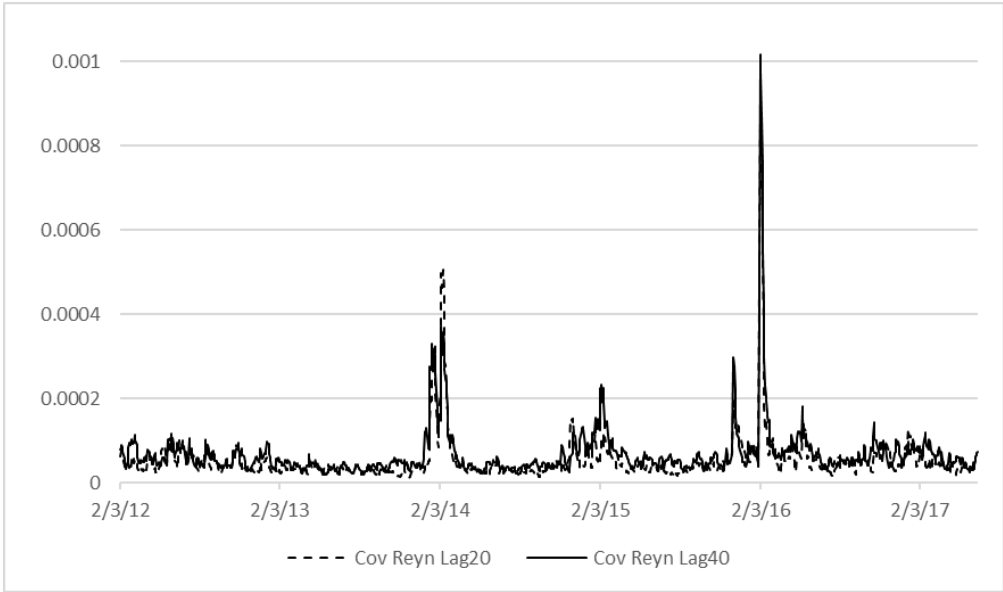
$$\sigma^2_{2,t-20} = -0.0000201 + 0.0391612 \varepsilon^2_{2,t-21} + 0.9315916 \sigma^2_{2,t-21} \quad (3.9)$$

$$\sigma^2_{3,t-40} = -0.0000178 + 0.0508818 \varepsilon^2_{3,t-41} + 0.9248127 \sigma^2_{3,t-41} \quad (3.10)$$

With these values of conditional variances, the new optimal hedge ratios h_j^* can be estimated and done on a recurring basis, as new information is obtained, in the manner of Gannon and Liu (2013). The model in Table 3.6 and equations (3.8), (3.9) and (3.10) can be used both to forecast conditional variances of future periods, and to estimate conditional variances for the historical data period itself and, with conditional variances, calculate the optimal hedge ratios h_j^* by applying equation (3.5).

Figure 3.6 graphically shows the conditional covariances estimated in the study period obtained using the MGARCH VCC model. The estimation of conditional variances is dynamic, that is, even if the determined coefficients are applied to current data, the value of these coefficients is updated as new information is received and these new values are applied to the following current data.

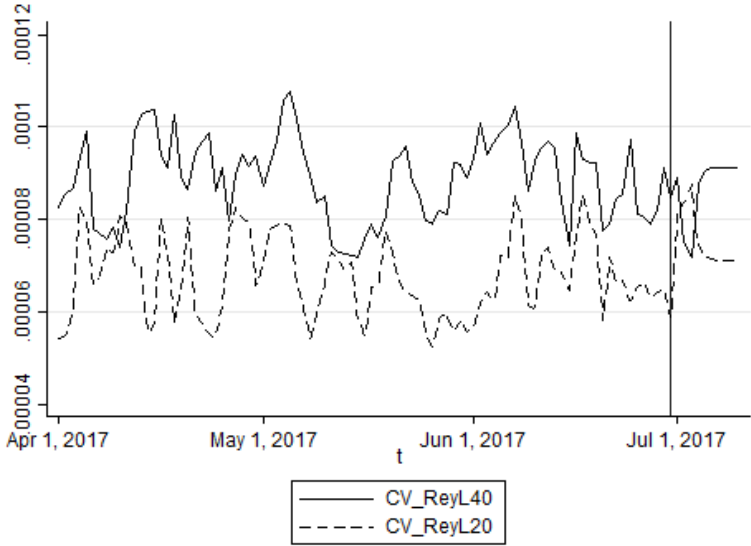
Figure 3.6. Estimated conditional covariances between the daily growth of the PVPM in Reynosa in dollars and the daily growth of the two-month Henry Hub futures with lags of 20 and 40 days in the study period.



Source: Own elaboration with data from CRE and NYMEX.

Figure 3.7 shows the estimates of the conditional covariances between the variations of the PVPM in Reynosa and those of the two-month Henry Hub futures, with lags of 20 and 40 days, for the last 100 days of the series, which include 10 forecasted days. The predicted conditional covariances are those that appear after the vertical line. The covariances of the last 100 days are shown in Figure 3.7, however, the data of the 1,384 days of the study period were used to obtain them. The purpose of Figure 3.7 is to depict in greater detail the last part of the estimated conditional covariances.

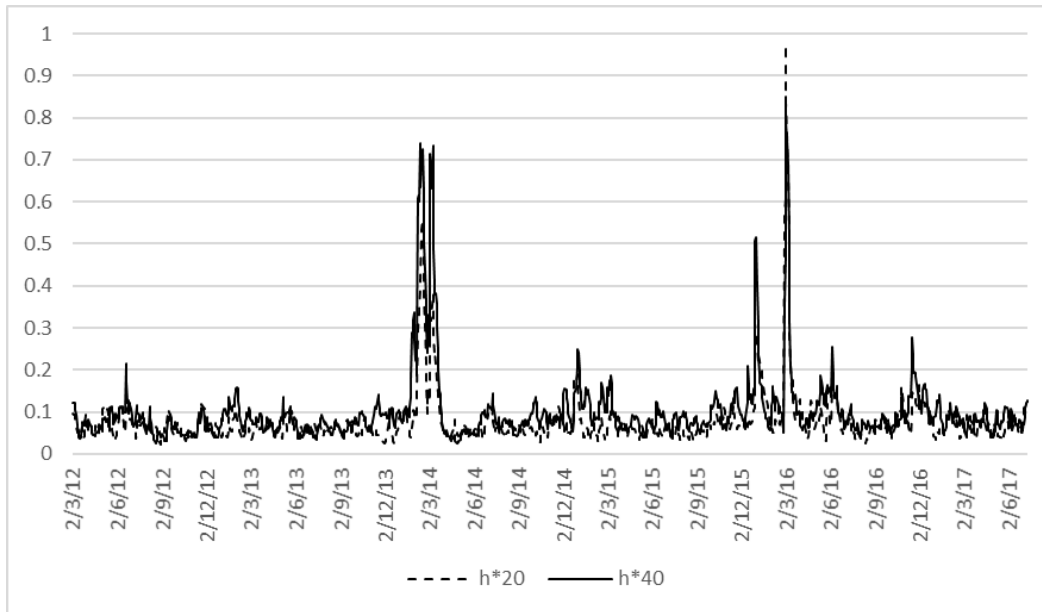
Figure 3.7. Estimated conditional covariances between the daily growth of the PVPM in Reynosa in dollars and the daily growths of the two-month Henry Hub futures, with lags of 20 and 40 days in the period of the last 90 days of the historical series and the first 10 days forecast.



Source: Own elaboration with data of the CRE and NYMEX

As already stated from the data of the conditional variance matrix, the optimal hedge ratios h_j^* are obtained using equation (3.5). Figure 3.8 shows the graph of the hedge ratios between the spot position and the futures with lags of 20 and 40 days in the study period.

Figure 3.8. Optimal hedge ratios h_j^* between the daily growth of the PVPM in Reynosa in dollars and the daily growth of the two-month Henry Hub futures, with lags of 20 and 40 days in the study period.



Source: Own elaboration with data from CRE and NYMEX.

Table 3.7 lists the optimal hedge ratios h_j^* predicted for the 10 days following the last date with historical data and, as support, the conditional covariances between the spot position and the futures are detailed.

Table 3.7. Relationship of conditional covariances and optimal hedge ratios h_j^* predicted for 10 days with the two-month Henry Hub futures, with lags of 20 and 40 days.

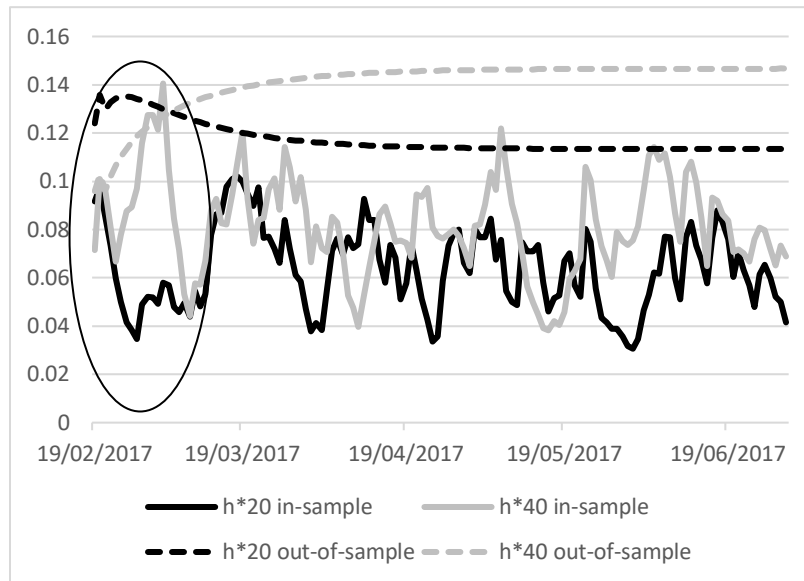
Day Forecast	Cov Reyn Lag20	Cov Reyn Lag40	h*20	h*40
1	0.000032	0.000032	0.060609	0.074616
2	0.000047	0.000034	0.086592	0.075478
3	0.000060	0.000033	0.109885	0.071314
4	0.000061	0.000043	0.109619	0.088827
5	0.000061	0.000051	0.108659	0.101764
6	0.000062	0.000057	0.107585	0.111066
7	0.000062	0.000062	0.106600	0.117609
8	0.000062	0.000066	0.105759	0.122111
9	0.000062	0.000069	0.105057	0.125129
10	0.000062	0.000072	0.104472	0.127080

Source: Own elaboration with data of the CRE and NYMEX

3.5.4. Backtesting in the VCC model

Forecasts of conditional variances and optimal hedge ratios h_j^* say little about the goodness of the estimate. Figure 3.8 above shows the conditional covariances predicted for a period of 10 days, however, how much do the out-of-sample covariances forecasts approximate the estimated in-sample covariances? To resolve this uncertainty, we performed a backtesting; first, we used the first 90% of historical data (in-sample) to forecast the last 10% of the information (out-of-sample). Figure 3.9 graphically shows the results in the forecast period; in that figure, the first current and forecast optimal hedge ratios appear within the ellipse.

Figure 3.9. Optimal hedge ratios h_j^* in-sample and forecasted out-of-sample in the last 10% period of the data observed through backtesting.



Source: Own elaboration with data of the CRE and NYMEX

Notice that the out-of-sample predicted hedge ratios h_j^* do not closely follow short-term changes in the in-sample ratios; however, the order of the predicted ratios is the same as that of the current ones, that is, the in-sample and out-of-sample h_{20}^* predicted hedge ratios are lower than the in-sample and out-of-sample h_{40}^* predicted ratios. Table 3.8 shows the backtesting statistics, both for in-sample data and in the out-of-sample forecast period.

Table 3.8. Statistics of the optimal h_{20}^* and h_{40}^* in-sample ratios and out-of-sample forecasts in the entire period and the estimation period in the backtesting.

	First 1,192 days (90%) (In-sample)		Remaining 132 days (10%) (Out-of-sample)			
	h_{20}^* actual	h_{40}^* actual	h_{20}^* actual	h_{40}^* actual	h_{20}^* predict	h_{40}^* predict
Mean	0.082095	0.112268	0.063728	0.081820	0.117371	0.140759
Std Dev	0.076295	0.103794	0.017128	0.020148	0.006357	0.010954
Kurtosis	83.675187	25.423120	-0.689331	0.100092	1.587913	5.993851
Skewness	7.824266	4.612342	0.158550	0.129760	1.683808	-2.502337

Source: Own elaboration with data of the CRE and NYMEX

The absolute differences between the actual and predicted out-of-sample optimal hedge ratios h_{20}^* and h_{40}^* are 0.05364 and 0.05894, respectively. Table 3.8 shows the “memory” that the estimated hedge ratios retain, since they do not fully reflect the decline in actual hedge ratios in the forecast period, within the backtesting.

In order to reduce this “memory” period in the estimated hedge ratios, we reduced the period of actual data to a minimum in which the MGARCH VCC estimates were convergent with the Newton-Raphson method and we sought to make forecasts for a shorter period (10 days). Table 3.9 shows the results for this shorter backtesting period.

Table 3.9. Statistics of the optimal h_{20}^* and h_{40}^* in-sample ratios and out-of-sample forecasts for the period of the last 252 days in the backtesting.

