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An Empirical Econometric Model for the Mexican Target Rate and its application to determine the Interest Rate Curve

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Breno Lorenzo Madero Salmerón
November 19th ,2021

EGADE BUSINESS SCHOOL
Mexico City

Dedication

To Mayan, my deep love. Your support, your help and patience have been crucial to the fulfillment of this work. We are in this together, as in the rest of our life.

To Sabina, my darling. You are my inspiration to keep going forward. I love you deeply and will do forever.

To Tere, my mom, my joy. This work is for you to feel proud of yourself. We are together now and will be always together.

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"An Empirical Econometric Model for the Mexican Target Rate and its application to determine the Interest Rate Curve"

By

Breno Lorenzo Madero Salmerón

Abstract

The term structure of interest rates is an extremely important subject in Finance. The rates' value influences the pricing of practically any asset, and it is also important to determine the value of several liabilities. Its study has been a field of intense research, both from the theoretical point of view as well as from the purely applied point of view. In this work, we review the different approaches that have been taken to describe it with the idea of better understand the reasons subjacent to its assessment.

We also propose, for the Mexican case, a novel approach to model empirically the Target Rate that Banco de México (Banxico) sets as part of their monetary policy. We propose a model based on the difference of two Poisson distributions in terms of public data: the monetary decisions taken by the Federal Reserve, the exchange rate, the inflation rate and its expectation, and the economic growth results and expectations. The addition of the expectations, which come from surveys made from Banxico as well, improved the results in a significant manner, showing the relevant value of "soft" data in this type of analysis. Our model also determines the probability of a rise or of a lower of the rate and the amount of this movement as shown in an example of application.

Since the Target Rate serves as basis for the other interest rates, modeling it is of extreme importance. After the modelling of this rate was completed, we used this rate as the one-day rate to determine the prices of short-term rate bonds to estimate the given prices with excellent statistical significance.

However, for medium-term rates we noticed that this approach was not enough. So, after a deeper analysis of the rate curve, we used our rate together with some macroeconomic variables (namely, public spending without financial costs as a proportion of GDP.) Incorporating this, we established the corresponding behavior of mid-term rates with a very good statistical significance as well.

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Some discussion of the arithmetic of longer-term yields provides a useful perspective on recent developments in bond markets. The ten-year Treasury yield, for example, can be viewed as a weighted average of the current one-year rate and nine one-year forward rates, with the weights depending on the coupon yield of the security. [E]ach of these forward rates can be split further into (1) a portion equal to the one-year spot rate that market participants currently expect to prevail at the corresponding date in the future, and (2) a portion that reflects additional compensation to the bondholder for the risk of holding longer-dated instruments. Current and near-term forward rates are particularly sensitive to monetary policy actions, which directly affect spot short-term interest rates and strongly influence market expectations of where spot rates are likely to stand in the next year or two.

—Ben S., Bernanke, Former Chairman of the Federal Reserve,
Speech before the Economic Club of New York,
New York, March 20, 2006

Chapter 1

Introduction

We cannot overstate the importance of interest rates in Finance. Base interest rates reflect growth and inflation expectations, the investors' appetite for holding government bonds (hence the trustworthiness of that government), as well as several additional factors which affect their determination, see for example (Peterson Drake & Fabozzi, 2010) and (Schweser, 2001) for the classical approaches to analyze interest rates where we can see how different risks are incorporated into the rate, as well as expectations of the market.

The base interest rates in turn affect the value of all possible assets, since the expected future cash flows must be discounted using these rates to mount for the time value of money as well explained in (Cochrane J. , 2001). This is, interest rates are important independently of other financial assets, but they are part of the building blocks needed to value the rest of these assets.

At the beginning of the 1970's there was a boom of Derivatives pricing following the publication of the works by (Merton, 1973) and (Black & Scholes, 1973). These models of asset pricing assumed at the beginning that interest rates were constant, which was argued to be true "at least in the short-term". However, there are moments where interest rates develop very stochastic behavior as shown in the following graph of the Mexican 1-month bond (CETE 28):

