Theoretical and Methodological Proposal on the Development of Critical Thinking through Mathematical Modeling in the Training of Engineers*

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ABSTRACT

The present work constitutes a research advance, which takes up previous work where the teaching of Mathematics in higher education is modified through experimentation in the classroom using the modeling of real phenomena and / or simulation. The idea is to rethink Mathematical modeling as a didactic strategy that aims to develop not only disciplinary but also transversal competencies such as Critical Thinking. A review of ideas around Critical Thinking is shown, and this is intended to show that Mathematical modeling can help develop this competency in the Mathematics classroom. A theoretical proposal of how we conceive Mathematical modeling and methodological proposals is presented along with elements to look at these processes in the classroom. Considering that this study focuses on the study of the articulation of mathematical modeling as a didactic strategy and on critical thinking, based on mathematical modeling activities, the level of critical thinking of engineering students who perform these modeling activities in a Differential Equations course will be evaluated.

CCS CONCEPTS

•Applied computing ~ Engineering • Computing methodologies ~ Modeling methodologies

KEYWORDS

Mathematical teaching, mathematical modeling, critical thinking, educational innovation, higher education

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1 Introduction

According to the data presented by the Population Division of the United Nations Department of Economic and Social Affairs (UN DESA), the world population reached 7.2 billion inhabitants, and it is expected that by the year 2025, a billion more people will add to this number [1]. This growth trend implies that many difficulties will have to be confronted.

The population growth brings global challenges such as water supply, agricultural production, energy supply, and many others, which must be resolved by the employment of scientific and technological solutions; therefore, there is a deep interest in the capabilities of the workforce in the world [2].

These challenges of the 21st century that must be overcome mean that the training of engineers is of profound interest and concern to society. The questioning in the educational field about student training, especially in engineering, focuses on what domains and competencies professional students must develop so that they can be successful in working life and cope with the challenges posed by the population growth [3].

Hirers in the workforce will expect that, in addition to mastering their areas of study, the graduating students will come equipped with the competencies of analysis, problem-solving, and decision making [4]. Therefore, in addition to technical knowledge, students must develop skills and attitudes that allow effective participation in professional work activities.

The following sections present the research advances which takes up the previous work of Rodríguez [5] where the teaching of Mathematics in higher education is modified through experimentation in the classroom using the modeling of real phenomena and / or simulation. In the problem statement, the relevance between the mathematical modeling strategy and the competence of critical thinking is established. The theoretical framework of the research is described in State-of-the-art. In the method section, the methodology to be carried out and the next steps of the investigation are disclosed. Finally, in Final considerations, some conclusions are offered in given that research is still in its early stages.

2 Problem Statement

Pressured by the rapid development of critical information and technological changes, society has altered its vision about the training of people, especially in the educational sphere [6]. According to Coskun [7], the most important objective of educational programs and the entire nation is the development of students in such a way that they can access, interpret, process, and use knowledge. Moreover, he states that "people who can use their mental processes effectively and creatively can obtain and give meaning to knowledge and, therefore, advance society more than their contemporaries"[7].

Mathematics is a formal language that allows us to express our abstract thoughts [8]. We have seen that one way to be open to developments and changes in technology and science is through the ability to use mathematical techniques in mental processes. Because of this, society expects that mathematics teachers can ensure the formation of students so that they are trained to achieve effective solutions. Therefore, one of the challenges that teachers must face is how to interest students in solving problems in complex systems within an interdisciplinary context [9].

For some years, many researchers have seen the integration of mathematical modeling in the teaching of mathematics as an option to encourage analytical thinking, solve problems, and use technologies that are common in society [10].Mathematical modeling can be thought of as interpreting real information, identifying potential problems, establishing mathematical models, considering the possible errors in the mathematical-solution process, improving the mathematical models, and validating them with the reality. This process goes beyond the traditional teaching that only poses the representation of problems and the subsequent mathematical solutions [11].

Modeling as a strategy in the teaching of mathematics leads the student to relate the real-life world to the mathematical world, thereby involving the student in problems of society. The cyclical process of this modeling puts students to work actively in the development of critical thinking skills so that they interpret reality, translate it into a mathematical model, solve the problem mathematically, and validate the solution in the real world [12].

Nowadays, in addition to the mastery of knowledge, students must learn to apply the knowledge reflectively and critically [13] in such a way that they can solve the problems of daily life through deliberations about their ideas and judgments [14].

Because problems in real life are complex, it is necessary that students, especially those in the field of engineering, develop the ability to think [15]. In addition, the challenges that society must face today, such as the continuous technological changes and globalization, demand in an integral way the training of engineers from the educational and labor point of view [16].

Considering the great relevance between the strategy of mathematical modeling and the competency of critical thinking, the purpose of this research is to answer several questions that allow us to propose a theoretical and methodological framework for the development of critical thinking through modeling mathematics in the training of engineers. In this proposal, we intend to contribute a theoretical perspective on how to conceive mathematical modeling and, from a methodological perspective, we can design elements to look at these processes in the classroom.