	First 242 days (In-sample)		Remaining 10 days (Out-of- sample)			
	h_{20}^* actual	h_{40}^* actual	h_{20}^* actual	h_{40}^* actual	h_{20}^* predict	h_{40}^* predict
Mean	0.012957	-0.049824	-0.01765	-0.075207	0.019034	-0.073572
Std Dev	0.039929	0.036346	0.010531	0.014862	0.014705	0.023248
Kurtosis	-0.602354	0.613126	0.507476	2.186578	5.099191	-0.442972
Skewness	0.279302	-0.558116	0.214191	1.365924	2.206246	-0.968940

Source: Own elaboration with data of the CRE and NYMEX

The absolute differences between the actual and predicted out-of-sample optimal hedge ratios h_{20}^* and h_{40}^* in the 252 days period are 0.036685 and 0.001635, respectively, which implies reductions in the differences of the estimates of 31.61% and 97.23% for hedge ratios of 20 and 40 days. That is, the proximity of the actual data and the shortage of the predicted period result in better predictions if the historical data is enough for a convergent solution.

3.6. Conclusions and final considerations

This study focuses on the study of the dynamic hedging of NG, in particular of PVPM in Mexico. It is paradoxical that being NG a fuel of such broad use, it has attracted so little attention among researchers in the field. This study confirms, at least during the study period, that volatility in the prices of NG usually exceeds exchange rate volatilities. During the study period, the volatility of the NG prices in NYMEX was 6.7 times the volatility in the peso-dollar exchange rate; however, the correlation between variations in the price of NG and the exchange rate is close to zero. This was also true for two subperiods of the sample which show different volatility patterns.

The optimal hedging of NG first-hand sale prices (PVPMs) proposed considers the purchase of futures, months before the hedging date, which may allow arbitrage. Considering the opening of the oil and gas market in Mexico, if PVPMs are re-established, the pricing schemes must be reviewed to reflect in a timelier manner the international price levels and avoid arbitrage. A similar recommendation applies wherever PVPMs are used.

Dynamic hedging is a necessary tool for exposures to changing levels of risk, so that hedging is updated as new information is obtained. To obtain more reliable forecasts of variances, it is necessary to "filter" historical price information, so that the importance of some abrupt changes can be properly assessed and whether they are matched in other markets.

The MGARCH VCC method of forecasting conditional variances was an adequate tool for estimating optimal hedge ratios for the case analyzed. This tool improves its efficiency when the predicted period is short and the actual sample data is close and they result in a convergent solution in the estimation method.

The proposed hedging analysis and scheme is extensible to other fuels and other international markets, with little effort since the regulation of NG prices is an international regulatory practice and many countries are net importers of hydrocarbons. An immediate case is the gasoline market where gasolines spot positions can be hedged with crude oil or RBOB (reformulated blendstock for oxygenate blending) futures. Another case of great importance is the generation of electricity from NG where both markets, the power market and the NG's have their own intricacies.

The hedging strategy adopted in this investigation minimizes the variance of the hedge portfolio which it is not necessarily the most adequate approach for an investor, especially when he or she has an opinion on the price trends, in the presence of transaction costs or with a more rational attitude towards risk. In these cases, the optimal hedge solution should consider the expectations of the returns

and risk measures as well as a function to deliver the investor’s preferences under such expectations.

Finally, the field looks promising; NG pricing for a period, even without the PVPM scheme, implies costs and risks that someone must bear: the final consumer, the distributor, the importer, and/or the local gas producer. Hedging strategies allow the distribution of this risk and cost among other participants with capital structures and market views that may be different. Having a different view of the risk as a result of a forecast and, at the same time, having the hedging a cost, it is convenient to evaluate whether it is appropriate to rebalance the hedging, however, this would be subject to further study.

3.7 Post Data

This section includes materials which were not part of the original essay, however, in the view of the thesis’ examiners and my own, we consider they are pertinent to include.

3.7.1 Effectiveness of the hedging results with the backtesting

To better explain the results obtained in section 3.5.4 ‘Backtesting in the VCC model’, I am going to make use of some elements which were already included in that part of the essay.

The ellipse in Figure 3.9 intends to raise the attention to the out-of-sample short-term forecast of the model, which I reproduce in the following Table 3.10 that includes the optimal hedge ratios calculated for the 132-day out-of-sample period. Some of the figures in Table 3.10 were obtained from Table 3.8.

Table 3.10. Effectiveness of the optimal hedge ratios h_{20}^* and h_{40}^* forecasts for the 132-day out-of-sample period in the backtesting.

	Remaining 132 days (10%) (Out-of-sample)			
	actual	predict	% Abs Diff	Abs Diff
h_{20}^*	0.063728	0.117371	84.17%	0.053643
h_{40}^*	0.08182	0.140759	72.03%	0.058939

Source: Own elaboration with data of the CRE and NYMEX

In the case of the estimates of the out-of-sample ratios, even though the h_{20}^* actual value is smaller than the h_{40}^* actual value and this order in size is maintained in the forecasted values, the percentage absolute difference between the actual and the forecasted values ranges from 72.03% to 84.17%, which can be improved.

To enhance the effectiveness of the forecasts, a shorter and more recent period of historic data was used, and the forecast period was reduced to 10 days.

Table 3.11 shows the predicted hedge ratios and their effectiveness, calculated for the new out-of-sample period of 10 days, based on the in-sample information of only 242 days. Some of the figures in Table 3.11 were obtained from Table 3.9.

Table 3.11. Effectiveness of the optimal hedge ratios h_{20}^* and h_{40}^* forecasts for the 10-day out-of-sample period in the backtesting.

	Remaining 10 days Out-of-sample			
	actual	predict	% Abs Diff	Abs Diff
h_{20}^*	-0.01765	0.019034	207.84%**	0.036684
h_{40}^*	-0.07521	-0.073572	2.17%	0.001638

Source: Own elaboration with data of the CRE and NYMEX

** Without considering the difference in sign, the percentage would have been 7.84%.

Even though there is a better forecast for the h_{40}^* hedge ratio (the % Abs Diff is 2.7%), we should be cautious with negative hedge ratio figures. This means that instead of having short positions in the Henry Hub Future#1 we should be long. This is because the short-term log variations of the NG Futures are contrary in nature to the PVPM log variations. This effect happened occasionally but could not last more than a few days and was due to differences in the short-term trend of the Henry Hub NG prices and the short-term trend of the PVPM.

The hedging model proposed does not limit the sign of the optimal hedge ratio.

In terms of absolute differences there are enhancements in both forecasts. The absolute difference in the h_{20}^* forecast improved; it decreased 31.61% from the 132-day out-of-sample case to the 10-day out-of-sample case. The reduction is clearer in the h_{40}^* forecast; it was 97.22%.

There is an enhancement with the forecast capabilities of the model when the forecast period is shorter, and the historic data used are more proximate. The model can be improved introducing a positive sign constraint in the optimal hedge ratios, but just for the PVPM case, due to the time lag in the Henry Hub prices used to set the PVPM.

Chapter 4

Hedging and optimization of energy asset portfolios

Abstract

Due to the economic value and the volatility of energy markets, hedging strategies and portfolio optimization are useful tools that allow to reduce non desired levels of risk or to avoid unnecessary costs. Based on the share price data of six Latin American and other regions energy companies and two crude oil futures, this study proposes the integration of hedging portfolios and the calculation of efficient frontiers under different risk measures. The original financial series are transformed into new ones to increase granularity and extension. With the new series obtained through simulation with the support of the Extreme Value Theory and Copulas-t, different conditional risk measures are calculated. These conditional risk measures are used in the solutions of the hedging and optimization problems. To obtain these solutions, we use non-linear integer programming techniques. Additionally, we present the programming codes used to generate the new series and to solve the hedging and optimization problems.

Key Words: Financial hedging, Risk measures, Portfolio optimization.

JEL Classification: C61, G15, G17, Q49

4.1. Introduction

Hedging and portfolio optimization problems are closely related. The optimal hedging solution, which minimizes risk for all possible cases, is a specific solution to the portfolio optimization problem. The goal is to minimize a risk measure, subject to a return and other conditions, or vice versa, to maximize the portfolio return, subject to a level of risk and other conditions. The "efficient frontier" (EF) concept developed by Markowitz in 1952, is the set of optimal portfolios for a mean-variance model. In this study, we will extend this concept to other mean-risk models, in which mean is the expected value of returns and risk is a measurement of the randomness of returns that can have different traits, as will be seen later.

Energy commodities are natural resources that have become highly valuable because, on the one hand, they are non-perfect irreplaceable goods, and, on the other, they are scarce and sometimes non-renewable. Hydrocarbons, such as crude oil or natural gas, are non-renewable energy sources from which fuels and other widely used chemicals are derived. Oil and gas fields are concentrated in some regions of the world; some countries have these resources in abundance while others do not.

This study focuses on portfolios with shares or American Depositary Receipts (ADRs) of specific oil and gas companies that have attracted less attention than the world's five biggest oil consortiums, as named by Forbes magazine. In particular, we will analyze portfolios that include securities from six energy companies that trade on international markets: Amerisur Resources Plc (AMER.L) from Colombia, Ecopetrol (EC) from Colombia, Petro Matad Ltd (MATD.L) from Mongolia, Petrobras (PBR) from Brazil, Sasol Ltd (SSL) from South Africa and Yacimientos Petrolíferos Fiscales (YPF) from Argentina. The six companies' total revenues in 2019 were US \$1,112.7 billion. Also, our analysis includes short positions in two crude oil futures, Brent Crude (-BRENT FUT) and West Texas Intermediate (-WTI FUT), which are listed on the New York Mercantile Exchange (NYMEX).

Based on portfolios that include these eight securities - six long positions in stocks and two short positions in futures - we will seek to obtain optimal hedging portfolios under three risk measures: standard deviation, Conditional Value at Risk (CVaR) and Mean Absolute Deviation (MAD) and develop the efficient frontiers of these portfolios using those risk measures.

This chapter has several objectives:

1. We will obtain a new series with greater granularity and extension through a simulation from the historical price series for the securities mentioned above. The main characteristics of the original series are preserved so that we can interpolate and extrapolate the results and, with the simulated data, use numerical tools to calculate conditional risk measures. we will apply the model proposed by Nyström & Skoglund (2002), which uses the Extreme Value Theory (EVT) and t-copulas, to develop a new series from historical data.
2. We will obtain the best hedging solutions for a portfolio comprising the six selected stocks and ADRs plus short positions in the two crude oil futures with the three different risk measures with the simulated series.
3. We will obtain the efficient frontiers for portfolios composed of long positions in stocks and ADRs and short positions in crude oil futures under the risk measures mentioned above.

We will develop codes in MATLAB® to generate the new series and to obtain the hedged portfolios and efficient frontiers.

This study's importance stems from its analysis of portfolios that include international energy companies that have received less coverage from industry analysts. Besides, new data analysis techniques are applied, less conventional "coherent" risk measures are obtained, and investment portfolios are integrated along efficient frontiers. Employing these techniques is useful for diversifying risks and obtaining better investment returns.

4.1.1. Organization of the study

The organization of this study is as follows. Section 4.1 includes the introduction and research objectives. Section 4.2 describes the relevant literature to the study. Section 4.3 presents the study's methodology (i) for generating new data series; (ii) with regards to risk measures; and (iii) overall, in terms of the optimization problems to be addressed. Section 4.4 describes the data used and presents the results for the problem of hedging and for portfolio optimization to obtain efficient frontiers under different risk measures. Section 4.5 introduces a computational alternative to estimate CVaR from MAD. Finally, section 4.6 states conclusions and final considerations. A bibliography is included, and annexes detail the codes developed in MATLAB® that support data transformation, hedging, and optimization calculations at the end of this thesis.

4.2. State of the art

The literature on risk measurement is extensive, see, for example, Glosten et al. (1993), Artzner et al. (1998), Hult et al. (2012), Nyström and Skoglund (2002), Rockafellar et al. (2002), Khokhlov (2016), Du et al. (2016), Isaksson (2016), among others. In particular, Glosten et al. (1993) find support for a negative relationship between conditional expected monthly returns and the conditional variance of those returns using a Generalized Autoregressive Conditional Heteroscedastic (GARCH-M) model modified to allow (1) adjustment for seasonal volatility, (2) positive and negative innovations to returns, and (3) nominal interest rates to predict conditional variance. Artzner et al. (1998) analyze risk measurement methods, and present and validate four desirable properties for measuring risk, satisfying the "coherent" risk measures. Similarly, Hult et al. (2012) pinpoint the properties that risk measures should have. Nyström and Skoglund (2002) develop a mathematical model of the evolution of joint risk factors over time and the concept of an information hypercube. On the other hand, the distribution in the tails can have different properties from the main distribution. Rockafellar et al. (2002) establish the advantage of CVaR over VaR as a measure of risk for financial loss distribution that can involve discreteness. In this line, Khokhlov (2016) derives closed-form CVaR formulas for certain elliptical distributions. In a parallel line of research, Du et al. (2016) note that Expected Shortfall (ES) has better risk measuring properties than VaR and proposes a back-testing use of ES.

Because natural gas prices, like other commodity prices, cannot be predicted, management must handle portfolios and risk exposures. The forecasting of commodity prices is a difficult problem because these markets, in general, are efficient. For example, Mishra & Smyth (2015) conclude that natural gas (NG) futures prices are not good predictors of hydrocarbon spot prices and that NG spot and futures prices are weak-form efficient and, therefore, non-predictable using market information.