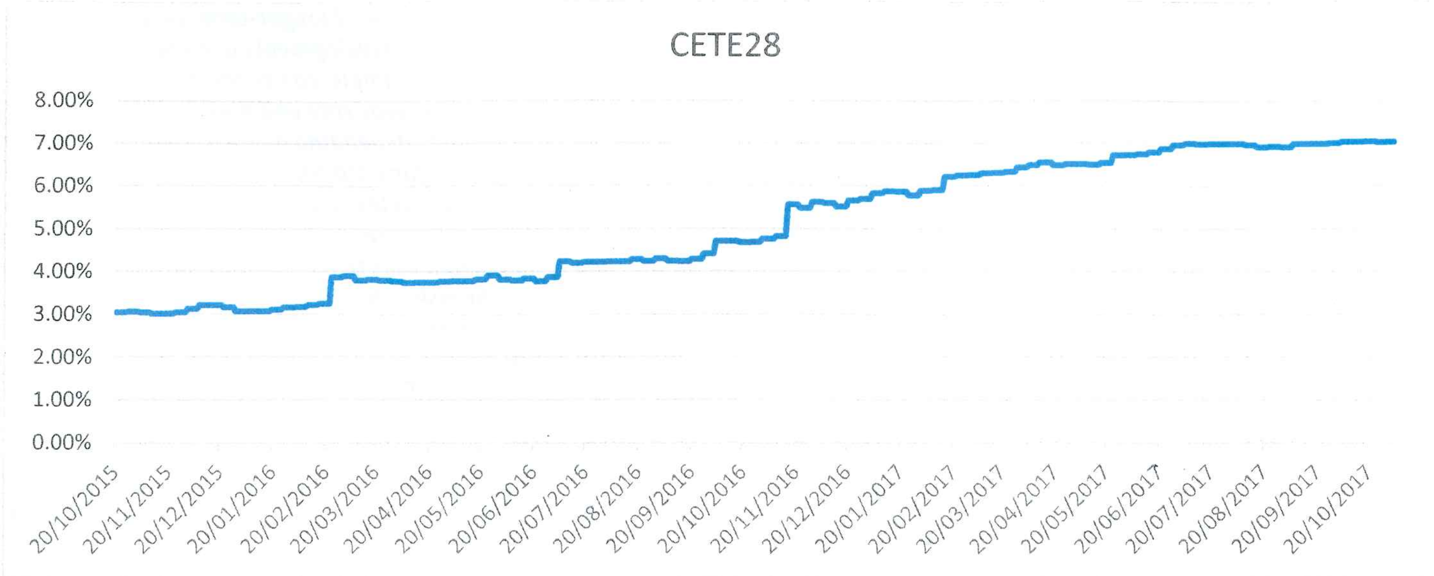

Figure 1: CETE 28 yield (October 2015 to October 2017)

We see here that there are special moments where the yield jumps in spontaneous ways, hence making impossible to ignore these effects even "in the short-term".

After these considerations and the effectiveness of the stochastic approach in valuing derivatives, much interest started to develop in stochastic models for the interest rate. These models were immediately implemented by practitioners to accurately price financial securities, like interest rate derivatives. Their pricing, however, was mathematically far more demanding than equity pricing since the whole term structure of interest rates and its development over time was required.

However, as we will see in the following section, most stochastic models fail to consider that the basic element that influences short term rates is monetary policy, like the above quote of Ben Bernanke mentions. Central Banks fix a so-called Target Rate at which they inject or withdraw liquidity to the interbank loans. In the Mexican case, Banco de Mexico (Banxico) sets this rate, and we see that it impacts the short-term bonds in an almost direct way as shown in the following graph and compare it with the previous one:

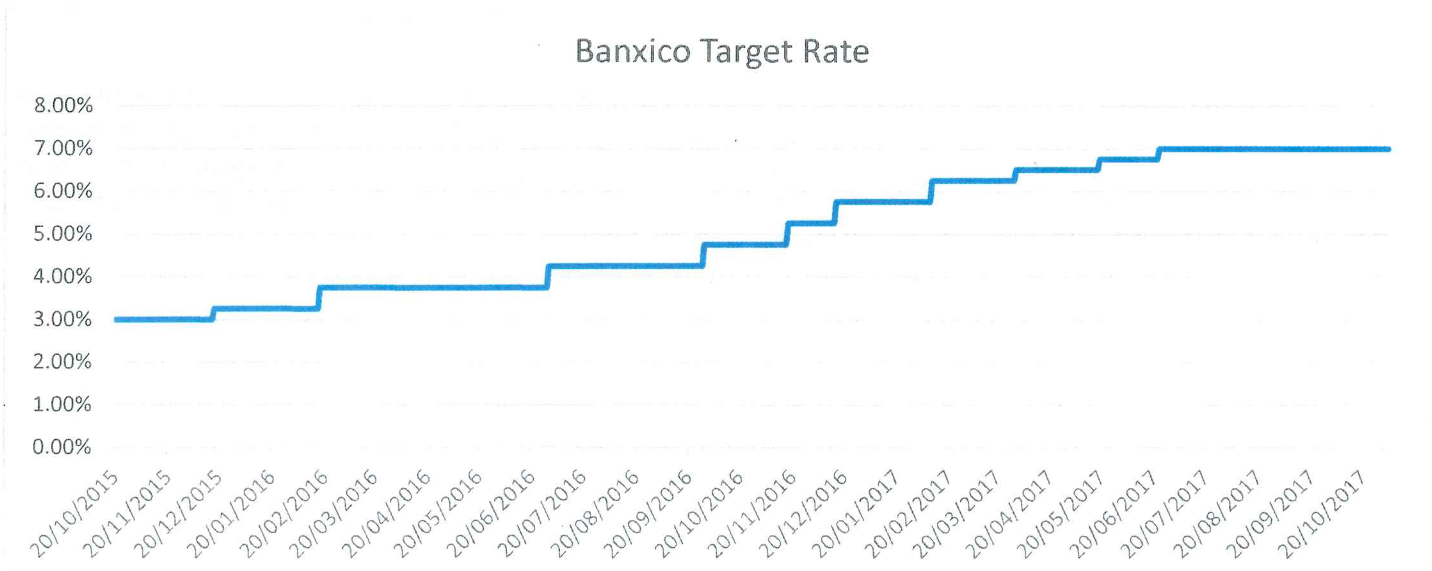

Figure 2: Banxico Target Rate (October 2015 to October 2017)

We can see that the jumps set by monetary policy trigger the jumps that yields of short-term bonds experience. Hence, to develop models for interest rate, we must start with the Target Rate.

In this work, we will focus on the Mexican rates, so first we will propose a model for the Target Rate established by Banco de México. To do so, first we will make a thorough review of the publications on monetary policy that Banco de México delivers to determine the variables needed to this model. We will obtain as a byproduct the probabilities for changes in the rate related to the values given by this set of variables. After considering the model for the Target Rate, we will analyze how to use it as the basic determinant of the values of short and medium-term bonds and the validity of this approach.

1.1 Literature Review

As mentioned before, the boom of Derivatives pricing and particularly the stochastic calculus approach given by (Merton, 1973) inspired the development of many stochastic models for short term interest rates. Each of these models propose different ways trying to capture their main behavior in such a way to be consistent with the prices of tradable bonds. Following (Brigo & Mercurio, 2006) we have that the first model that was regarded as relevant to this end was that of O. Vašiček from 1977 based upon ideas of Ornstein-Uhlenbeck:

$$dr_t = k(\theta - r_t)dt + \sigma dWt \tag{1}$$

Where r_t is short term interest rate, θ is the long-term average, k is a regression toward the mean factor, σ is standard deviation and dWt a standard Wiener process.

This model was proposed by Vašiček to guarantee that there is a mean reversing property for the rates, which was seen in practice. Consequently, such model allows for negative interest rates, which at that time was consider a mayor flaw.

However, now we experience negative rates given by some mayor Central Banks which make us rethink on the validity of this type of model. More precisely, now we are experiencing negative interest rates in Japan, Switzerland, Scandinavia, and Europe which are aimed to boost investment and therefore growth.

In the Mexican case, we are still far from experiencing negative interest rates in nominal terms, however in real terms after factoring for inflation we are now experiencing them sometimes.

To avoid these negative interest rates, after Vašiček's, another important model proposed was the square root model introduced by Cox-Ingersoll-Ross (CIR) in 1985

$$dr_t = k(\theta - r_t)dt + \sigma\sqrt{r_t}dWt \tag{2}$$

The drift factor, $k(\theta - r_t)$, is the same as in the Vašiček model. It ensures mean reversion of the interest rate towards the long run value θ, with speed of adjustment governed by the strictly positive parameter k. The standard deviation factor, $\sigma\sqrt{r_t}$, avoids the possibility of negative interest rates for all positive values of k and θ. An interest rate of zero is also precluded if the condition $2k\theta \geq \sigma^2$ is met. More generally, when the rate r_t is close to zero, the standard deviation $\sigma\sqrt{r_t}$ also becomes very small, which dampens the effect of the random shock on the rate. Consequently, when the rate gets close to zero, its evolution becomes dominated by the drift factor, which pushes the rate upwards (towards equilibrium).

These models are called endogenous one-factor models with regression to the mean, where the actual market curve is a byproduct and not an input for the model, and they contain just one source of randomness expressed by the dWt term. Unfortunately, this type of models fails to capture adequately all forms that actual market curves present.