The research problem can be established from the following general question: How can critical thinking competence be developed through the implementation of mathematical modeling in a mathematics course for future engineers?

Five subordinate questions emerge from the general question:

- What is and what could be the level of mathematical modeling that is developed during a Mathematics course for engineering students?
- What is the type of critical thinking that is and can be developed in a Mathematics course?
- What are the skills that characterize a critical thinker who has developed them through mathematical modeling?
- How can the different disciplines of science, technology, engineering, and mathematics (STEM) be integrated when using the mathematical modeling strategy?
- What are the disciplinary and transversal advantages of using mathematical modeling as a collective activity in teaching?

3 State–of–the–art

In this section, we describe concepts of the mathematical modeling strategy, the transversal competency of critical thinking, and the rubrics used to assess mathematical modeling.

3.1 Mathematical modeling

Mathematical modeling has great current relevance; this is evidenced in many scientific journals, specialized books, associations, and institutions that focus on this field [17]. For a large majority of students from the first years of studies to professional levels, the study of mathematics is not to their liking. This situation is the result of several factors, among them, the fact of not knowing what practical purposes there are in the study of Mathematics [18]. This need to relate mathematics to reality leads us to consider contexts, models and modeling in the classroom in such a way that we knock down the limitations and barriers of rigid programs and train the students to take leading roles in the construction of knowledge [19].

A model is a representation of reality, so it can be said that mathematical modeling tries to characterize a part of reality through mathematics [20]. For Dundar, Gokkurtb and Soyluc [12], "to model" means to represent the realities that are part of physical life through symbols and to make the complex become simple because, in everyday life, there are situations where access to reality is difficult, sometimes impossible. Mathematical modeling is the process of creating mathematical representations when trying to solve real-life problems [21]; the model is an element of mathematical modeling. Specifically, it represents the product [22]. In the words of Huang [11], Mathematical modeling focuses on converting and interpreting contextual information, identifying potential problems, establishing models, and reinterpreting the premise, hypothesis, and possible errors of mathematical solutions. These processes are usually described in the form of stages. By following these processes, students can refine and constantly develop their mathematical models in a cyclical manner. In addition, students should be able to participate in mental activities when they move from one stage to another during the modeling process.

From the perspective of relating the mathematical world and the everyday world, diverse representations of cycles have been constructed that try to show the mathematical activity that takes place in mathematical modeling. An example of these cycles is what Blum and Borromeo [23] use in their studies of a modeling cycle that has seven steps:

1. Building.
2. Simplifying and structuring.
3. Mathematizing (making calculations, solving equations, etc.).
4. Working mathematically.
5. Interpreting.
6. Validating.
7. Making an exhibition.

In this seven-step cycle, the student must first understand the problem or the situation that presents itself. Then the problem must be simplified, structured, and specified to take what is relevant for its solution. This problem, when translated into a mathematical model, must work with calculations and equations to have a result that will later be interpreted in the real world. Finally, this result will be validated with reality and, if not validated, there must be a return to the previous phases until a resolution is achieved. Once validated, the solution will be exposed to others. This cycle of mathematical modeling can be observed in Figure 1.

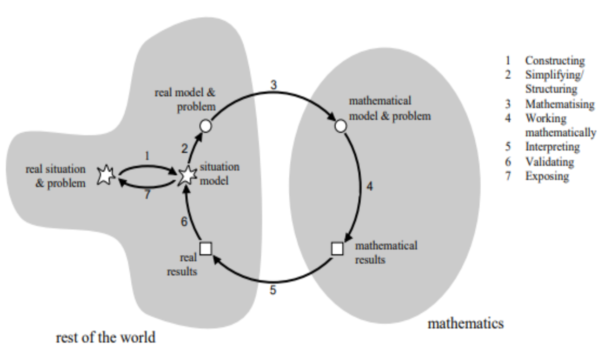

Figure 1: Blum and Leib modeling cycle [1]

For Rodríguez [5], mathematical modeling is a cyclical process where four domains can be recognized: the real domain, the pseudo-concrete domain, the physical domain, and the mathematical domain. To relate the real domain to the mathematical world, the mathematician proposes activities that establish a problem of the real world that, when understood, the students transform into a physical model and from there to a mathematical model to solve the problem. The proposed solution will be in both mathematical and pseudo-concrete terms in such a way that it can be validated and, where appropriate, changes can be made. This cyclical process that Rodríguez proposes where the real world (real domain) is connected to mathematics (mathematical domain) can be seen in Figure 2.

In the cycle of mathematical modeling, in carrying out their processes, students refine and constantly develop their mathematical models in a circular manner; thereby, they participate in mental activities when moving from one stage to another [11]. This high-level cognitive process is one of the elements that distinguishes mathematical modeling [24].