Related to portfolio risk management, there is extensive literature considering different risk measures and optimization techniques, after the initial efforts of Markowitz (1952) mean-variance analysis. For example, Konno & Yamazaki (1991) show that the optimization of a portfolio using Mean Absolute Deviation (MAD) as a risk measure can be carried out with less computational effort than the Markowitz model. Rockafellar et al. (2000) compare the minimization of Value at Risk (VaR) and Conditional Value at Risk (CVaR) as risk measures in the optimization of investment portfolios. Xu et al. (2008) use a non-linear integer programming model to find a minimum cost rebalancing solution in portfolios where the risk measure is CVaR. Wang and Zheng (2010) investigate the use of rebalancing using fat-tailed distribution functions in optimization models with downside risks. Weng et al. (2010) analyze and compare the performance of various portfolio optimization models employing different risk measures and conclude that the Mini-Max model is superior to the others. Babazadeh et al. (2019) use VaR with extreme values to measure risk in portfolio optimization models, employing a Non-dominated Sorting Genetic Algorithm (NSGA-II) and conclude that use of this algorithm allows results superior to those of other mean-VaR models.

Some recent efforts have been oriented to optimize portfolios using CVAR risk measures and multivariate distributions at the mean or the tails, for example, Forghieri (2014), Isaksson (2016), and Lönnquist (2018). Forghieri (2014) performs portfolio optimization by minimizing CVaR as a measure of risk, which he rates as a robust method, and proves that Generalized Hyperbolic distribution (GH) delivers the best fit for the real distribution of returns and is the most accurate in minimizing risk and calculating optimal weights. More recently, Isaksson (2016) develops a robust optimization method with CVaR and logarithmic returns for both elliptical and asymmetric marginal distributions with normal copulas. Lönnquist (2018) evaluates the importance of Multivariate Generalized Autoregressive Conditional Heteroskedastic (MGARCH) constant conditional correlation (CCC), dynamic conditional correlation (DCC) and varying conditional correlation (VCC) models in the context of optimal portfolios and concludes that these models have relevance for purposes of minimum-variance portfolios.

Hedging is a related risk management problem to the portfolio risk management problem. In this line, Fu (2002) explores the non-linear relationship between optimal hedge ratios and transaction costs under different spot market returns and concludes that optimal hedge ratios are relatively sensitive to transaction costs when spot market returns are low. More recently, Chen et al. (2013) propose a new spot-futures hedging method that determines optimal hedge ratios by minimizing the riskiness of hedged portfolio returns, where the Aumann & Serrano (2008) index measures risk. Regarding transaction costs, Andrade et al. (2018) evaluate the influence of stochastic transaction costs on hedging decisions.

Concerning oil hedging, among the recent contributions are Zhao et al. (2018), Batten et al. (2019), and Wang et al. (2019). Zhao et al. (2018) develop a Fractionally Integrated Generalized Autoregressive Conditional Heteroskedastic (FIGARCH)-EVT-copula-VaR model for optimal hedge ratios including hedging crude oil spot and futures markets and conclude that this model gives superior results to those of other approaches. Batten et al. (2019) study the feasibility of hedging stocks with crude oil using the GARCH DCC model and show that there are economic benefits to hedging with oil. However, the effectiveness of the hedging varies over time and depends on the state of the market. Wang et al. (2019) compare variance reduction versus other risk reduction methods in selecting the optimal hedged portfolios for crude oil markets and find results differ depending on whether the aim is to minimize variance or minimize risk.

4.3. Methodology

4.3.1. Data adjustments and simulation

Zhao et al. (2018) refer to eighteen different methods to simulate series using GARCH models in conjunction with the Extreme Value and Copula Theory. In this study, we will start with the log-returns of the original price series. Using simulation, we will generate a new series of log returns with greater granularity and extension. To do so, we will use the method proposed by Nyström and Skoglund (2002) that employs EVT and t-copulas to obtain the new series and, with those, calculate risk measures. See Mathworks (2020-1).

4.3.1.1 Nyström & Skoglund (2002) basic theoretical assumptions

According to these authors, EVT and in particular the Generalized Pareto Distribution (GPD) give an asymptotic theory for the tail behavior of a distribution. They change the focus from modelling the whole distribution to the modelling of the tail behavior. One key assumption in EVT is that extreme returns are independent and identically distributed. As there is ample evidence of volatility clustering in extreme returns they propose to apply EVT to the filtered conditional residuals. Specifically, the authors assume that the continuously compounded return process, y_t , follows a stationary ARMA-(asymmetric) GARCH model

$$\Phi(L)y_t = \Theta(L)\varepsilon_t \quad (4.1)$$

where $\Phi(L) = \sum_{i=1}^p \phi_i L^i$, $\Theta(L) = 1 + \sum_{j=1}^q \xi_j L^j$ are the lag polynomials and with ε_t decomposed as, $\varepsilon_t = z_t h_t$. The conditional variance process, h_t^2 , is governed by the recursive equation

$$h_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + a_2 \text{sgn}(\varepsilon_{t-1}) \varepsilon_{t-1}^2 + b h_{t-1}^2 \quad (4.2)$$

with the filtered conditional innovation, z_t , being independent and identically distributed with mean zero and unit variance.

4.3.1.2 Data adjustments and simulation process

New series of log returns are generated due that the original data series have step patterns and include minimum and maximum limits. The limitations of the original data render the calculation of risk measures inadequate because the tails' effects are not fully captured. The collection of new series seeks to address these limitations.

The process first extracts the filtered residuals from the original return series with an asymmetric Glosten-Jagannathan-Runkle-GARCH (GARCH-GJR) model that eliminates autocorrelation and heteroscedasticity. Then, a cumulative sample distribution function is constructed for each asset using an estimated Gaussian kernel for the interior and a Generalized Pareto Distribution (GPD) to estimate the upper and lower tails. The data are then adjusted with a t-copula that incorporates correlations between the simulated residuals of each asset.

To generate a series of independent and identically distributed (i.i.d.) observations, a first-order autoregressive model is initially applied to the conditional mean of the logarithmic returns of each asset.

$$r_t = c + \theta r_{t-1} + \epsilon_t \quad (4.3)$$

and an asymmetric GARCH-GJR model to the conditional variance.

$$\sigma_t^2 = k + \alpha \sigma_{t-1}^2 + \phi \epsilon_{t-1}^2 + \psi [\epsilon_{t-1} < 0] \epsilon_{t-1}^2 \quad (4.4)$$

The first-order autoregressive model seeks to compensate for autocorrelation, while the GARCH model compensates for heteroscedasticity. In particular, the GARCH-GJR introduces asymmetry (leverage) in the variance through a Boolean indicator that takes a value of 1 if the residual of the previous model is negative and 0 otherwise. See Glosten et al. (1993).

Also, the standardized residuals of each asset are modeled as a standardized Student's t-distribution to compensate for the fat tails associated with securities returns:

$$z_t = \epsilon_t / \sigma_t \text{ i.i.d. } t(\nu) \text{ distributed} \quad (4.5)$$

Given the i.i.d. standardized residuals of the previous step, the Cumulative Distribution Function (CDF) of each asset's return must be estimated with a Gaussian kernel, which smoothes out the estimated CDFs, eliminating a step pattern of unsmoothed CDFs from the sample. Although the CDFs estimated with a non-

parametric kernel are appropriate for the interior of the distributions where most of the data is located, they tend not to behave well when applied to the upper and lower tails. To better estimate the tails of the distribution, the EVT is applied to the tails' residuals.

Specifically, we will look for the upper and lower thresholds so that 10% of the residuals are included in each tail. We will then adjust the amount by which these extreme residuals fall into the tails, beyond the thresholds associated with a parametric GPD using the maximum likelihood method. This approach is often referred to as the "distribution of exceedances" or "peaks over the threshold."

Given the exceedances for each tail, the negative log-likelihood function is optimized to estimate the tail index (zeta) and the beta scale parameter of the GPD.

The proposed method allows interpolation inside the CDFs (Gaussian kernel) and extrapolation in the tails (GPD). Extrapolation is desirable since it allows the estimation of quantiles apart from historical data, which is highly desirable in risk management applications.

Moreover, Pareto tail objects also allow methods to evaluate CDFs and inverse CDFs and to numerically calculate cumulative probabilities and quantiles in each segmented part of the distribution.

Given the standardized residuals, the degrees of freedom, and the correlation matrix (R) for the t-copula need to be estimated, which can be done using two different methods.

The primary method is the maximum likelihood method conducted as a two-step process. The first step maximizes the log-likelihood function with respect to the linear correlation matrix, given an initial value for the degrees of freedom. This conditional maximization is allowed a gap of one degree of freedom, and the log-likelihood function is maximized again by varying the other parameters. The function that is maximized in this second step is known as the log-likelihood profile for the degrees of freedom.

In contrast, the code used in this study employs an alternative method that approximates the log-likelihood profile for the degrees of freedom parameter. Although this method is significantly faster than the primary method, it should be used with caution, as the estimates and confidence limits may not be suitable for small or medium samples.

Nyström and Skoglund (2002) propose that the user provides the degrees of freedom parameter as a specific piece of data in the simulation process, which allows the user to induce extension in the tail dependencies between the assets. In

particular, the authors recommend a relatively low value for the degrees of freedom, something between 1 and 2. This method is useful for stress tests, where degrees of extreme codependency are of critical importance.

Given the parameters of the t-copula, the returns on assets with shared dependency can now be simulated by first simulating the standardized residuals. The simulation allows us to generate what we believe are sufficient elements for the case study: from 1,242 original returns, we will simulate 2,000 new ones.

To do this, first, we simulate the dependent uniform variables. Then, we extrapolate the tails (GPD) and interpolate the internal segment (Gaussian kernel), which transforms the uniform variations into standardized residuals through the inversion of the semi-parametric marginal CDF for each asset. The procedure generates simulated standardized residuals consistent with those obtained from AR(1) + GJR(1,1) (asymmetric GARCH) already described. These residuals are independent in time but dependent at each point in time. Each set of simulated standardized residuals for each asset represents a vector with a stochastic i.i.d process if viewed in isolation, while at each point in time, the assets' returns have a copula-induced correlation.

With the use of the simulated standardized residuals as an i.i.d. noise process, the autocorrelation, and heteroscedasticity observed in the original asset returns are reintroduced, and the new simulated returns are obtained. These simulated returns will be used in the risk measure calculations in this study. As such, the risk measures are not those corresponding to historical data but rather risk measures conditional to the new series obtained as indicated.

The process described above is developed in a MATLAB® code called 'stepzero' that is included in Annex 1 of this work.

4.3.2. Risk measures

According to Isaksson (2016), financial risk can be measured in different ways, most frequently by variance. As a risk measure, the variance has the disadvantage that it does not distinguish between positive deviations from the mean (portfolio gains) and negative deviations (losses). The standard deviation, the square root of the variance, can only be considered as a measure of risk if the future value of the portfolio is distributed in a normal or elliptical way. In general, it is better to use a risk measure that distinguishes between "good" and "bad" deviations from the expected future value of the portfolio.

Artzner et al. (1999), Hult, et al. (2012) and other authors stress that risk measures must gather specific properties to be considered suitable measures. The authors call measures that meet these properties "coherent."

Let $\rho(X)$ be a function that measures the risk of a stochastic variable X . Coherent risk measures are those that have the following properties:

1. *Translational invariance.* $\rho(cR_0 + X) = -c + \rho(X)$ for $c \in \mathbb{R}$. This characteristic means that adding an amount c with an interest rate R_0 to a portfolio reduces the portfolio risk by the same amount.
2. *Monotonicity.* If $X_2 < X_1$, then $\rho(X_1) \leq \rho(X_2)$. The property implies that if it is known for sure that an X_1 portfolio is larger than an X_2 portfolio in the future, the first portfolio is considered less risky.
3. *Convexity.* $\rho(\lambda X_1 + (1 - \lambda)X_2) \leq \lambda\rho(X_1) + (1 - \lambda)\rho(X_2)$, for any $\lambda \in [0,1]$. The risk measure rewards diversification, which means that it is better to invest in more than one risk position than in one.
4. *Normalization.* $\rho(0) = 0$. This property means that it is acceptable not to invest in risky assets; that is, the empty portfolio is risk-free.
5. *Positive homogeneity.* $\rho(\lambda X) = \lambda\rho(X)$, for any $\lambda \geq 0$. The property implies, for example, that investing twice as much in a risk position doubles the risk.
6. *Subadditivity.* $\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$. This property implies that the risk measure rewards diversification. A set of two risk assets is less risky than having the two assets separately.

4.3.2.1. Standard deviation, CVaR and MAD

For purposes of this study, we will use three risk measures: (1) standard deviation, (2) CVaR, and (3) MAD.

The concept of standard deviation is widely understood and needs not to be defined here. The same could be said about MAD; in short, it is the average of the absolute deviations from the mean. While the standard deviation is differentiable in almost all its points, the algebraic treatment of MAD is complicated. It is the differentiability of the standard deviation (and variance) that enables it to be a measure of risk in a large number of optimization models in which it is assumed that the returns on financial assets can be fully explained by the first two moments of their distribution functions. As will be seen later, the use of MAD as a risk measure in numerical models is based on the simplicity of its handling. Neither standard deviation nor MAD is a coherent risk measure as they do not meet the requirement of subadditivity. The same is true of variance, nor does the variance meet the property of positive homogeneity.