With the idea of obtaining results closer to the market data, other type of models were proposed, namely exogenous models that start with similar stochastic behavior, but that allow parameters to be adjusted by econometric or statistical technics to better reflect the actual temporal structure of interest rates and market volatility. With this idea in mind, many one factor models like Hull-White or other extensions of the Vašiček or CIR models have been proposed as shown in the following table taken from (Álvarez del Castillo, 2018):

Table 1: Interest rate one-factor stochastic models

Model	Dynamics	Positive Rate always	Distribution
Vašiček (1977)	$dr_t = k(\theta - r_t)d_t + \sigma dW_t$	No	Normal
CIR (1985)	$dr_t = k(\theta - r_t)d_t + \sigma\sqrt{r_t}dW_t$	Yes	Chi-Squared
Dothan (1978)	$dr_t = r_t(ad_t + \sigma dW_t)$	Yes	Log-Normal
Vašiček Exp (1990)	$dr_t = r_t(\eta - a ln(r_t))d_t + \sigma dW_t$	Yes	Log-Normal
Black-Derman-Toy (1990)	$d\ln(r_t) = \left(\theta_t - \frac{\sigma_t'}{\sigma_t} ln(r_t)\right)d_t + \sigma_t dW_t$	Yes	Log-Normal
Hull-White (1990)	$dr_t = k(\theta_t - r_t)d_t + \sigma dW_t$	No	Normal
Black-Karasinski (1991)	$dr_t = r_t(\eta_t - a ln(r_t))d_t + \sigma dW_t$	Yes	Log-Normal
CIR++ (2000)	$r_t = x_t + \phi_t \; dr_t = k(\theta - r_t)d_t + \sigma\sqrt{r_t}dW_t$	Yes	Chi-Squared

In (Aït-Sahalia, 1996) we see a deep analysis of many of these models testing the possibility of the model to be accurate. As he mentions, the imposed linearity of the drift and several other specification problems make impossible for many of these models to reflect the true form of the curve and to keep its specifications valid for a long time. He also includes the analysis of the following one with 8 parameters which he proposes to capture the main behavior of interest rate curve

$$dx = \left(\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \frac{\alpha_3}{x}\right)dt + \left(\beta_0 + \beta_1 x + \beta_2 x^{\beta_3}\right)dW_t \quad (3)$$

However, this model also shows problems with zero or negative interest rates and it has the additional problem that it is hard to make a sensible interpretation of the drift and volatility.

From all these results, multifactor models have been proposed where we have not just one Wiener process but two or three, not necessarily independent, which help to better reflect the form of curve and the fact that short-term rates and long-term rates are not as strongly correlated in practice that what we obtain from one factor models. Normally these models are also based on Vašiček and CIR, but allowing for the volatility term to be stochastic, for instance the Fong- Vašiček model

$$\begin{aligned} dr &= k_1(\theta_1 - r)dt + \sqrt{v}dW_{1,t} \\ dv &= k_2(\theta_2 - v)dt + \sigma\sqrt{v}dW_{2,t} \end{aligned} \tag{4}$$

or the Hull-White two factor model

$$\begin{aligned} dr_t &= (\theta_t + u_t - ax_t)dt + \sigma_1 dW_{1,t} \\ du_t &= -bu_t dt + \sigma_2 dW_{2,t} \\ d\langle W_{1,t}, W_{2,t}\rangle &= \rho dt \end{aligned} \tag{5}$$

Where a, b, σ_1, σ_2 are positive constants, r_t is the short-term rate, $\theta(t)$ a time dependent deterministic function which works as a parameter to fit the actual interest rate term structure, and $dW_{1,t}, dW_{2,t}$ are two Wiener processes with instantaneous correlation ρ. These models were extended and or generalized to include more factors by (Langetieg, 1980), (Duffie & Kan, 1996), (Dai & Singleton, 2000) and (Duffee, 2002). The canonic notation for these models is given by $A_m(N)$ corresponding to an N factor dynamic and an m factor volatility process where $m = 0$ corresponds to a constant volatility.

(Litterman & Scheinkman, 1991) employed empirical research to determine the common factors that affect returns on Treasury-based securities and found that most of the variation in returns can be explained in terms of three "factors" or attributes of the yield curve, which they called *level, steepness,* and *curvature.* See for instance (Martínez & Núñez, 2012) for a corresponding study of the Mexican case.

These results show that normally three factors would be enough. Although this approach has proved to be particularly useful for hedging purposes, given that it is obtained through empirical approaches with several bonds or principal component analysis, the actual structure of the stochastic processes as well as the interpretation of its nature is not clear, and it is not easy to find suitable economic or financial variables that explain them.