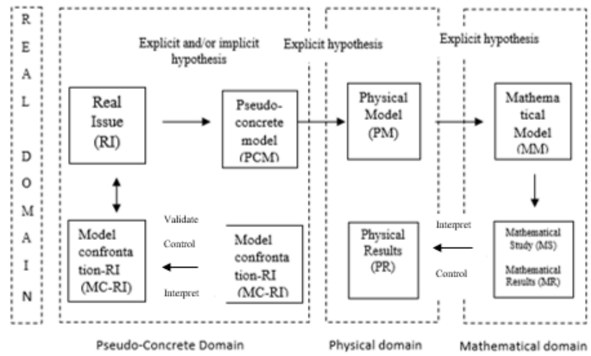

Figure 2: Rodríguez modelling cycle [5]

3.2 Instruments to evaluate the phases of mathematical modeling.

Qualitative studies on mathematical modeling use rubrics to evaluate the strategy. Associations such as the Association of American Colleges and Universities (AAC & U) [25] and the Consortium for Mathematics and Its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM) offer rubrics for the assessment of problem-solving and mathematical modeling. Some of the rubrics and their features are:

Problem-Solving VALUE Rubric [25]. This rubric is part of a compendium of 15 rubrics called VALUE developed by teams of faculty experts from colleges and universities throughout the United States via a process that examined many rubrics in these institutions. Rubrics articulate fundamental criteria for each learning outcome and have performance descriptors that demonstrate more sophisticated levels of achievement. The rubrics are intended for use at the institutional level for discussions and evaluations of student learning.

The descriptors of the rubric to assess this Value competency are:

1. Define the problem
2. Identify strategies
3. Propose solutions
4. Evaluate possible solutions
5. Implement the solution and
6. Evaluate the results.

Although the rubric was designed to solve problems, it has been adopted because its design permits that the problem-solving by the students could include problems that go from well-defined to ambiguous in a simulated, laboratory context or in real-world settings. In addition, the definition of this competency from which the students start their design includes the process of designing, evaluating, and implementing a strategy to answer an open question or achieve a desired objective. As is well observed, the elements of activities in a simulated or real-world context and the open response to a problem are part of the mathematical modeling. Table 1 shows some of the descriptors of the rubric.

Table 1: Some descriptors of the Problem-solving VALUE Rubric.

	Capstone 4	Milestones 3	2	Benchmark 1
Define Problem	Demonstrates the ability to construct a clear and insightful problem statement with evidence of all relevant contextual factors.	Demonstrates the ability to construct a problem statement with evidence of most relevant contextual factors, and problem statement is adequately detailed.	Begins to demonstrate the ability to construct a problem statement with evidence of most relevant contextual factors, but problem statement is superficial.	Demonstrates a limited ability in identifying a problem statement or related contextual factors.
Identify Strategies	Identifies multiple approaches for solving the problem that apply within a specific context.	Identifies multiple approaches for solving the problem, only some of which apply within a specific context.	Identifies only a single approach for solving the problem that does apply within a specific context.	Identifies one or more approaches for solving the problem that do not apply within a specific context.

GAIMME: a generic rubric used to assess stand-alone executive summaries [26]. In 2015, the leaders of SIAM and COMAP met to produce a report called *GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education [26]*. This group of leaders received the support and cooperation from the National Council of Teachers of Mathematics (NCTM) and Moody's Mega Math (M3) Challenge.

One of the reasons for the creation of this report was that, despite the usefulness and value of demonstrating how mathematics can help analyze and guide decision-making for complicated real-world problems, very few people have real experience with mathematical modeling and know what is and what is not a process. This report, in addition to offering the main arguments for the inclusion of mathematical modeling in all educational levels, also offers activities for each level and provides useful examples of rubric and evaluation tools that can be used when teaching mathematical models or teaching mathematics through modeling.

Among these examples of rubrics, the generic rubric for written activities was discussed in the report. The descriptors of the rubric are: a) Definition of the modeling problem, b) Model construction: making assumptions and recognizing limitations, c) Model construction: defining variables and identifying parameters, d) Solution: The model uses significant mathematics, e) Solution: The results are accessible to the audience, f) Analysis, g) Evaluation of the model and h) Writing style and organization. Table 2 shows some of the descriptors of the rubric.

Table 2: Some descriptors of the GAIMME generic rubric used to assess stand-alone executive summaries.

DEFINE THE MODELING PROBLEM (3 POINTS TOTAL)			
IDEAL	SATISFACTORY	NEEDS IMPROVEMENT	INCOMPLETE
(3 points) Concise problem statement that indicates exactly what the output of the model will be and, if appropriate, identifies the audience and/or perspective of the modeler. Statement is presented early in the paper.	(2 points) Problem statement is easily identifiable but not precise or consistent with other statements in paper.	(I point) Problem statement is difficult to understand or is buried in the text.	(0 points) No problem statement is given.