CVaR, also known as Expected Shortfall (ES), is the expected value of VaR. VaR is defined as follows:

Let X be a function of the distribution of utilities (or loss). The VaR at a level $\alpha \in (0,1)$ is the smallest number y so that the probability of $Y := -X$ does not exceed y is at least $1 - \alpha$. Mathematically, $VaR_\alpha(X)$ is the quantile $(1-\alpha)$ of Y , that is:

$$VaR_\alpha(X) = -\inf\{x \in \mathbb{R}: F_X(x) > \alpha\} = F_Y^{-1}(1 - \alpha) \quad (4.6)$$

where F_X is the CDF of X and is well defined.

VaR is also not a coherent measure of risk as it does not comply with the principle of subadditivity and, as some authors have pointed out, does not provide information about the number of losses beyond the VaR threshold. Besides, numerically, in terms of optimization, VaR calculated under different scenarios results in a non-convex, "non-smooth" function with multiple local extremes, making it difficult to use optimization algorithms.

CVaR is a coherent risk measure proposed by Rockafellar & Uryasev (2000), which lacks the limitations described for the other risk measures. CVaR has computational advantages over VaR while maintaining consistency with VaR in terms of results when normal or elliptical distributions are used. In these cases, VaR and CVaR are consistent with variance results, opening the possibility for them to participate as alternative risk measures in the Markowitz optimization model.

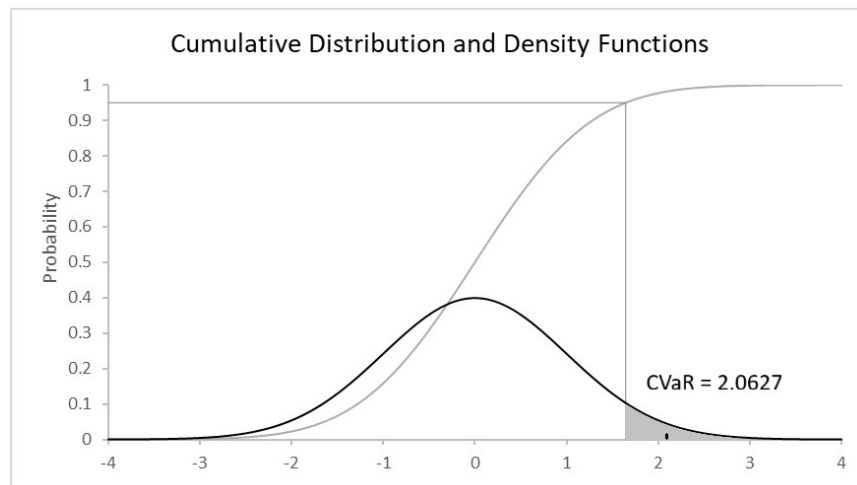
CVaR is defined as follows for the continuous case:

Let X be a continuous random variable representing a loss. Given $\alpha \in (0,1)$, the $CVaR_\alpha(X)$ is:

$$CVaR_\alpha(X) := E[X|X \geq VaR_\alpha(X)] \quad (4.7)$$

Figure 4.1 shows the VaR and CVaR of a standard normal distribution for an α of 95%.

Figure 4.1. $\text{VaR}_{95\%}$ and $\text{CVaR}_{95\%}$ of a random variable standard normally distributed.



Source: Own elaboration.

4.3.3. Optimization of functions

Problems of portfolio hedging or obtaining efficient frontiers, in which a risk measure with minimum levels and associated returns are sought in addition to the composition of specific portfolios, almost always require algorithms for their solution. Sometimes these are linear optimization problems, although, for example, when using CVaR as a risk measure, optimization problems are non-linear and require integer solutions.

Optimization problems can be linear or non-linear, depending on the nature of the objective function and its constraints. For example, the problem of minimizing MAD is usually a linear programming problem. However, depending on the type of constraints, it can become an integer programming problem, i.e., one where the solutions or constraints require integer values. CVaR minimization or standard deviation problems are usually non-linear problems of integer solutions.

To solve a complex optimization problem, such as a non-linear integer problem, there are two paths to take: choose a general solver or choose a problem solver. A general solver addresses a technique or set of general application optimization techniques, in which problems can be fragmented and addressed in a serial or parallel manner. A problem solver addresses typical cases, such as those in this study: minimizing a risk measure like CVaR, subject to linear constraints, including integers.

In this study, we will use programming objects to obtain the desired solutions, for example, and in the case of MATLAB®, mean-standard deviation, mean-CVaR, or mean-MAD optimization objects.

4.3.4. Portfolio hedging

The purpose of portfolio hedging is to create a minimal risk portfolio. In our case, the portfolio is composed of a 'long' position in the spot market that we want to hedge with a 'short' position in the futures market. The purpose of minimal risk hedging is to find a combination of hedging elements that replicates the behavior of the long positions with the least possible risk (e.g., variance). This objective is particularly attractive when it comes to creating synthetic instruments.

Batten et al. (2019) point out that the optimal hedge ratio β_t , i.e., the ratio of futures to spot assets, is defined by the following:

$$\beta_t = \frac{\text{cov}(R_{S,t}, R_{F,t} | \mathcal{F}_{t-1})}{\text{var}(R_{F,t} | \mathcal{F}_{t-1})} \quad (4.8)$$

where $R_{S,t}$ and $R_{F,t}$ are the returns of positions in the spot and futures markets respectively, and \mathcal{F}_{t-1} is the filtration with information for the period $t-1$.

Equation (4.8) is manageable when spot and futures positions maintain their composition, and when futures are not correlated to each other. However, the problem increases in complexity when compositions vary, that is, when it comes to changing the portfolio composition of spot and future assets, alternatively, as in the case of this study, when futures are highly correlated. Also, equation (4.8) is used when trying to minimize the hedged portfolio's variance. In the case of normal or elliptical distributions, this method cannot be used in a generalized manner to find the minimums of other risk measures.

As an alternative to equation (4.8), Zhao et al. (2018) describe at least six different hedging methods based on autoregressive conditionally heteroscedastic (ARCH) models, which seek to obtain an explanation of the conditional variances of the series in order to calculate optimal hedge ratios.

Another way to solve portfolio hedging is to treat it as an optimization problem that attempts to identify the minimum value of the risk measure under all feasible scenarios. In other words, the minimum of minimal risks.

We pose the following optimization problem:

$$\text{Min } \rho(X_1, X_2, \dots, X_n) \quad (4.9)$$

s.t.

$$\begin{aligned}\sum_i \omega_i r_{X_i} &\geq R_p, \\ \sum_i \omega_i &= 1, \\ \omega_i &\geq 0, \quad j = 1, \dots, n\end{aligned}$$

where $\rho(\cdot)$ is a risk measure to be minimized; X_i is a vector of random variables that represent stocks or futures; r_{X_i} is a vector of expected returns on the variables X_i ; R_p is the desired return, at the very least, on the portfolio p ; and ω_i is the weight of the value X_i in the portfolio (note that all weights must be positive and add up to one).

The code in MATLAB® that solves the hedging problem described is called 'stephedge' and is found in Annex 2 of this study.

4.3.5. Portfolio optimization

In 1952, Markowitz proposed the mean-variance (MV) model in which the risk measure is the variance, and the mean return is the key risk indicator of the expected portfolio return. The objective function of equation (4.9) above takes the following form for the MV model:

$$\text{Min } \rho(\cdot) = \text{Min } \sum_i \sum_j \omega_i \omega_j \sigma_{ij} \quad (4.10)$$

where σ_{ij} is the covariance of the returns r_{X_i} and r_{X_j} ; σ_{ii} is the variance of the return r_{X_i} , and ω_i and ω_j are the weights of the values described in equation (4.9) above. The other constraints of equation (4.9) also apply to this problem.

The MV model obtains the portfolio of minimum variance given a portfolio return R_p . However, if this return varies throughout its domain and if the constraints of equation (4.9) are met, the EF will be the location where the minimum variance portfolios lie. In other words, the EF for the MV model is the geometric place of the portfolios of minimum variance for any allowable R_p return, when the model's restrictions are met.

As mentioned above, risk measures other than variance will be used in this study. However, the generalization of the EF is applicable, i.e., the EF is the location of the least risk portfolios rather than the location of least variance portfolios, all other things being equal.

One of the drawbacks of the Markowitz model is the computational effort it requires since, for example, $\frac{n(n+1)}{2}$ covariances need to be calculated to obtain its solution.

To simplify the calculations of the risk measure, Konno & Yamazaki (1991) proposed the MAD method that gives solutions equivalent to those of the MV model for normal distributions but only using an average value.

According to Konno & Yamazaki (1991), the use of MAD instead of variance as the risk measure to obtain the EF has the following computational advantages:

1. There is no need to calculate the covariance matrix to set up the optimization model.
2. It is easier to update the model when new data are added.
3. The use of computational resources is less demanding, which enables to calculate the EF on a real time basis even when the model incorporates thousands of securities.

Isaksson (2016) points out that portfolio optimization when using CVaR as a risk measure shows a significant advantage over the MV approach when log-returns are modeled with asymmetric distributions. The disadvantage of using CVaR with simulated yields is that it introduces statistical uncertainty into the problem, beyond the calculation of historical covariances of MV and that the calculation of t-copulas involves more steps and processing time than only taking a sample from an elliptical distribution.

The objective of obtaining the EF is different from that of hedging. The EF marks different points for the risk-return combination, leaving it up to the investor to decide which point to choose, depending on their risk preferences. In contrast, as noted above, minimal risk hedging seeks to identify elements that come as close as possible to the risk pattern of the underlying or long-held securities.

For the solution to optimization problems and for obtaining the EF under different risk measures, the code 'stepone' was developed in MATLAB®, the text of which appears in Annex 3.

4.4. Data and Results

4.4.1. Data

For this study, five years of daily price data from November 10, 2014, to November 7, 2019, were used. These were obtained from the New York (EC, PBR, SSL, and YPF) and London (AMER.L and MATD.L) stock exchanges, the CME Group (BRENT FUT, and WTI FUT) and Bloomberg. Prices from the New York Stock Exchange (NYSE), as well as those of the Chicago Mercantile Exchange (CME), are in US dollars; those of the London Stock Exchange (LSE) are in pounds sterling. Bloomberg was the source for the exchange rates from pounds to dollars. All prices were converted to US dollars.

Table 4.1 shows statistics of log returns from the daily price series in dollars, with the understanding that data related to futures are already expressed as short positions. As can be seen, the AMER.L, MATD.L, and YPF series are markedly

leptokurtic, and those of MATD.L and YPF are asymmetrical, the former being positive and the latter negative. The observed asymmetry indicates that some of the series are not normally distributed.

Table 4.1. Statistics of the daily log-returns of the energy assets during the period of analysis.

	Mean	Median	Std Dev	Kurtosis	Skewness	Min	Max	Count
AMER.L	-0.0009	-0.0022	0.0400	9.3377	0.3589	-0.3122	0.3146	1,242
- BRENT FUT	0.0002	-0.0008	0.0232	2.9240	-0.0822	-0.1364	0.1029	1,242
EC	-0.0001	0.0000	0.0262	3.1804	-0.0982	-0.1608	0.1417	1,242
MATD.L	0.0002	-0.0027	0.0870	41.7854	2.3895	-0.7383	1.1757	1,242
PBR	0.0004	0.0015	0.0351	2.4016	-0.1449	-0.1852	0.1555	1,242
SSL	-0.0006	0.0000	0.0228	3.6265	-0.5096	-0.1617	0.0898	1,242
- WTI FUT	0.0002	0.0010	0.0244	2.6632	-0.1304	-0.1369	0.1079	1,242
YPF	-0.0010	-0.0018	0.0276	41.9005	-2.5939	-0.4163	0.1216	1,242

Source: Own elaboration with data of NYSE, LSE, CME, and Bloomberg.

Table 4.2 shows the correlations from the original log returns series before further processing. The futures series are strongly correlated (0.9264), as well as the series of futures and returns of almost all companies, except for the MATD.L ADR. It can also be observed that the WTI future correlates more with Latin American companies than the Brent future, except for YPF, the Argentine oil company.

Table 4.2. Correlation matrix of the original daily log-returns of the energy assets during the period of analysis.

	AMER.L	- BRENT FUT	EC	MATD.L	PBR	SSL	- WTI FUT	YPF
AMER.L	1	-0.1896	0.1957	0.0222	0.1743	0.1983	-0.1948	0.1551
- BRENT FUT	-0.1896	1	-0.6534	-0.0332	-0.4987	-0.5167	0.9264	-0.3817
EC	0.1957	-0.6534	1	0.0388	0.6118	0.5691	-0.6668	0.4240
MATD.L	0.0222	-0.0332	0.0388	1	0.0330	0.0221	-0.0307	0.0441
PBR	0.1743	-0.4987	0.6118	0.0330	1	0.5005	-0.5106	0.4454
SSL	0.1983	-0.5167	0.5691	0.0221	0.5005	1	-0.5067	0.3965
- WTI FUT	-0.1948	0.9264	-0.6668	-0.0307	-0.5106	-0.5067	1	-0.3720
YPF	0.1551	-0.3817	0.4240	0.0441	0.4454	0.3965	-0.3720	1

Source: Own elaboration with data of NYSE, LSE, CME, and Bloomberg.

4.4.2. Results

From the historical data, a new series of log returns were generated through simulation using the Nyström y Skoglund (2002) method with the 'stepzero' code. Table 4.3 shows the statistics of the simulated daily log returns series. In these new returns, the AMER.L and MATD.L ADRs maintain a marked leptokurtosis, and, the latter has a skew, now negative.

Table 4.3. Statistics of the daily log returns generated by the model for the period of analysis.