Considering the possible need to incorporate additional sources of stochasticity, models that incorporate jumps have proved to be very relevant. It has been well documented, for instance in (Nuñez Mora & Lorenzo Valdés, 2008) and in (Albanese & Trovato, 2007), that jumps in the short-term part of the interest rate curve cannot be overlooked. As we will see later in this work, most of the jumps that the short-term part of the curve show, reflect the jumps coming from monetary policy.

One of such models that incorporate jumps is the so called JCIR, which is based on the CIR model with the addition of a pure jump process with intensity $\alpha > 0$ and size determined by some distribution π on $\mathbb{R}^+$:

$$dr = k_1(\theta_1 - r)dt + \sigma\sqrt{r}dW_{1,t} + dJ_t \tag{6}$$

Finally, (Kim & Wright, 2016) recently proposed an affine model of three factors, free of arbitrage, where jumps can occur in each of the factors and not only for the short-term rates as previous models suggest. These jumps can occur at known moments on time, but their magnitude is random. They apply this model to the US rate curve and the given events they consider are the publication of employment data.

One of the few models that incorporate monetary policy in the modeling of the interest rate is that given by (Albanese & Trovato, 2007). They propose a model that includes asymmetric jumps, and they assume both, the drift and volatility, are stochastic

$$\mathrm{d}L_t = \mu_{a_t}(L_t)dt + \sigma_{b_t}(L_t)dW_t + \mathrm{dJ_t} \tag{7}$$

where W_t is Brownian, and drift and volatility are determined by processes a_t and b_t respectively, which allows the model to adopt the most usual forms the interest rate curve shows (but not all, for instance humped yield curve shapes as they mention in their work). Although very interesting, it fails to capture all the elements that the Target Rate reflects, not incorporating information about inflation and growth, for example.

Prior to the stochastic models, there were a good number of econometric models that tried to explain and predict interest rates. Following (Elliot & Baier, 1979) the econometric interest rate models roughly fall into three categories:

(a) those that emphasize distributed lags of past rates to explain current rates,

(b) those that are derived from some version of liquidity preference theory, and

(c) those that are the reduced forms of multisector econometric models.

For each of those categories they analyze two examples. The first distributed lags model they consider is that given by (Modigliani & Sutch, 1966)

$$R_t = a + b_0 B_t + \sum_{n=1}^{23} b_n B_{t-n} + K_t + e_t; \text{ with } \sum_{n=1}^{23} b_n > 0. \tag{8}$$

where R_t and B_t are the nominal rates of interest on long-term and short-term securities respectively, K_t is a term representing the effect of relative government supplies of bonds with long and short maturities, a, b_i are fixed coefficients, and e_t is an error term. The (MS) model concludes that the long rate is a distributed lag on the pattern of the short rate, modified by the effects of relative bond supplies. The intended role of relative bond supplies in the (MS) model is to reflect the effects of institutional "preferred habitats" upon current interest rates.

A second distribution model is that given by (Modigliani & Shiller, 1973)

$$R_t = a + b_0 B_t + \sum_{n=1}^{23} b_n B_{t-n} + c_0 \dot{P}_t + \sum_{n=1}^{23} c_n \dot{P}_{t-n} + dL_{t+}e_t; \text{ with } \sum_{n=1}^{23} b_n > 0, d > 0 \tag{9}$$

where $\dot{\boldsymbol{P}}_t$ is the rate of change in general prices, $\boldsymbol{L_t}$ is a liquidity premium variable defined as a moving standard deviation of short rates, $\boldsymbol{a}, \boldsymbol{b}_i, \boldsymbol{c}_i, \boldsymbol{d}$ are fixed coefficients, and all other variables are as previously defined. This last equation is a generalization of the previous one in that it accounts for the effects of inflationary expectations by adding a distributed lag on past inflation rates. Use of the distributed lags in the (MSH) model asserts that past patterns of short rates and inflation rates contain information useful in setting the current long rate.

For the other two categories, we will only show one of the models to explain the variables they consider and the logic behind the modelling. A liquidity preference approach to explaining the interest rate focuses upon the factors that determine the interrelated supplies and demands for money and bonds. The first of these models used is that of (Feldstein & Eckstein, 1970). Their model focuses upon the relationship between two assets, bonds, and money. Accordingly, the demand for money is positively influenced by income changes and negatively influenced by changes in the real interest rate and perceived capital gains on bonds. When the demand function for money is equated with the supply of money, the following model for the interest rate results:

$$R_t = a + b_0 \ln(M_t^R) + b_1 \ln(Y_t^c) + \sum_{n=0}^{23} b_{n+2} \dot{P}_{t-n} + b_{24}(R_{t-1} - R_{t-2}) + e_t;$$