BUILDING THE MODEL: MAKE ASSUMPTIONS AND ACKNOWLEDGE LIMITATIONS (3 POINTS TOTAL)			
IDEAL	SATISFACTORY	NEEDS IMPROVEMENT	INCOMPLETE
(3 points) Primary assumptions used to develop the model are clearly identified, easy-to-read and well justified. Limitations due to simplification are stated when appropriate.	(2 points) Primary assumptions are noted; justification or readability is lacking.	(I point) Assumptions and justification exist, but are difficult to identify in the text.	(0 points) No assumptions –or justification for lack of assumptions is provided.

In general, it is concluded that one of the most widespread problems in the study of mathematics is that many students do not know the practical purpose of this discipline. The learning of mathematics is very necessary for daily life, work, and in society; Mathematical modeling has become relevant due to its ability to develop real-world problem-solving skills.

Based on studies conducted in the field of education for more than forty years, it can be said that mathematical modeling promotes the understanding of mathematics by applying this discipline to reality; in this way, it can motivate students of all educational levels to experience active learning in solving problems [11, 27, 28, 29, 30]. Therefore, teachers play a role of great importance in the development of mathematical modeling skills of the students. However, research reveals that teachers often have difficulties in working with their students on modeling activities and establishes that there is a prevailing need for the teachers to be trained [21, 31, 32, 33, 34].

The research reported in this current article takes up the work previously by Rodríguez [1] that modifies the teaching of mathematics in higher education through experimentation in the classroom of modeling real phenomena. The objective is to rethink the mathematical modeling as a didactic strategy that aims to develop not only disciplinary but also transversal competencies such as Critical Thinking; therefore, for this study, a rubric was designed that involves the evaluation of critical thinking that derives from using mathematical modeling. This rubric was developed from the literature review and the critical thinking rubrics mentioned below.

3.3 Critical thinking

- Fahim and Masouleh [35] state that critical thinking can be defined from three different approaches [35, 36]:
- The *philosophical approach* focuses on the application of formal rules of logic and what a person needs to think. Therefore, critical thinking attitudes are the most important. These critical thinkers are characterized by seeking and revealing the facts, having an open mind, self-confidence, a great curiosity, and a skeptical mentality.

- The *psychological approach* defines critical thinking in terms of the actions or behaviors that critical thinkers have. They seek to find a relationship between critical thinking, problem-solving, and high levels of thinking skills. In this way, they focus on critical thinking skills. These skills are analysis, inference, reasoning, comparison, formulation of hypotheses, synthesis and the creation of new ideas, tests, and exhaustive conclusions.
- The *educational approach* establishes that critical thinking must consider both skills and dispositions since it is a mental process that is not activated automatically and, therefore, requires a stimulus for its activation [37].

Ennis [38] states the following definition of critical thinking: "...(it) is reflective and reasonable thinking that focuses on deciding what to believe or do". According to the researcher, this conception of critical thinking competency involves both dispositions and skills that are integrated into the process of deciding what to believe or do.

Critical thinking skills are the most important in the 21st century because they are considered to be innovative and learning skills and, therefore, are a requirement to be successful today [39, 40]. Employees are expected to have analysis and problem-solving skills to make decisions in the workplace [5].

Training students to become critical thinkers is the primary objective of all educational institutions [41]. According to Elder and Paul [42], if students learn to think critically and impartially, in addition to achieving academic learning, they can aspire to become effective citizens, capable of reasoning ethically and acting in favor of the common good. They can participate in active citizenship that is part of a plural and democratic society [43].

3.4 Instruments to assess critical thinking

There are several studies that have designed and validated tests for measuring the critical thinking competency of the participating students. There is also a large amount of research where tests or rubrics are already established and validated. Some of the most representatives are:

Watson-Glaser Critical Thinking Test [44]. This evaluation is a psychometric test produced by Pearson's evaluations since 1960. Currently, the test is used for two main purposes: on the one hand, for the selection of work of graduates and professionals in the fields of law, finance, and others, and for talent management to evaluate managers; on the other hand, for academic evaluations of students in advanced degree courses or seminars.

The assessment is administered by employers as an online test or as a paper version in an evaluation center. Two versions are available, one with 80 items and another with 40; they are completed in 60 and 30 minutes, respectively. The evaluation is divided into five sections, and each section has its own type of question that assesses particular skills; namely, a) Inference, b) Recognition of assumptions, c) Deduction, d) Interpretation, and e) Evaluation of arguments.

Halpern Critical Thinking Assessment [45]. This is a test to assess levels of critical thinking in people, ages 15 to adult. The evaluation consists of 20 daily scenarios, each of which is briefly described and presented in a common language. For each scenario, respondents are first asked an open question, followed by a forced-choice question (for example, multiple choice, classification, or classification of alternatives). Cognitive psychologists differentiate between free memory and recognition processes in memory, and these two types of questions are designed to take advantage of different cognitive processes. The total score is (approximately) equally weighted between the constructed responses and the forced-choice questions.