	Mean	Median	Std Dev	Kurtosis	Skewness	Min	Max	Count
AMER.L	-0.0015	-0.0022	0.0284	9.6822	0.7477	-0.2229	0.2411	2,000
- BRENT FUT	0.0016	0.0007	0.0181	1.3107	0.3939	-0.0612	0.0883	2,000
EC	-0.0005	-0.0001	0.0156	2.1953	-0.0676	-0.0702	0.1163	2,000
MATD.L	0.0024	-0.0021	0.1347	95.7780	-3.0099	-2.6245	1.3405	2,000
PBR	-0.0001	0.0001	0.0205	5.8334	-0.6048	-0.2093	0.0684	2,000
SSL	-0.0020	-0.0010	0.0315	4.2357	-0.6863	-0.2736	0.1017	2,000
- WTI FUT	0.0017	0.0011	0.0178	0.5699	0.2086	-0.0590	0.0786	2,000
YPF	-0.0004	-0.0004	0.0288	1.4766	-0.1628	-0.1871	0.1044	2,000

Source: Own elaboration with data of NYSE, LSE, CME, and Bloomberg.

Table 4.4 shows the correlations between the simulated returns series. In the simulated series, there remains a strong correlation between futures, and between futures and all stocks except for the MATD.L ADR, as in the original series. In addition, the high degree of correlation between WTI futures and Latin American series is maintained, except for YPF, as in the original correlations.

Table 4.4. Correlation matrix of the daily log returns generated by the model for the period of analysis.

	AMER.L	- BRENT FUT	EC	MATD.L	PBR	SSL	- WTI FUT	YPF
AMER.L	1	-0.1792	0.1499	0.0442	0.1394	0.1571	-0.1830	0.1504
- BRENT FUT	-0.1792	1	-0.6294	-0.0857	-0.4978	-0.5331	0.9268	-0.3872
EC	0.1499	-0.6294	1	0.0585	0.5871	0.5751	-0.6477	0.4196
MATD.L	0.0442	-0.0857	0.0585	1	0.0826	0.0577	-0.0590	0.0876
PBR	0.1394	-0.4978	0.5871	0.0826	1	0.4763	-0.5137	0.4446
SSL	0.1571	-0.5331	0.5751	0.0577	0.4763	1	-0.5180	0.4112
- WTI FUT	-0.1830	0.9268	-0.6477	-0.0590	-0.5137	-0.5180	1	-0.3723
YPF	0.1504	-0.3872	0.4196	0.0876	0.4446	0.4112	-0.3723	1

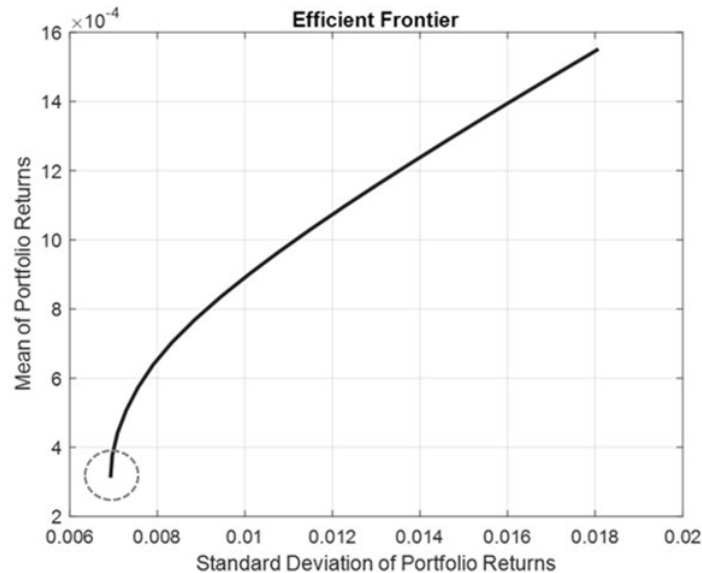
Source: Own elaboration with data of NYSE, LSE, CME, and Bloomberg.

The simulated data obtained allow risk measures such as CVaR to be calculated other than through the historical method. These same simulations can be used in the valuation of contingent assets whose underlying are the examined series.

To solve the hedging problem, a portfolio comprising the six energy stocks or ADRs, each with the same weight, was evaluated and hedged with short futures positions in Brent and WTI crudes. As noted above, short futures were introduced in the data from the beginning (negative returns, positive variances) so that the weights in the portfolio compositions were positive, which is a common constraint on optimization problem solvers.

Figure 4.2 shows the EF for the hedged portfolio, taking standard deviation as a risk measure. Note the point of minimum risk value within the circle; this is the desired hedged portfolio.

Figure 4.2. Mean-Standard Deviation Efficient Frontier for the Equally Weighted Portfolio during the period of analysis.



Source: Own elaboration with data of NYSE, LSE, CME, and Bloomberg.

Table 4.5 shows the composition of the hedged portfolios under the three risk measures considered: standard deviation, CVaR, and MAD. It also includes data on the value taken by the risk measure at that point, which is shown in **bold** and the daily portfolio return at the minimum risk point. When the risk measure is the standard deviation, the hedged portfolio is composed of 6.82% in stock and ADRs, 43.60% in a short position in Brent futures and 49.58% in a short position in WTI futures. The minimum standard deviation value is 0.0069, and the return on the hedged portfolio is 0.0003. As can be observed, given the selected hedging (short futures position in crude oil), a significant short position is required to minimize the standard deviation, which does not happen in the other two cases considered.

In each risk model, the values of the other risk measures are included. For example, the portfolio of optimal hedging in the mean-standard deviation model also provides CVaR and MAD results. However, the values in bold are the minimum values of the risk measure for each case. The portfolio of mean-standard deviation hedging is the one with the lowest standard deviation, and the portfolio of mean-CVaR is the one with the lowest CVaR, which does not imply, at least in an obvious way, that the portfolio of the standard deviation model is better hedged than the portfolio of mean-

CVaR. Each hedged portfolio is the one with the lowest risk measure, based on how it is measured. However, since CVaR is the only coherent risk measure used, its use is recommended, especially if the proportions of the portfolios should change. It should be noted that CVaR was calculated at 95%.

Table 4.5. Composition and risk levels of the hedged portfolios by risk model for the period of analysis.

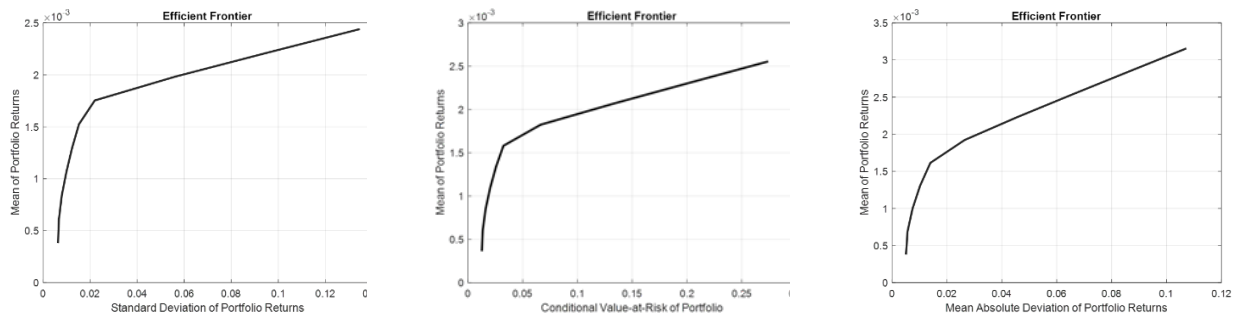
Risk Model	Weights			Risk level			Return
	Portfolio	-Brent Fut	-WTI Fut	Std Dev	CVaR	MAD	
Mean-Std Dev	0.0682	0.4360	0.4958	0.0069	0.0314	0.0126	0.0003
Mean-CVaR	0.3331	0.3458	0.3211	0.0105	0.0235	0.0094	0.0010
Mean-MAD	0.3438	0.3597	0.2965	0.0106	0.0235	0.0094	0.0010

Source: Own elaboration with data of NYSE, LSE, CME, and Bloomberg

The values obtained should be interpreted as the best values, given the series simulated from the historical series. The use of these values as predictors of future values depends on the permanence of the series' parameters.

Regarding portfolio optimization, Figure 4.3 shows the EFs graphically for the energy portfolios, using standard deviation, CVaR at 95%, and MAD as risk measures.

Figure 4.3. Efficient frontiers for the portfolios under different risk measures for the period of analysis.



Source: Own elaboration with data of NYSE, LSE, CME, and Bloomberg.

Table 4.6 shows the composition of 10 portfolios that lie on the EF using different risk measures in which, in addition to the weights, the values of risk and daily return are indicated. Note that in all cases, the minimum return is 0.0004 using the corresponding measure (standard deviation, CVaR, or MAD). There is a small difference in the maximum return (0.0024, 0.0026, and 0.0032), which should be the

same since it corresponds to having the entire portfolio invested in the MATD.L ADR. The result is due to the different values produced by the simulations.

Table 4.6. Weights, Returns, and Risk levels of Portfolios lying on the Efficient Frontiers for the period of analysis.

Mean - Standard Deviation Model										
Port #	1	2	3	4	5	6	7	8	9	10
AMER.L'	0.0536	0.0117	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
- BRENT FUT	0.1683	0.1395	0.1016	0.0588	0.0073	0.0000	0.0000	0.0000	0.0000	0.0000
EC'	0.3609	0.3321	0.2248	0.1019	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MATD.L	0.0015	0.0048	0.0097	0.0149	0.0206	0.0293	0.1213	0.4142	0.7071	1.0000
PBR	0.0871	0.1208	0.1573	0.1952	0.2092	0.0868	0.0000	0.0000	0.0000	0.0000
SSL	0.0271	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
- WTI FUT	0.2787	0.3642	0.4871	0.6191	0.7628	0.8839	0.8787	0.5858	0.2929	0.0000
YPF	0.0228	0.0269	0.0196	0.0101	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Return	0.0004	0.0006	0.0008	0.0011	0.0013	0.0015	0.0018	0.0020	0.0022	0.0024
Risk Level	0.0063	0.0067	0.0079	0.0099	0.0123	0.0152	0.0220	0.0561	0.0951	0.1347
Mean – CVaR Model										
Port #	1	2	3	4	5	6	7	8	9	10
AMER.L'	0.0515	0.0094	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
- BRENT FUT	0.1892	0.1582	0.1441	0.1155	0.0296	0.0000	0.0000	0.0000	0.0000	0.0000
EC'	0.3475	0.3020	0.1677	0.0270	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MATD.L	0.0023	0.0063	0.0111	0.0153	0.0234	0.0329	0.2297	0.4865	0.7432	1.0000
PBR	0.0960	0.1323	0.1858	0.2327	0.1697	0.0334	0.0000	0.0000	0.0000	0.0000
SSL	0.0258	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
- WTI FUT	0.2673	0.3585	0.4579	0.5775	0.7773	0.9337	0.7703	0.5135	0.2568	0.0000
YPF	0.0204	0.0333	0.0334	0.0320	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Return	0.0004	0.0006	0.0009	0.0011	0.0013	0.0016	0.0018	0.0021	0.0023	0.0026
Risk Level	0.0126	0.0135	0.0161	0.0202	0.0255	0.0323	0.0664	0.1331	0.2031	0.2741
Mean – MAD Model										
Port #	1	2	3	4	5	6	7	8	9	10
AMER.L'	0.0546	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
- BRENT FUT	0.1925	0.1536	0.1619	0.1527	0.1499	0.0000	0.0000	0.0000	0.0000	0.0000
EC'	0.3537	0.2987	0.1178	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
MATD.L	0.0020	0.0078	0.0177	0.0307	0.0494	0.2306	0.4229	0.6153	0.8076	1.0000
PBR	0.0828	0.1476	0.2180	0.1862	0.0071	0.0000	0.0000	0.0000	0.0000	0.0000
SSL	0.0314	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
- WTI FUT	0.2582	0.3744	0.4846	0.6304	0.7936	0.7694	0.5771	0.3847	0.1924	0.0000
YPF	0.0249	0.0179	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Return	0.0004	0.0007	0.0010	0.0013	0.0016	0.0019	0.0022	0.0025	0.0028	0.0032
Risk Level	0.0051	0.0056	0.0074	0.0102	0.0139	0.0264	0.0456	0.0659	0.0865	0.1072

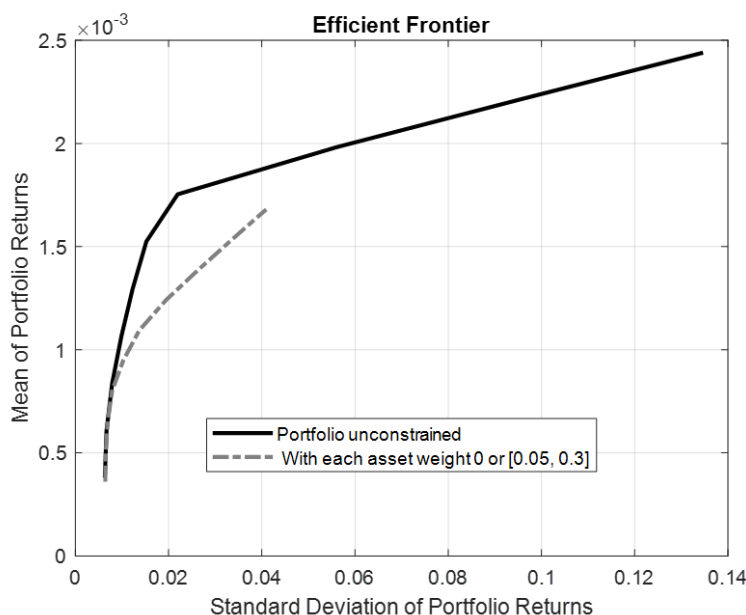
Source: Own elaboration with data of NYSE, LSE, CME, and Bloomberg.