$$\text{with } b_0 < 0;\ b_1 > 0; \textstyle\sum_{n=0}^{23} b_{n+2} > 0, b_{24} > 0 \qquad (10)$$

where $\boldsymbol{M_t^R}$ is the real monetary base per capita, $\boldsymbol{Y_t^c}$ is real personal income per capita, $\boldsymbol{a}, \boldsymbol{b_i}$ are fixed coefficients, and other variables as before. In order to make the assumptions of the (FE) model more nearly correspond to actual investment markets, Feldstein (with Chamberlain) reformulated their demand for money function as an excess demand for bonds model that considered four financial assets: money, bonds, equities and short-term bills and they do empirical experiments to determine another larger model.

The multi-sector approach to explaining interest rate movements emphasizes the interrelated impact of commodity market influences upon real rates of interest and monetary influences upon nominal rates due to inflationary expectations. The first such model is by (Sargent, 1969). In Sargent's model, investment is a function of the real rate of interest, $\boldsymbol{\rho_t}$, and the current change in real income $\Delta \boldsymbol{Y_t}$, the latter reflecting a crude accelerator hypothesis. Saving is a function of real income and the real rate of interest. Equating total investment and total saving produces the following expression for the real rate:

$$\rho_t = f(Y_t, \Delta Y_t), \quad \partial_1 f < 0, \partial_2 f > 0 \qquad (11)$$

To this model of the equilibrium real rate, Sargent adds a distributed lag on past prices to approximate an inflationary expectation term. This produces a model for the equilibrium nominal rate. To this model is added a market disequilibrium term accounting for the possibility that the market nominal rate may not be equal to the natural rate. The change in the nominal money supply is used to approximate this disequilibrium term. The resulting model is:

$$R_t = a + b_0 \Delta Y_t + b_1 Y_t + b_2 \dot{M}_t + L_1 \dot{P}_t + e_t;\ b_0, L_1 > 0;\ b_1, b_2 < 0 \qquad (12)$$

where $\dot{\boldsymbol{M}}_t$ is the rate of change in the nominal money supply narrowly defined and other variables are as before. The second multi-sector interest rate model they analyze expands on Sargent's structure by including a government expenditure and taxing sector along with the private investment and saving sectors.

After analyzing all these models, (Elliot & Baier, 1979) found that they generally explain long term interest rates very well by relating interest rate movements to concurrent movements in various macroeconomic variables. However, the ability of the models to explain next month's interest rates is very poor since fluctuations in next month's rate depend very little upon fluctuations in this month's economic variables. They even show that predictions of the best of the econometric models analyzed are not as accurate as if we assume that next month's interest rate will be the same as the current rate.

These results are very relevant in many ways: they suggest that variables analyzed helped explain the long part of the curve, not the short; without relevant economic information, the best possible model for the rate is that it does not change but for stochastic fluctuations coming from a Weiner process unrelated to economic or financial variables.

Following these results, attention in econometric models for interest rates decreased sharply and helped boomed the stochastic type models. However, as we will see in the next section, we will take some of the lessons obtained from the econometric models to develop a model for the Target Rate and to apply it to analyze the structure of the yield curve.

1.2 Problem statement and Modelling Rationale

According to (Banco de México, Política Monetaria, 2020) the central Bank has as its top priority to maintain the stability of the purchasing power of the Mexican Peso. To achieve this objective, starting on January 2008, Banco de México adopted as its operational objective the one-day Interbank Interest Rate (Target Rate).

With this operational objective, Banco de México injects or withdraws daily the missing or extra liquidity of the system through Open Market Operations (liquidity auctions). The rates at which surpluses in checking accounts are paid is zero while overdrafts are charged at twice the Target Rate for one-day bank funding.

By using the target rate for collecting overdrafts and as the basis for the Open Market Operations, Banco de México provides incentives for the funding operations between banks being carried out at rates close to that Target Rate.

The operational Target Interest Rate has several advantages:

- Facilitates understanding of monetary policy actions and their effectiveness.
- Provides greater stability to short-term interest rates and greater relevance of monetary policy over the entire Rate Curve.

- It makes the instrumentation of monetary policy in like manner as several of the most important central banks in the world.

This Target Rate is the basic building block to determine the Interest Rate Curve, so it is very relevant to understand which factors are the most important in determining this rate.

In the Mexican case, the Monetary Policy decisions are taken most months in specific dates that are published at previous year's end. On those days, the Governing Board take the decision to raise, maintain or decrease the Target Rate and if the decision is to raise it or to lower it, they also determine the amount which always comes in multiples of a quarter interest point.