The test assesses five dimensions; namely, verbal reasoning, argument analysis, thinking as hypothesis testing, probability and uncertainty, and decision making and problem-solving.

Cornell Critical Thinking Test [46]. This instrument is offered for two levels, the Cornell Critical Thinking Test, Level X, and the Cornell Critical Thinking Test, Level Z. Level X is intended for students in grades 4-14; Level Z for advanced high school students and gifted college students, graduate students, and other adults.

Level X is a test consisting of 71 multiple-choice items, which assess the abilities of a) induction, b) credibility of a source, c) observation, d) semantics, e) deduction and f) identification of hypotheses. The Z level has 52 items, also multiple choice, and it evaluates the following skills: a) induction, the b) credibility of a source, c) semantics, d) prediction and e) experimentation, fallacies, deduction, definition and identification of hypotheses. Each of the tests must be taken within a period of 50 minutes. Each item in each test has three options and a key answer.

To analyze and sub-categorize critical thinking skills, this approach defines three types of inferences to beliefs (induction, deduction and evaluation of judgments) and four types of bases for such inferences, which are: a) the results of other inferences, b) observations, c) statements made by others and d) assumptions. Given certain characteristics of this test, it was chosen to be used in the quantitative methodology of this study.

The AACU Critical Thinking VALUE Rubric [25]. This rubric is part of the compendium of 15 rubrics called VALUE that was mentioned in the rubrics section to evaluate mathematical modeling. It was designed with the intention of being transdisciplinary since the experts who created it agree that success in all disciplines requires research and analysis habits that share common attributes. In addition, they state that many studies suggest that successful critical thinkers of all disciplines need increasingly to be able to apply those habits in diverse and changing situations in all areas of life. This rubric can be used with many different types of tasks, such as those that require students to perform analyses of text, data, or problems.

The rubric of critical thinking at the University of Deusto [47]**.** The University of Deusto, through a vast team of professors from all areas, designed and developed a system for the evaluation of generic competencies. This system presents rubrics for each one of the competencies, detailing the definition of each one, its descriptors, and three levels of "expertise;" among them is the competency of critical thinking. The descriptors of the rubric to evaluate this competency are:

1. The formulation of own judgments: questions are asked about the reality that exists; the student reflects on it, formulates his own judgments and argues them.
2. The analysis of the judgments of others: the judgments of others are analyzed (differences of opinion, data, strengths, and weaknesses), and from there the student identifies ideas, principles, and underlying values).
3. The use of well-founded criteria for the analysis of judgments adopting a constructive attitude: internal criteria (consistency, coherence, reliability), and external criteria (usefulness, viability, validity, etc.).
4. Awareness of the practical implications of judgments and the assumption of responsibility regarding these implications: a) the pros and cons of decisions, b) the consequences on others, and c) the rights of people.

Finally, it can be affirmed that currently, critical thinking can be defined from three approaches, namely, the philosophical, the psychological, and the educational. The latter considers both the critical thinking skills and the dispositions towards it.

The main characteristic of critical thinking is its evaluative nature; it is related to what is believed and the decisions made. Therefore, critical thinking is a transversal competence that leads to questioning things and acts; is interested in the deepening of ideas, actions, and personal judgments; and analyzes the information from different situations. It aims to analyze, evaluate, infer, and explain, according to the evidence, contexts, concepts, and methodologies in which it supports its judgments, to finally make decisions and solve situations.

The challenges that society must face require that university students develop not only disciplinary skills but also those transversal skills that allow them to make decisions and solve problems generated by the challenges of a population in constant growth. Critical thinking is considered one of the most important transversal competencies for the present with such importance that to train students as critical thinkers is one of the primary objectives of every educational center.

The challenges faced by university students are complex problems that arise from daily life and, due to this complexity, professionals from a scientific standpoint must perform critical thinking processes to be able to arrive at complex solutions.

A great variety of tests and rubrics can be found to evaluate critical thinking. However, this is a difficult competency to implement in the school curricula. Therefore, there is a turning to research strategies that favor the development of this competency.

4 Method

In this study, the mixed method of equitable status will be used. This is located within the post-positivist paradigm and is considered the ideal because it establishes that even if there is a reality, it cannot be fully achieved [48].

Considering that this study focuses on the study of the articulation of mathematical modeling as a didactic strategy and on critical thinking, based on mathematical modeling activities, the level of critical thinking of engineering students who perform these modeling activities in a Differential Equations course will be evaluated. For the evaluation, two instruments will be used:
a) The Cornell Critical Thinking Test and
b) a rubric that evaluates both critical thinking (light blue) and the phases of mathematical modeling (blue). Table 3 shows some of the descriptors of the rubric.

Table 3: Some descriptors of the rubric of mathematical modeling and critical thinking.