As can be seen in both Figure 4.3 and Table 4.4, the efficient frontiers' endpoint is composed of a single stock or ADR (MATD.L) that has a risk and return higher than all other combinations in the portfolio. The corner solution is a common problem in

optimization, usually solved by placing additional limits on portfolio composition, e.g., minimum, and maximum investment weights, a minimum number of stocks that must make up the portfolio, and so on. These limitations have a cost: ultimately, new frontiers are obtained that are ‘sub-efficient’ compared to the original frontier.

Figure 4.4 shows the EF of a mean-standard deviation portfolio, with and without restrictions. To the original constraints of equations (4.7) and (4.8) was added another in which the weights of the stocks or ADRs in the portfolio must be zero or in the range of 0.05 to 0.3. The later constraint requires at least four stocks or short positions in the portfolio.

Figure 4.4. The efficient frontier of the Mean-Std Dev portfolio with and without constraints for the period of analysis.



Source: Own elaboration with data of NYSE, LSE, CME, and Bloomberg.

4.5 A computational alternative to estimate CVaR based on MAD results

It has already been said in this essay that the computational effort to calculate CVaR can be highly demanding. In the model developed in this work, we calculate 2,000 points for the two tails and the mid-section of each historic data distribution (6,000 points in total per distribution), then we simulate 2,000 values for the eight t -connected distributions and then the model performs 20,000 internal trials to calculate a single CVaR value. All this to obtain a good estimate of CVaR without abrupt steps. By contrast, the calculation of MAD and the Standard Deviation can be performed directly from historic data.

Once we have done the initial effort to calculate CVaR and MAD from a series of data, we can *estimate* new values of CVaR from new calculated MAD values with the use of mapping and Monte Carlo simulation techniques.

1. Let P_i be a specific portfolio i of n weighted securities which fulfills the constraints of equation (4.9).
2. Portfolio P_i has a, r_{P_i} , return and two risk measures associated, MAD_{P_i} , and, $CVaR_{P_i}$.
3. For any two portfolios, P_a and P_b , that fulfill the constraints of equation (4.9) it is necessary that they accomplish the principle of ordinality: $MAD_{P_a} \leq MAD_{P_b} \Leftrightarrow CVaR_{P_a} \leq CVaR_{P_b}$.
4. A pair (MAD_{P_i}, r_{P_i}) can be mapped to a risk measure $CVaR_{P_i}$, because they correspond to the same portfolio P_i .
5. With the use of Monte Carlo simulation, we generate m portfolios, P_i , and calculate m triads $(CVaR_{P_i}, MAD_{P_i}, r_{P_i})$. These triads integrate a matrix \mathbf{M} with a dimension $(m \times 3)$.
6. When we create a new portfolio P_k for which we calculate its return, r_{P_k} , and MAD_{P_k} values, we enter matrix \mathbf{M} and obtain the corresponding $CVaR_{P_k}$ estimate value. As matrix \mathbf{M} is integrated by discrete real triads, we need to use interpolation techniques to find the approximate $CVaR_{P_k}$ value that corresponds to the (MAD_{P_k}, r_{P_k}) pair.

This proposed computational alternative has some limitations: (1) it works mostly with elliptical distributions; (2) new data should not change significantly the statistical characteristics and relations of the distributions; and (3) MAD does not only capture 'negative' variations of the returns in non-elliptical distributions as CVaR does, so the mapping of CVaR and MAD may be distorted by this characteristic of the risk measures.

This proposed method can be improved with other techniques that are going to be suggested as 'work to be developed' in the conclusions part of this thesis.

4.6. Conclusions and final considerations

It is not unusual for financial series to be limited and to have staggering returns. The algorithm developed in this study, based on the model by Nyström & Skoglund (2002), makes it possible to simulate new series with more extension and granularity to calculate risk measures that are difficult to compute, such as CVaR and, eventually, to evaluate contingent assets.

Of the risk measures used, CVaR is the only coherent one. It shows advantages in risk assessment compared to the standard deviation or MAD. However, these advantages come at a cost: the computational effort to obtain it is considerable compared to the other two.

If we live in a world of normal or elliptical distributions, MAD produces results consistent with those of standard deviation when measuring risk, although with less computational effort.

Energy assets are highly volatile, including the stocks, ADRs, and oil futures used in this study, as demonstrated by the abrupt changes that occurred in March 2020. This volatility, coupled with the economic importance of the sector, underlines the need for efficient hedging and portfolio optimization. Between May 1, 2018, and March 9, 2020, the Morgan Stanley Capital Initiative (MSCI) global energy index fell 48.5%, while the five largest global oil companies lost US\$ 593.3 billion in capitalization value from January 1 to March 9, 2020. Much of this loss could have been offset by short positions in oil futures, as discussed in this study. Paradoxically, the trigger for this abrupt change in March 2020 was a sudden drop in oil price, followed by the unexpected COVID-19 pandemic.

Solving optimization problems is complex. Fortunately, there are tools already developed in different programming languages that enable us to get closer to the specific solution we seek.

The solution proposed here allows us to solve a problem with historical data and, although new series are generated with more extension and granularity, it is still today's vision based on what has already happened. Nevertheless, what do we expect to happen tomorrow or in the following days? A logical approach would be to develop a scheme of dynamic hedges and optimizations based on volatility predictions or other risk measures. We already have techniques to make these predictions. The market provides information about implied volatilities, which can be good predictors of volatilities in the immediate future.

Chapter 5

Conclusions

5.1. Contributions

This thesis contributes to the field of risk management in energy asset investments. As noted in the introduction, energy assets are important for their economic value but also volatile because of their price fluctuations.

For the case of NG PVPs in Mexico, a hedge model was developed using Henry Hub NG futures. This allowed the volatility of prices to be replicated to a great extent while minimizing the risk of combining long positions in NG PVP and short positions in futures, with a result that benefits the investor and other participants in the value chain. Despite the fact that the PVPs ceased to be published in July 2017, the problem of hedging NG in Mexico remains, as the urban distributor sells the molecule at a fixed price in pesos for a period and acquires the commodity at a fluctuating price denominated in dollars. For this reason, the model proposed remains applicable even if there is no longer an official maximum price. Application of the methodology developed here can be extended to other petroleum products and to other markets that still use price capping, such as the agricultural market. The model and methodology are also applicable to other commodities and in other countries.

The model proposed for determining variances and conditional correlations and, therefore, for obtaining optimum hedging ratios, attains high likelihood and produces estimates with a high level of confidence. Backtesting proved to be useful in calibrating the model to achieve more accurate forecasts. The dynamic hedge model developed allows the composition of the hedging portfolio to be adjusted against forecasts of changes in the short-term future.

The application of Nyström & Skoglund's (2002) method for obtaining new series based on historical yields expands the number of data while retaining the main statistical properties of the original series. This makes the calculation of risk measures for a portfolio of international energy assets more reliable. With the new series and the use of the non-linear programming tools that were developed, hedging solutions and optimal risk-return combinations for a portfolio of energy stocks and ADRs are obtained. This same tool simplifies the calculation of coherent risk measures. These same hedging and optimization techniques can be applied to other financial assets.

5.2. Conclusions

Financial hedging problems are dynamic because hedging decisions are recurrent. When a hedge is adopted, it is done with immediate and future effects in mind, even if the solution has been obtained with historical data. In the best-case scenario, one would expect that if optimal hedging ratios are maintained, combinations of long and short positions in assets would result in a lasting degree of low risk. When new information

arrives, new combinations of assets will be calculated to minimize the level of risk. In situations such as those discussed in Chapter 3, historical data are used for forecasting purposes and updating these data with new observations results in new forecasts. The objective is for these new forecasts to yield combinations that are superior to the alternative of maintaining the current hedge, as was achieved in the model introduced.

The hedging and energy portfolio optimization solutions in Chapter 4 are the best, considering historical data. The data generated from historical data allows for more reliable risk measurements by avoiding step patterns. The CVaR is a coherent measure of risk, and the portfolios obtained with this and other measures show minimal risk for both hedging and efficient frontiers.

5.3. Future work

As noted above, the hedging model can be extended to other commodities and other countries. The situation is analogous to a power producer that sells kWhs (kilo-watt hours) at a fixed price in pesos and buys NG at floating prices in other currencies or for an LPG distributor that sells at fixed prices in pesos and imports propane at floating prices. The model can also utilize other non-futures hedging instruments, such as options, which are suitable when interested in managing the returns. Hedging can also incorporate risk measures other than variance, such as CVaR, which performs better when risks are added or multiplied.

MGARCH models have several variants that could be tested for specific situations. Other statistical forecasting methods can also be used, not only for conditional variances and correlations, but for alternative measures of risk. Moreover, markets provide values that can be estimates of short-term variances, such as implied volatilities of derivative instruments or volatility indexes.

By adopting a hedging or optimization solution, one would assume that the results of the models could be preserved. However, there is no guarantee that this will happen. Therefore, one of the challenges that arise from hedging and optimization proposals is how to make the tool work dynamically and, since changing the composition of a portfolio entails costs, the dilemma is whether the benefits of reducing risk or seeking a different risk-return positioning outweigh the costs. Beyond the certainty of risk forecasts, it is necessary to decide whether to modify portfolios in the face of any new expectation. Since rebalancing is costly, the benefit of a probable decrease in risk must be evaluated against the cost certainty of modifying the investment position. This involves comparing the expected utility value of a benefit (the reduction in risk) against a certainty equivalent (the cost of the rebalancing).

Nyström & Skoglund's (2002) method can be used in financial series other than energy. The GJR model implicit in the method can be changed to one that better represents the volatility of the residuals, depending on the specific case. The series generated from historical data can be used to obtain other risk measures. These same series can be

generated through ad-hoc functions built from historical data with the help of deep learning tools.

The availability of computational resources for the researcher or the investor is each time greater. Nowadays personal computers can handle optimization problems with portfolios of hundreds of securities. Even, some software products allow parallel computing capabilities which enable to work on different parts of a problem simultaneously. Technological improvements do not eliminate the need of developing new algorithms or methods that bring more efficiency to problem-solving, like the case of estimating CVaR from MAD.

Energy markets are likely to remain valuable and volatile, so the development of new approaches to improve risk management is likely to find fertile ground.

References

- Agnolucci, P. (2009). Volatility in crude oil futures: a comparison of the predictive ability of GARCH and implied volatility models. *Energy Economics*, 31, 316–321. doi:10.1016/j.eneco.2008.11.001
- Andrade, E., Mattos, F. & Arruda, R. (2018). New Insights on Hedge Ratios in the Presence of Stochastic Transaction Costs. *Risks*, 2018, 6, 118. doi:10.3390/risks6040118
- Artzner, P., Delbaen, F., Eber, J. & Heath, D. (1999). Coherent measures of risk. *Mathematical Finance*, 9 (3), 203-228. doi.org/10.1111/1467-9965.00068
- Asche, F., Oglend, A., & Osmundsen, P. (2017). Modeling UK natural gas prices when gas prices periodically decouple from the oil price. *Energy Journal*, 38(2), 131-148. doi:10.5547/01956574.38.2.fasc
- Aumann, R. J. and Serrano R. (2008) An Economic Index of Riskiness, *Journal of Political Economy* 116(5) pp. 810:836. doi:10.1086/591947
- Babazadeh, H. & Esfahanipour, A. (2019). A novel multi period mean-VaR portfolio optimization model considering practical constraints and transaction cost. *Journal of Computational and Applied Mathematics*. doi.org/10.1016/j.cam.2018.10.039
- Barrera-Rivera, R., & Valencia-Herrera, H. (2019). Estrategias de Cobertura de Precios de Gas Natural de Primera Mano en México en Mota, B., Ortiz E., Lopez-Herrera, F.(eds.) *Economía Financiera: Teoría, Modelos e Investigación Aplicada*, Universidad Autónoma Metropolitana, Unidad Iztapalapa. Publication in process.
- Barrera-Rivera, R., & Valencia-Herrera, H. (2020). Dynamic hedging of prices of Natural Gas in Mexico. *Revista Mexicana de Economía y Finanzas*. 15 (3), 355-374. https://doi.org/10.21919/remef.v15i3.478.
- Barrera-Rivera, R., & Valencia-Herrera, H. (2021). Hedging and optimization of energy asset portfolios. Manuscript submitted for publication.
- Batten, J., Kinatader, H., Szilagyi, P. & Wagner, N. (2019). Hedging stocks with oil. *Energy Economics*. doi.org/10.1016/j.eneco.2019.06.007.
- Blazsek, S. & Villatoro, M. (2015). Is Beta- t -EGARCH(1,1) superior to GARCH(1,1)? *Applied Economics*, 47(17), pp. 1-11. doi:10.1080/00036846.2014.1000536
- Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics*, 31, 307–327. doi:10.1016/0304-4076(86)90063-1