(Litterman & Scheinkman, 1991) mention that it is widely believed that, in the U.S. case, changes in Federal Reserve policy are a major source of changes in the shape of the yield curve, then they mention that there is no clear idea of how to measure "policy". In what follow we will try to "measure" policy to determine the Target rate and then use that to determine at least the short and medium-term parts of the yield curve.

Following (Piazzesi, 2001) we could consider that the Target Rate R solves the stochastic differential equation

$$dR(t) = .0025(dN_U - dN_D) \tag{13}$$

where N_U and N_D are counting processes with stochastic intensities λ_U and λ_D, respectively. Jumps in the first process increase the target while jumps in the second lower the Target Rate by multiples of 25 bp.

However, we will follow a different approach to better reflect the actual behavior of the Target Rate. This rate is determined not in continuous time; instead, it moves in discrete jumps which are determined by discrete data as well.

In the following graph we show the actual Target Rate that has set Banxico's Board since August 2009. As mentioned before, starting in 2008 this mechanism has been the main tool for Monetary Policy. We decided to focus on data from one year after this implementation to allow for possible adjustments during the first year of its usage and we stop at December 2019 reflecting two periods of decreases and one long one for increases in the rate.

Figure 3: Banxico Target Rate (2009-2019)

As we can see, the Target Rate didn't change for a long period of time since August 2009 up to March 2012 where it was diminished in a quarter point. Since then, we have seen a period of diminishes in the rate, up to January 2016 where it started a period of increases that ended in 2019.

To model this behavior, we propose a discrete econometric model for the Target Rate. We will consider that the Target Rate for the nth month, R_n, is given by

$$R_n = R_{n-1} + .25C_n \tag{14}$$

with C_n being a discrete random variable with values on the integers and which responds to a group of external variables.

After analyzing the minutes of Monetary Policy decisions (Banco de México, Minutas, 2020), we realized that the rational to raise or to lower the Target Rate respond to different factors. Since the main mandate of the central bank is to maintain the stability of the purchasing power of the Mexican Peso, and standard monetary policy teach us that higher interest rates help curve inflation, the Board first looks at conditions that can affect inflation to determine if a hike is needed. In case these factors are under control, then they look at factors that affect growth to determine if there is a need to lower the rate to help boost growth. After this finding, we propose to consider $C_n = U_n - D_n$ with both U_n and D_n being non-negative integer valued random variables depending on two different sets of external variables, although some variables affect both.

At different moments, the Board can decide to not increase, increase in a quarter of point the rate, increase in half a point and so on and the same with the decreases. We concluded that this behavior is well captured by two Poisson distributions, one corresponding to U_n and another one corresponding to D_n given that the second one is restricted to be non-zero only if the first one is zero (hence they are by construction not independent.)

The analysis of the above-mentioned minutes helped us determine which variables are the most relevant to the Governing Board to raise, keep, or lower the Target Rate. We found that the information that Banxico gathers, either hard data coming from actual results or data obtained through surveys the Bank itself carries away, are considered as relevant to the determination of the Target Rate.

From the information provided by the minutes we determined that there is a group of economic and financial data that influence the decisions of the Governing Board heavier than others, namely: the decisions of the US Federal Reserve to raise or lower their rate, the exchange rate Mexican Peso-Dollar, the inflation data and expectations, the GDP growth data and expectations, and the market volatility.

1.3 Structure

In chapter 2 we define the full econometric model proposed, as well as the specific considerations we needed to introduce to reflect most of the analysis made to Banxico minutes as explained above. To avoid the problems found in (Piazzesi, 2001) we will propose the intensities to be given as the exponential of some auxiliary function that will be the one affected by financial and economic data and expectations.

After analyzing the Board minutes and testing the best way to incorporate the relevant variables we realized that additional information other than the one obtained from pure economic and financial data was needed. Also deriving from information given at the Board minutes, we decided to incorporate Banxico's surveys results. Here I would like to thank my advisor, Dr. Núñez Mora since his recommendation for this addition was crucial to obtain the good results we are going to show in this chapter.

Once defined the relevant variables, to determine the set of parameters in the model proposed we make an optimization using some evolutionary algorithm and show that the approximation we found is very consistent with the behavior of the decisions of the Governing Board.

Using the model found, after incorporating January 2020 data, we can use it to forecast for the decision of the Board that considers this new information. Moreover, we can determine the relevant data that was considered by them to take that decision. Here we find that our model predicts well the decision taken, as well as captures the relevant information that is considered for this decision.