Phase	Indicadores	1	2	3	4
Formulation	**Identify the real-world problem or situation**	The problem statement is not established	The problem statement is difficult to understand	The problem statement is easily identifiable, but it is not precise or consistent with other approaches of the activity	Concise problem statement that indicates exactly what the result of the model will be
	Explanation of the problem or situation	The situation / problem to be considered critically is exposed without clarification or description.	The situation / problem to be considered critically is indicated, but the description leaves some terms undefined, the ambiguities unexplored, the boundaries undetermined and the background unknown	The situation / problem to be considered critically is established, described and clarified so that omissions do not seriously hamper understanding.	The situation / problem to be considered critically is clearly established and described in a comprehensive manner, communicating all the relevant information necessary for a complete understanding.
	Identify the relevant parts or data of the problem for solution	All the data of the problem are considered relevant	Take some data of the problem as relevant when they are not	Identify some relevant data of the problem	Identify all the relevant data of the problem
	Identify strategies to address the problem for its solution	Identify one or more ways to approach the problem to solve it, which does not apply within a specific context.	Identify only one way to approach the problem to solve it which is applied within a specific context.	Identify various ways to approach the problem to solve it, of which only some are applied within a specific context.	Identify various ways to approach the problem to solve it applied within a specific context.
	Influence of context and assumptions	Shows an awareness of current assumptions (sometimes assertions are labeled as assumptions). Begin to identify some contexts when presenting his point of view	Question some assumptions. Identify several relevant contexts when presenting his point of view. He may be more aware of others' assumptions than his own (or vice versa).	Identify your own and others' assumptions and various relevant contexts when presenting his point of view.	It thoroughly analyzes (systematically and methodically) his own and others' assumptions and carefully evaluates the relevance of the contexts in presenting his point of view.
	Proposes solutions / hypotheses	Proposes a solution / hypothesis that is difficult to assess because it is vague or only indirectly addresses the problem statement.	It proposes a solution / hypothesis that is "ready to use" instead of being individually designed to address the specific contextual factors of the problem.	It proposes one or more solutions / hypotheses that indicate the understanding of the problem. The solutions / hypotheses are sensitive to contextual factors, as well as one of the following: ethical, logical or cultural dimensions of the problem.	It proposes one or more solutions / hypotheses that indicate a deep understanding of the problem. The solutions / hypotheses are sensitive to contextual factors, as well as to all the following: ethical, logical and cultural dimensions of the problem.

Two pilot tests were carried out with engineering students for the validation of the instruments. Once the instruments are validated, the study will be carried out in an educational institution in northern Mexico, the participants are engineering students from different disciplines, they are distributed in two groups and they study the course of Differential Equations during the August-May 2019 semester. In addition, classroom observations and student interviews will be conducted to complement the data during the modeling activities in the course.

5 Final considerations

Currently, the educational sector is committed to a vision of STEM education, which is teaching and learning in the fields of science, technology, engineering, and mathematics [49, 50]. This vision focuses on using science and mathematics as strategies for technology and engineering [51].

According to Kertil and Gurel [10], the different perspectives on integrated STEM education generally emphasize the use of contextual tasks which involve students in multiple processes such as design, construction, analysis, mathematization, verification, review, and communication; thereby, mathematical modeling can become a process that is involved in all applications related to STEM.

As mentioned, on the one hand, modeling is an excellent strategy for the implementation of integrated STEM education and, on the other hand, critical thinking is considered one of the most important competencies in the training of students, especially in the field of engineering. It is due to these benefits that the study of the development of critical thinking in mathematical modeling for the training of engineers is considered of great importance.

This study aims to show that mathematical modeling can help develop this competence in the Mathematics classroom through a theoretical proposal of how we conceive mathematical modeling and methodological proposals along with elements to observe these processes in the classroom. Currently working with the validation of instruments and will be applied to engineering students during the second half of 2019.

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REFERENCES

[1] UN DESA. 2013. *World Population Prospects: The 2012 Revision, Highlights and Advance Tables.* ONU, United Nations Department of Economic and Social Affairs/Population Division. New York: United Nations. Retrieved from http://esa.un.org/unpd/wpp/Documentation/pdf/WPP2012_HIGHLIGHTS.pdf

[2] T. R. Kelley and J. G. Knowles. 2016. A conceptual framework for integrated STEM education. *International Journal of STEM Education,* 3-11. DOI 10.1186/s40594-016-0046-z

[3] F. M. Esteve and M. Gisbert. 2013. Explorando el potencial educativo de los entornos virtuales 3D. *Teoría de la Educación. Educación y cultura en la era de la información. 14(3),* 5-327.

[4] L. M. Murawski. 2014. Critical Thinking in the Classroom…and Beyond. *Journal of Learning in Higher Education, 10(1),* pp. 25-30.

[5] R. Rodríguez. 2017. Repensando la enseñanza de las matemáticas para futuros ingenieros: actualidades y desafíos. *Revista de Investigación Educativa de la REDIECH*, 8(15), 69-85.