- Brown, S.P.A., & Yücel, M. K. (2008). What drives natural gas prices? *The Energy Journal*, 29(2), 45-60. doi:10.5547/ISSN0195-6574-EJ-Vol29-No2-3
- Chen, Y., Ho, K. & Tzeng, L. (2013). Riskiness-minimizing spot-futures hedge ratio. *Journal of Banking & Finance*. 40, 154-164. doi.org/10.1016/j.jbankfin.2013.11.038
- Comisión Reguladora de Energía (CRE), (2016). RESOLUCIÓN por la que la Comisión Reguladora de Energía expide la metodología para la determinación de los precios máximos de GN objeto de venta de primera mano. RES/998/2015. *Diario Oficial de la Federación*. DOF 15-02-2016.
- Díaz Contreras, J. A., Macías Villalba, G. I., & Luna González, E. (2014). Estrategia de cobertura con productos derivados para el mercado energético colombiano. *Estudios Gerenciales* 30(130) pp. 55-64. doi:10.1016/j.estger.2014.02.008
- Du, Z. & Escanciano, J. (2016). Backtesting Expected Shortfall: Accounting for Tail Risk. *Management Science*. 63. 10.1287/mnsc.2015.2342. doi.org/10.1287/mnsc.2015.2342
- Engle, R. F. (1982). Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica*, 50:4, pp. 987-1007. <https://doi.org/10.2307/1912773>
- Forghieri, S. (2014). Portfolio Optimization using CVaR, Master's Thesis. Libera Università Internazionale degli Studi Sociali Guido Carli. Department of Economics and Finance.
- Fu, L. (2002). Three essays on future markets. Doctoral Thesis. Graduate College of the University of Illinois at Urbana-Champaign.
- Gannon, G.L. & Liu, R. (2013). Intraday dynamic hedging and futures market volatility. *Review of futures markets*, 21(1), pp. 9-32. Consulted in April, 26th, 2020 at <http://dro.deakin.edu.au/view/DU:30057117>
- Ghoddusi, H. y Emamzadehfard S. (2017). Optimal hedging in the US natural gas market: The effect of maturity and cointegration. *Energy Economics*, 63, pp. 92-105, doi: 10.1016/j.eneco.2017.01.018
- Glosten, L., Jagannathan, R. & Runkle D. (1993). On the Relation between Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance*. 48, 1779–1801. doi.org/10.21034/sr.157
- Godínez Plascencia, J. A. (2007). Causalidad del precio futuro de la Bolsa de Chicago sobre los precios físicos de maíz blanco en México. *Estudios Sociales* 15(29), pp. 205-223. Consulted in April 27th, 2020 at <http://www.scielo.org.mx/pdf/estsoc/v15n29/v15n29a6.pdf>

- Guízar Mateos, I., Martínez Damián, M. A. & Valdivia-Alcalá, R. (2012). Cobertura óptima en el mercado de futuros bajo riesgo de precio y rendimiento. *Revista mexicana de ciencias agrícolas*, 3(6), pp. 1275-1284. Consulted in April 27th, 2020 at http://www.scielo.org.mx/scielo.php?script=sci_arttext&pid=S2007-09342012000600016
- Gulay, E. & Emec, H. (2018). Comparison of forecasting performances: Does normalization and variance stabilization method beat GARCH(1,1)-type models? Empirical evidence from the stock markets. *Journal of Forecasting*, 37(2), pp. 133-150. doi.org/10.1002/for.2478
- Gutiérrez, R. de J. (2016). Estrategias dinámicas de cobertura cruzada eficiente para el mercado del petróleo mexicano: Evidencia de dos modelos garch multivariados con término de corrección de error. *Economía: Teoría y Práctica*, 44(1), pp. 115-146. Consulted in April 27th, 2020 at <https://www.redalyc.org/articulo.oa?id=281145721005>
- Hull, J.C. (2009). *Options, Futures, and other Derivatives*. New York. 7th edition, Prentice Hall.
- Hult, H., Lindskog, F., Hammarlid, O. & Rehn, C. (2012). Risk and portfolio analysis: Principles and methods. Springer Science & Business Media. doi.org/10.1007/978-1-4614-4103-8
- Isaksson, D. (2016). Robust Portfolio Optimization with Expected Shortfall, Master's Thesis. KTH School of Engineering Sciences, Mathematics Department.
- Jin, Y., & Jorion, P. (2006). Firm value and hedging: Evidence from U.S. oil and gas producers. *Journal of Finance*, 61(2), 893-919. doi:10.1111/j.1540-6261.2006.00858.x
- Kaufmann, R. K., & Ullmann, B. (2009). Oil prices, speculation and fundamentals: interpreting causal relations among *spot* and futures prices. *Energy Economics*. 31 (4), 550–558. doi:10.1016/j.eneco.2009.01.013
- Khokhlov, V. (2016). Conditional Value-at-Risk for Elliptical Distributions. *Evropský časopis Ekonomiky a Managementu*. 2 (6): 70–79.
- Konno, H. & Yamazaki, H. (1991). Mean-Absolute Deviation Portfolio Optimization Model and Its Applications to Tokyo Stock Market. *Management Science*, 37 (5), 519-531. doi.org/10.1287/mnsc.37.5.519
- Laurent, S., Rombouts, J. & Violante, F. (2012). On the forecasting accuracy of multivariate GARCH models. *Journal of Applied Econometrics*, 27(6), pp. 934-955. doi:10.1002/jae.1248

- Lönnquist, A. (2018). The economic relevance of multivariate GARCH models. Master's Thesis. Örebro University School of Business, Department of Statistics.
- Lv, X., & Shan, X. (2013). Modeling natural gas market volatility using GARCH with different distributions. *Physica A: Statistical Mechanics and Its Applications*, 392(22), 5685–5699. doi:10.1016/j.physa.2013.07.038.
- Markowitz, Harry (1952). Portfolio selection, *The Journal of Finance* 7(1): 77-91. doi:10.2307/2975974
- Mathworks (2020-1). Using Extreme Value Theory and Copulas to Evaluate Market Risk. <https://www.mathworks.com/help/econ/examples/using-extreme-value-theory-and-copulas-to-evaluate-market-risk.html>. doi.org/10.2139/ssrn.1160167
- Mishra, V. & Smyth, R. (2016). Are natural gas spot and futures prices predictable?. *Economic Modelling*. 54, 178-186. doi.org/10.1016/j.econmod.2015.12.034
- Nomikos, N., & Andriosopoulos, K. (2012). Modelling energy spot prices: Empirical evidence from NYMEX. *Energy Economics*, 34(4), 1153-1169. doi:10.1016/j.eneco.2011.10.001
- Nyström, K. & Skoglund, H. (2002). Univariate Extreme Value Theory, GARCH and Measures of Risk. Preprint, submitted 2002. <http://gloria-mundi.com/UploadFile/2010-2/knjsug.pdf>.
- Orskaug, E. (2009). Multivariate DCC-GARCH Model – With various errors distributions. Norwegian University of Science and Technology. *Master of Science in Physics and Mathematics Thesis*. Consulted in April 27th 2020 at <https://core.ac.uk/display/30864702>
- Ortiz Alvarado, A., & Girón, L. E. (2015). Predicción de volatilidad de la rentabilidad diaria del mercado del azúcar y su aplicación en la razón de cobertura. *Semestre Económico*, 18(38), pp. 105-136. doi:10.22395/seec.v18n38a4
- Ortiz Arango F. and Montiel Guzman, A. N. (2017). Transmisión de precios futuros de maíz del Chicago Board of Trade al mercado spot mexicano. *Contaduría y Administración*, 62, pp. 924-940. doi:10.1016/j.cya.2016.01.004
- Pindyck, R. (2003). Volatility in natural gas and oil markets. MIT Center for energy and environmental policy research. *Working paper*, 03-012 WP. Consulted in April 27th, 2020 at <https://dspace.mit.edu/bitstream/handle/1721.1/45005/2003-012.pdf%3Bjsessionid%3D9F0DFFF8029E50DC43A56FBDB7FA8C03?sequence%3D1>
- Rockafellar, T. & Uryasev, S. (2000). Optimization of conditional value-at-risk. *Journal of risk*, 2:21–42.

- Rockafellar, T. & Uryasev, S. (2002). Conditional value-at-risk for general loss distributions. *Journal of Banking & Finance*, 26:1443–1471. doi.org/10.1016/s0378-4266(02)00271-6
- Rosellón, J. (2008). Investigación académica que sustenta la toma de decisiones: El convenio CIDE-CRE. *Gestión y Política Pública*, 17(1), 71-99.
- Scholten, B., & van Goor, H. (2014). Modeling natural gas price volatility: The case of the UK gas market. *Energy*, 72, 126-134. doi:10.1016/j.energy.2014.05.016
- Suenaga, H., Smith, A., & Williams, J. (2008). Volatility dynamics of NYMEX natural gas futures prices. *Journal of Futures Markets*, 28(5), 438-463. doi:10.1002/fut.20317
- Tolmasky, C. & Hindanov, B. (2002). Analysis for correlated curves and seasonal commodities, the case of the petroleum market. *The Journal of Futures Markets*, 22(11), 1019-1035. doi:10.1002/fut.10046
- Troncoso-Sepúlveda, R. & Cabas-Monje, J. (2019). Factibilidad del uso de contratos de futuros del Chicago Mercantile Exchange para la cobertura del riesgo de precio en el ganado bovino chileno. *Lecturas de Economía*, 90(1), pp. 9-44. doi:10.17533/udea.le.n90a1
- Tse, Y., & Tsui, K. (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time varying correlations. *Journal of Business and Economic Statistics*, 20: 351-362. doi:10.1198/073500102288618496
- Wang, D. & Zheng, X. (2010). Rebalancing, Conditional Value at Risk, and t-Copula in Asset Allocation. Honors Thesis. Trinity College, Duke University.
- Wang, Y., & Wu, C. (2012). Forecasting energy market volatility using GARCH models: Can multivariate models beat univariate models? *Energy Economics*, 34(6), 2167–2181. doi:10.1016/j.eneco.2012.03.010.
- Wang, Y., Geng, Q. & Meng, F. (2019). Futures hedging in crude oil markets: A comparison between minimum-variance and minimum-risk frameworks. *Energy* 181 815-826. doi.org/10.1016/j.energy.2019.05.226
- Wei, Y., Wang, Y., & Huang, D. (2010). Forecasting crude oil market volatility: further evidence using GARCH-class models. *Energy Economics*. 32, 1477–1484. doi:10.1016/j.eneco.2010.07.009
- Weng, L, Saiful, J. & Zaidi, I. (2010). An empirical comparison of different risk measures in portfolio optimization. *BEH - Business and Economic Horizons*, 1 (1), 39-45. doi.org/10.15208/beh.2010.06

- Wong-Parodi, G., Dale, L., & Lekov, A. (2006). Comparing price forecast accuracy of natural gas models and futures markets. *Energy Policy*, 34, 4115-4122. doi.org/10.1016/j.enpol.2005.08.013
- Woo, C. K., Olson, A., & Horowitz, I. (2006). Market efficiency, cross hedging and price forecasts: California's natural-gas markets. *Energy*, 31(8), 1290-1304. doi:10.1016/j.energy.2005.05.003.
- Xu, C., Kijima, K., Wang, J. & Inoue, A. (2008). Portfolio rebalancing with VaR as risk measure. *International Journal of Innovative Computing, Information & Control: IJICIC*. 4. 2147-2159.
- Zhao, L., Meng, Y., Zhang, Y. & Li, Y. (2018). The optimal hedge strategy of crude oil spot and futures markets: Evidence from a novel method. *International Journal of Finance & Economics* 24(1):186-203. doi.org/10.1002/ijfe.1656

Appendixes

Appendix 1

A.1. 'Stepzero'. Code in MATLAB® that generates a series of simulated returns from the historic price data of the portfolio assets.

```
% STEPZERO    Comments are preceded by '%'

% Program to generate simulated returns from the original returns of the data series.
% We can calculate standard deviations, CVaRs, and MADs from the new returns
% generated here.

% Load the data
% Warning: choose the right directory
filename = 'D:\Documentos\Doctorado en Ciencias Financieras\INVESTIGACIÓN\LIBRO
JANM\DATOS\Consolidado.xlsx';
sheet = 'LNSHORT'; % Load the price log returns without NaN

[num,txt]=xlsread(filename,sheet); % Separate the numeric and the text parts (headings
% and dates)
Return=num; % Security returns
[n,nsec]=size(Return); % Number of data and securities
Fecha=txt;
ncol=nsec+1;
Label=Fecha(1,2:ncol); % Labels of the securities
% Convert dates in numeric values
nd=n+1;
formatIn='dd/mm/yyyy';
Date=datenum(Fecha(2:nd,1),formatIn);

% Calculation of the correlation matrix of the original returns
c1 = corrcoef(Return);

% Transformation of the returns of the hedging portfolio
% Adjust the returns: they have high dispersion and extreme values
% We create a GARCH(1,1) model with a t function

model=arima('AR', NaN, 'Distribution', 't', 'Variance', gjr(1,1)); % nsec is the number of
% securities
residuals=NaN(n,nsec); % Create the array
variances=NaN(n,nsec); % Create the array
fit=cell(nsec,1); % Here goes the description of the model
```

```

% Options to generate estimates
options=optimoptions(@fmincon, 'Display', 'off', 'Diagnostics', 'off', 'Algorithm', 'sqp',
'Tolcon', 1e-7);
% We obtain the residuals and the variances of the new model of returns.
for sec=1:nsec
    fit{sec}=estimate(model, Return(:,sec), 'Display', 'off', 'Options', options);
    [residuals(:,sec), variances(:,sec)]=infer(fit{sec}, Return(:,sec));
end

% Standardized residuals
residuals=residuals./sqrt(variances);

% Estimation of the semi-parametric CDF

nPoints=2000; % Number of points in each segment of the CDF
tailFraction=0.1; % Decimal fraction placed in each tail
tails=cell(nsec,1); % Cell with the Pareto objects
% We separate the residuals into segments: two tails and one in the middle
for sec=1:nsec
    tails{sec}=paretotails(residuals(:,sec), tailFraction, 1-tailFraction, 'kernel');
end

% Transformation of the residuals into a df and a CDF
U=zeros(size(residuals)); % Clean the array where the generated values will be placed
for sec=1:nsec
    U(:,sec)=cdf(tails{sec}, residuals(:,sec)); % Transform the margin to uniform
end

% We use a t-copula because we want the series to be related to each other
[R, DoF]=copulafit('t', U, 'Method', 'ApproximateML'); % Adjust of the copula

% Generate a random series of returns

% Prepare to generate good pseudorandom data
s=RandStream.getGlobalStream();
reset(s)

% Number of data and, if necessary, the number of periods ahead.
nTrials=2000;
horizon=1;

z=zeros(horizon, nTrials, nsec); % Here we save the iid random values
U=copularnd('t', R, DoF, horizon*nTrials); % Generate values between 0 and 1 distributed