[6] A. Arseven, 2015. Mathematical modeling Approach in Mathematics Education. *Universal Journal of Educational Research, 3*(12), 973-980. DOI: 10.13189/ujer.2015.031204

[7] H. Coskun. 2017. Mathematical modeling research in Turkey: A content analysis study. *Educational Research and Reviews, 12*(1), 19-27. DOI: 10.5897/ERR2016.3077

[8] Ciltas and A. Isik. 2013. The Effect of Instruction through Mathematical modeling on modeling Skills of Prospective Elementary Mathematics Teachers. Educational Sciences: Theory & Practice, 13(2), 1187-1192.

[9] L. D. English. 2009. Promoting interdisciplinarity through mathematical modeling. *ZDM Mathematics Education, 41*, 161–181 DOI 10.1007/s11858-008-0106-z

[10] M. Kertil and C. Gure.l 2016. Mathematical modeling: A bridge to STEM education. International Journal of Education in Mathematics, Science and Technology, 4(1), 44-55. DOI:10.18404/ijemst.95761

[11] C.-H. Huang. 2012. Investigating engineering students' mathematical modeling competency. *World Transactions on Engineering and Technology Education, 10*(2), 99-104.

[12] S. Dundar, B. Gokkurt and Y. Soylu. 2012. Mathematical modeling at a glance: a theoretical study. *Social and Behavioral Sciences, 46*, 3465-3470.

[13] S. Maričić, K. Špijunović and B. Lazić. 2016. The Influence of Content on the Development of Students' Critical Thinking in the Initial Teaching of Mathematics. *Croatian Journal of Education. 18*(1/), 11-40. DOI: 10.15516/cje. v18i1.1325

[14] G. Aksu and N. Koruklu. 2015. Determination the effects of vocational high school students' logical and critical thinking skills on mathematics success. Eurasian Journal of Educational Research, 59, 181.

[15] S. Osman et al. 2016. Identifying Pertinent Elements of Critical Thinking and Mathematical Thinking Used in Civil Engineering Practice in Relation to Engineering Education. *The Qualitative Report 2016, 21(2),* 212-227.

[16] M. P. Kindelán and A. M. Martín. 2008. Ingenieros del siglo XXI: importancia de la comunicación y de la formación estratégica en la doble esfera educativa y profesional del ingeniero. *ARBOR Ciencia, Pensamiento y Cultura, CLXXXIV (732*), 731-742.

[17] J. A. Villa-Ochoa, M. Rosa and M. E. Gavarrete. 2018. Aproximaciones socioculturales a la modelación en Educación Matemática. Aportes de una comunidad latinoamericana. *Revista Latinoamericana de Etnomatemática, 11*(1), 4-12.

[18] P. Camarena. 2009. La matemática en el contexto de las ciencias. *Innovación Educativa, 9*(6),15-25. http://www.redalyc.org/pdf/1794/179414894003.pdf

[19] J. F. Molina-Toro, J. A. Villa-Ochoa and L. Suárez Téllez. 2018. La modelación en el aula como un ambiente de experimentación-con-graficación-y-tecnología. Un estudio con funciones trigonométricas. *Revista Latinoamericana de Etnomatemática, 11*(1), 87-115.

[20] M. Brito-Vallina, L. Alemán-Romero, E. Fraga-Guerra, J. Para-García and R. Arias-deTapia. 2011. Papel de la modelación matemática en la formación de los ingenieros. *Ingeniería Mecánica, 14*(2), 129-139.

[21] R. G. Jacobs. and Durandt. J. 2017. Attitudes of Pre-Service Mathematics Teachers towards modeling: A South African Inquiry. *EURASIA Journal of Mathematics Science and Technology Education, 13*(1), 61-84. DOI 10.12973/eurasia.2017.00604a

[22] S. Aztekin and T. Zehra. 2015. The content analysis of mathematical modeling studies in Turkey: a meta-synthesis study. *Education and Science. 40*(178), 139-161

[23] W. Blum and R. Borromeo. 2009. Mathematical modeling: Can It Be Taught And Learnt? *Journal of Mathematical modeling and Application, 1*(1), 45-58.

[24] N. Mousoulides, C. Christou and B. Sriraman. 2008. A Modeling Perspective on Teaching and Learning of Mathematical Problem Solving. Mathematical Thinking and Learning, 10(3),293-304 DOI: 10.1080/1098606080221813

[25] Association of American Colleges and Universities (AAC&U). 2009. *Critical Thinking VALUE rubric.* https://www.aacu.org/value/rubrics/inquiry-analysis

[26] *GAIMME: Guidelines for Assessment and Instruction in Mathematical Modeling Education, Second Edition.* 2019. Sol Garfunkel and Michelle Montgomery, editors, COMAP and SIAM, Philadelphia. https://www.siam.org/Publications/Reports/Detail/guidelines-for-assessment-and-instruction-in-mathematical-modeling-education

[27] L. Albarracín and N. Gorgorió. 2013. Problemas de estimación de grandes cantidades: modelización e influencia del contexto. *Revista Latinoamericana de Investigación en Matemática Educativa, 16* (3), 289-315.

[28] A. Carberry and A. Mc Kenna. 2014. Exploring student conceptions of modeling and modeling uses in engineering design. Journal of Engineering Education. 103(1), 77-90.

[29] J. Cozcher. 2017. How can emphasizing mathematical modeling principles benefits students in a traditionally taught differential equations course? *Journal of Mathematical Behavior, 45*, 78-94.

[30] L. Plaza. 2016. Obstáculos presentes en modelación matemática. Caso ecuaciones diferenciales en la formación de ingenieros. *Revista científica*, 25, 176-187.

[31] A. Aydogan., A. K. Erbas, E. Cakiroglu and C. Alacaci 2017. Developing teachers' models for assessing students' competence in mathematical modeling through lesson study. *International Journal of Mathematical Education in Science and Technology, 48*(6), 895-912.

[32] C. Paolucci and H. Wessels, H. 2017. An Examination of Preservice Teachers' Capacity to Create Mathematical Modeling Problems for Children. *Journal of Teacher Education, 68*(3), 330-344.

[33] J. A. Shahbari and M. Tabach. 2016. Developing modeling lenses among practicing. *International Journal of Mathematical Education in Science and Technology, 47*(5), 717-732.

[34] A. S. Zeytun, B. Cetinkaya, and A. K. Erbas. 2017. Understanding Prospective Teachers' Mathematical Modeling Processes in the Context of a Mathematical Modeling Course. *EURASIA Journal of Mathematics Science and Technology Education, 13*(3), 691-722.

[35] M. Fahim and N. S. Masouleh (2012). Critical Thinking in Higher Education: A Pedagogical Look. *Theory and Practice in Language Studies, 2(7),* 1370-1375.

[36] A. M. S. Atabaki, N. Keshtiaray and M. H. Yarmohammadian, (2015). Scrutiny of Critical Thinking Concept. *International Education Studies, 8(3)*, 93-102.

[37] C. Poondej and T. Lerdpornkulrat, (2015). The reliability and construct validity of the critical thinking disposition scale. *Journal of Psychological and Educational Research, 23 (1)*, 23-36.

[38] R. H. Ennis (1985). A Logical Basis for Measuring Critical Thinking Skills. *Educational Leadership,43*(2) p44-48.

[39] D. Vargas (2015). Evidence of Critical Thinking in High School Humanities Classrooms. *Gist Education and Learning Research Journal,* (11), 26-44

[40] H. Bie, H. P. Wilhelm and H. Van der Meij (2015). The Halpern Critical Thinking Assessment: Toward a Dutch appraisal of critical thinking. *Thinking Skills and Creativity, 17*, 33–44.

[41] S. Cargas, S. Williams and M. Rosenberg (2017). An approach to teaching critical thinking across disciplines using performance tasks with a common rubric. *Thinking Skills and Creativity, 26*, 24–37.

[42] L. Elder and R. Paul (2010). Critical Thinking: Competency Standards Essential for the Cultivation of Intellectual Skills, Part 1. *Journal of Developmental Education, 34(2),* 38-39.

[43] L. Bigozzi. et al. (2018) The Influence of Teaching Approach on Students' Conceptual Learning in Physics. *Frontiers in Psychology. 9,* (2474). DOI: 10.3389/fpsyg.2018.02474
[44] JobTestPrep (2019). https://www.jobtestprep.com/watson-glaser-test?utm_source=wikijob&utm_medium=nav-link&utm_content=watson-glaser&utm_campaign=jobtestprep-com
[45] The Halpern Critical Thinking Assessment. 2016. https://sites.google.com/site/dianehalperncmc/home/research/halpern-critical-thinking-assessment
[46] R. H. Ennis, J. Millman and T. N. Tomko. 1985. Cornell critical thinking tests level X & level Z: Manual. Boise, ID: Midwest Publications.
[47] A. Villa and M. Poblete, M. 2007. *Aprendizaje basado en Competencias. Una propuesta para la evaluación de las competencias genéricas*. Bilbao, España: Ediciones Mensajero.
[48] J. R. Valenzuela and. M Flores (2012). Fundamentos de investigación educativa. Monterrey, México: Editorial Digital del Tecnológico de Monterrey.
[49] T. J. Kennedy and M. R. L. Odell. 2014. Engaging Students In STEM Education Science *Education International, 25(3)*, 246-258.
[50] P. R. Hernández, et al. (2014). Connecting the STEM dots: measuring the effect of an integrated engineering design intervention. *International Journal of STEM Education, 24*, 107-120. DOI 10.1007/s10798-013-9241-0
[51] D. Bell. 2016. The reality of STEM education, design and technology teachers' perceptions: a phenomenographic study. *International Journal of Technology and Design Education (2016) 26*, 61–79. DOI 10.1007/s10798-015-9300-9