```



```

% in a t-Student form

for sec=1:nsec
    z(:,:,sec)=reshape(icdf(tails{sec}, U(:,sec)), horizon, nTrials); % Generate iid values
    % according the original profile further improved
end

% Simulation of the original residuals' Return'
% Simulate the hedging portfolio returns and prepare them to accumulate and further
% calculation of VaR and ES, if necessary

Y0=Return(end,:); % 'Return' presampled
Z0=residuals(end,:); % Presampled standardized residuals
V0=variances(end,:); % Presampled variances

simulatedReturn=zeros(horizon, nTrials, nsec); % Clean the matrix

% Simulate the portfolio returns
for sec=1:nsec
    simulatedReturn(:,:,sec)=filter(fit{sec}, z(:,:,sec), 'Y0', Y0(sec), 'Z0', Z0(sec), 'V0',
    V0(sec));
end

% Permute the order of the simulation columns
simulatedReturn=permute(simulatedReturn, [1 3 2]);

% Sort the results from small to large
simulatedReturn=sort(simulatedReturn,1);
simRes=zeros(nTrials,nsec);
simRes=reshape(simulatedReturn, [nsec, nTrials]);
simRes=transpose(simRes);

% Calculate the correlation matrix of the new generated results
c2 = corrcoef(simRes);

% Writing the results
% We write the simulated returns in a different Excel sheet of the original file.

sheet='SIMRES';
xlswrite(filename, Label, sheet, 'A1');
xlswrite(filename,simRes,sheet,'A2');

```


Annex 2

A.2. 'Stephedge'. Code in MATLAB® that calculates the Efficient Frontier values, including the one with minimum risk for the hedging portfolio. It also calculates the values of alternative risk measures for optimal hedge solutions.

```
% STEPHEDGE  Comments are preceded by '%'

% Program to generate the optimal hedge of a portfolio
% We create a portfolio with the returns generated in 'Stepzero.' We assign weights to
% the components of this % portfolio, and with the data of the efficient frontier, we
% determine the hedging portfolio with minimum risk.

% Load the data
% Warning: choose the right directory
filename = 'D:\Documentos\Doctorado en Ciencias Financieras\INVESTIGACIÓN\LIBRO
JANM\Datos\Consolidado.xlsx';
sheet = 'SIMRES'; % Load the returns generated in 'Stepzero'
[num,txt]=xlsread(filename,sheet); % Separate the numeric and the text parts (headings
% and dates)
Ret=num; % Security returns
[n,nsec]=size(Ret); % Number of data and securities
Label=txt; % Capture the name of the securities

% Integration of the portfolio to hedge

w=[1/6,1/6,1/6,1/6,1/6,1/6]; % Weights of the securities in the portfolio
Sel=Ret(:,[1 3:6 8]); % We exclude the columns with the futures
Port=(w*Sel)'; % We obtain the portfolio to hedge with the weights

% Include the futures in the portfolio
Targ=[Port Ret(:,2) Ret(:,7)];

% Modify the lables (titles)
Modlabel=['Port', Label(2), Label(7)];

% Create an object for the Mean-Standard deviation portfolio

p0=Portfolio('AssetMean',m,'AssetCovar',C); % 'p0' for the mean-std dev case
```

```
% Calculate the mean-std dev efficient frontier
```

```
p0=Portfolio(p0,'AssetList',Modlabel(1:3)); % Include the name of the securities  
p0=estimateAssetMoments(p0,Ret(:,1:3),'missingdata',true); % Calculate the moments  
% of the portfolio  
p0=setDefaultConstraints(p0);  
[prsk0,pret0]=plotFrontier(p0,20); % Include 20 portfolios in the efficient frontier, show  
% their risks and returns  
p0wgt=estimateFrontier(p0,20); % Return the weight of the components of each portfolio  
% in the efficient frontier  
result0=table(p0.AssetList', p0wgt); % Set a table with the weights and names of the  
% securities for the 20 portfolios
```

```
% Create an object for the Mean-CVaR portfolio
```

```
% Calculate the Mean-CVaR efficient frontier
```

```
p1=PortfolioCVaR; % Define the object 'p1' as a CVaR portfolio  
AssetScenarios=mvnrnd(m,C,20000); % Generate 20,000 scenarios  
p1=setScenarios(p1,AssetScenarios); % Include the scenarios in the object  
p1=PortfolioCVaR(p1,'AssetList',Modlabel(1:3)); % Include the name of the securities  
p1=setDefaultConstraints(p1);  
p1=setProbabilityLevel(p1,0.95); % Set the level of confidence for the CVaR (95%)  
[prsk1,pret1]=plotFrontier(p1,20); % Include 20 portfolios in the efficient frontier, show  
% their risks and returns  
p1wgt=estimateFrontier(p1,20); % Return the weight of the components of each portfolio  
% in the efficient frontier  
result1=table(p1.AssetList', p1wgt); % Set a table with the weights and names of the  
% securities for the 20 portfolios
```

```
% Create an object for the Mean-MAD portfolio
```

```
p2=PortfolioMAD; % Define the object 'p2' as a MAD portfolio
```

```
% Calculate the Mean-MAD efficient frontier
```

```
AssetScenarios=mvnrnd(m,C,20000); % Generate 20,000 scenarios  
p2=setScenarios(p2,AssetScenarios); % Include the scenarios in the object  
p2=PortfolioMAD(p2,'AssetList',Modlabel(1:3)); % Include the name of the securities  
p2=setDefaultConstraints(p2);  
[prsk2,pret2]=plotFrontier(p2,20); % Include 20 portfolios in the efficient frontier, show  
% their risks and returns  
p2wgt=estimateFrontier(p2,20); % Return the weight of the components of each portfolio
```

```
% in the efficient frontier
result2=table(p2.AssetList', p2wgt); % Set a table with the weights and names of the
% securities for the 20 portfolios
```

```
%Alternative risk measures for the hedging portfolios
```

```
%Mean-Std Dev Portfolio
```

```
prsk0cvar=estimatePortRisk(p1,p0wgt);
```

```
prsk0mad=estimatePortRisk(p2,p0wgt);
```

```
%Mean-CVaR Portfolio
```

```
prsk1stdev=estimatePortRisk(p0,p1wgt);
```

```
prsk1mad=estimatePortRisk(p2,p1wgt);
```

```
%Mean-MAD Portfolio
```

```
prsk2stdev=estimatePortRisk(p0,p2wgt);
```

```
prsk2cvar=estimatePortRisk(p1,p2wgt);
```


Annex 3

A.3. 'Stepone'. Code in MATLAB® that calculates the Efficient Frontier values for a securities portfolio under different risk measures.

```
% STEPONE  Comments are preceded by '%'

% Program to calculate efficient frontiers with the returns generated by the program
% 'Stepzero.' The efficient frontiers are with Standard deviations, CVaRs 95% and MADs
% as risk measures.

% Load the data
% Warning: choose the right directory
filename      =      'D:\PC\Documents\Doctorado      en      Ciencias
Financieras\INVESTIGACIÓN\LIBRO JANM\Datos\Consolidado.xlsx';
sheet = 'SIMRES'; % Load the returns generated in 'Stepzero'
[num,txt]=xlsread(filename,sheet); % Separate the numeric and the text parts (headings
% and dates)
Ret=num; % Security returns
[n,nsec]=size(Ret); % Number of data and securities
Label=txt; % Capture the name of the securities

% Create an object for the Mean-Standard deviation portfolio

p0=Portfolio('AssetMean',m,'AssetCovar',C); % 'p0' for the mean-std dev case

% Calculate the mean-std dev efficient frontier
p0=Portfolio(p0,'AssetList',Label(1:8)); % Include the name of the securities
p0=estimateAssetMoments(p0,Ret(:,1:8),'missingdata',true); % Calculate the moments
% of the portfolio
p0=setDefaultConstraints(p0);
p0wgt=estimateFrontier(p0,10); % Return the weight of the components of each portfolio
% (10) in the efficient frontier
result0=table(p0.AssetList', p0wgt); % Set a table with the weights and names of the
% securities for the 10 portfolios
[prsk0,pret0]=plotFrontier(p0,10); % Obtain the risk and returns of the 10 portfolios in the
% efficient frontier
pWithConditionalBound = setBounds(p0, 0.05, 0.3,'BoundType', 'Conditional'); % Apart.
% Include weight limits

% Plotting
```

```

figure;
plotFrontier(p0); hold on;
plotFrontier(pWithConditionalBound); hold off;
legend('Portfolio unconstrained', ' With each asset weight 0 or [0.05, 0.3]', 'location',
'best');

```

```

% Create an object for the Mean-CVaR portfolio

```

```

p1=PortfolioCVaR; % Define the object 'p1' as a CVaR portfolio

```

```

% Calculate the Mean-CVaR efficient frontier

```

```

AssetScenarios=mvnrnd(m,C,20000); % Generate 20,000 scenarios
p1=setScenarios(p1,AssetScenarios); % Include the scenarios in the object
p1=PortfolioCVaR(p1,'AssetList',Label(1:8)); % Include the name of the securities
p1=setDefaultConstraints(p1);
p1=setProbabilityLevel(p1,0.95); % Set the level of confidence for the CVaR (95%)
[prsk1,pret1]=plotFrontier(p1,10); % Include 10 portfolios in the efficient frontier, show
% their risks and returns
p1wgt=estimateFrontier(p1,10); % Return the weight of the components of each portfolio
% in the efficient frontier
result1=table(p1.AssetList', p1wgt); % Set a table with the weights and names of the
% securities for the 20 portfolios

```

```

% Create an object for the Mean-MAD portfolio

```

```

p2=PortfolioMAD; % Define the object 'p2' as a MAD portfolio

```

```

% Calculate the Mean-MAD efficient frontier

```

```

AssetScenarios=mvnrnd(m,C,20000); % Generate 20,000 scenarios
p2=setScenarios(p2,AssetScenarios); % Include the scenarios in the object
p2=PortfolioMAD(p2,'AssetList',Label(1:8)); % Include the name of the securities
p2=setDefaultConstraints(p2);
[prsk2,pret2]=plotFrontier(p2,10); % Include 10 portfolios in the efficient frontier, show
% their risks and returns
p2wgt=estimateFrontier(p2,10); % Return the weight of the components of each portfolio
% in the efficient frontier
result2=table(p2.AssetList', p2wgt); % Set a table with the weights and names of the
% securities for the 10 portfolios

```


Curriculum Vitae

Roberto Raymundo Barrera Rivera was born in Hermosillo, Mexico. He obtained a Master of Law degree and a Bachelor of Law degree from the Centro de Estudios Avanzados de las Américas (*Center for Advanced Studies of the Americas*), a Master of Science degree in Management from the Alfred P. Sloan School of Management (MIT), and an engineering degree from the Universidad Nacional Autónoma de México (*National Autonomous University of Mexico*). He has obtained the professional designations of Chartered Financial Analyst (CFA) and Financial Risk Manager (FRM).

In addition to the three academic publications included in this thesis and another on the Pension Fund System in Mexico, he has presented papers at the VII, VIII and IX FIMEF (*Foundation of the Mexican Institute of Finance Executives*) International Financial Research Conference, the 2nd World Forum on Energy Regulation, and the 10th Forum on Finance, Risk Management and Financial Engineering, among others. He has been a speaker on infrastructure financing issues at seminars organized by the OECD in Paris and the World Bank in Washington. He has published articles on investment in nationally circulated media. Notable among his achievements is recognition for participating in the 2019 IMEF-EY International Award for Financial Investigation.

He served as President of the MIT Club of Mexico, as a member of the MIT Educational Council, President of the CFA Society of Mexico, and chaired the Analysis Committee of the Asociación Mexicana de Intermediarios Bursátiles (*Mexican Association of Securities Brokers*).

He has held management positions in the areas of planning and control, bank administration, stock market analysis, investment fund administration, corporate finance, credit, international investment, energy policy, and energy regulation. Currently, he heads the Credit and Finance area at a Mexican government financial institution.

He has taught professional seminars as well as university courses at the undergraduate and graduate levels in areas related to his field of expertise.