In chapter 3 we show the results found. We will see that given our model proposed, a direct testing of the goodness of fit is extremely complicated what forced us to consider a different approach. As mentioned before, the Target Rate mechanism gives the market enough incentives so that the funding operations among banks are carried out at rates close to the Target. This in turn, given the liquidity of short and mid-term Government Bonds (CETES) and the easiness of their use in funding operations (repo transactions for instance), make the rates of these instruments also very closely determined by the Target Rate. However, the rates of longer-term Government instruments are influenced by some other factors that are not directly related to the Target Rate, like fiscal and macroeconomic balances and trends as shown in (Baldacci & Kumar, 2010) and explained in our literature review.

To check the validity of our approach, we test the results given by the model of the Target Rate using it as the one-day rate to determine the rates of CETES of all maturities showing that, for those bonds with maturities up to one year, our model fits very well the given rate (in all cases with $R^2 > 95\%$ and good model specifications). However, the power of adjustment and model specification decreases with the maturity so that for three-year bonds. To determine the extra factors to consider or else some different structure to overcome these difficulties, we review models for the Mexican yield curve to explore different ways to incorporate additional fiscal considerations. After analyzing the residuals, we incorporate another variable related to fiscal and macroeconomic data which helped us correct the model specification and allowed us to obtain an $R^2 > 93\%$ in this case.

Finally, in chapter 4 we give some conclusions and possible ways of further work which arise from the results found.

Chapter 2
Modelling the Target Rate

2.1 Basic econometric Model

We will consider that the Target Rate for the nth month, R_n, is given by

$$R_n = R_{n-1} + .25C_n \tag{15}$$

with C_n being a discrete random variable with values on the integers and which responds to a group of external variables. We propose to consider $C_n = U_n - D_n$ with both U_n and D_n being two random variables. following Poisson distributions depending on two different sets of external variables. given that the second one is restricted to be non-zero only if the first one is zero.

As we know, a Poisson distribution is determined by its intensity. We call λ_n^U the intensity corresponding to U_n and λ_n^D the one corresponding to D_n. Since intensities should necessarily be positive valued, we consider instead of λ's the variables μ so that

$$\lambda_n^U = e^{\mu_n^U} \tag{16}$$

and the same for D.

After analyzing the minutes, we noted that the variables of greatest interest to determine an increase in the Target rate were the decisions taken by the U.S. Federal Reserve, inflation, and the exchange rate with the US dollar. The decisions of the Federal Reserve were incorporated in a very direct way; however, it was not clear in which way exchange rate and inflation could be considered. The first thing we considered was to incorporate directly the inflation rate, however we noticed that only when it was above the target of 3% (actually above 3.5%) it was a relevant variable so we decided to incorporate it only when it was above target. We proved many ways to incorporate the exchange rate and finally we realized that it was needed to consider the percentage change of the rate with respect to previous moments.

After these considerations, we proposed the following model

$$\mu_n^U = \alpha_0 + \alpha_1 i_{n-1} + \alpha_2 \delta_{n-1}^{+} + \alpha_3 r_{n-1}^{3m} + \alpha_4 r_{n-1}^{1m} + \varepsilon_n^U \tag{17}$$

where i_{n-1} is the rate of inflation above the target rate of 3% reported on month $n-1$, δ_{n-1}^{+} is the amount of quarter points increased by the Fed immediately before the Governing Board decision, r_{n-1}^{3m}

is the quarterly percentage change on the exchange rate with respect to the US dollar, and finally r_{n-1}^{1m} is the monthly percentage change on the exchange rate with respect to the US dollar.

We see that the exchange rate is incorporated relative to previous values, both a quarter before and a month before. This reflects the interest given by the Board to the possibility of a transfer effect from the exchange rate to inflation.

As we mentioned before, after considering that there is no risk of spiking inflation, the Board analyzes the economic growth to see if they can help dynamize the economy if needed. The variables of greatest interest to determine a decrease in the Target rate were also the decisions taken by the Federal Reserve (to lower its reference rate, in this case), relative inflation, and GDP growth. However, to capture the effect of GDP growth, after too much analysis, we found that it was relevant to consider not actual figures of GDP, but better indicator-based data (namely the quarterly change on the IGAE index). After analyzing how these variables are affecting the decisions, we consider the following model for the natural logarithm of the intensity

$$\mu_n^D = \gamma_0 + \gamma_1 i_{n-1} + \gamma_2 p_{n-1} + \gamma_3 y_{n-1} + \gamma_4 \delta_{n-1}^{-} + \varepsilon_n^D \tag{18}$$

where i_{n-1} is as explained before, δ_{n-1}^{-} is the amount of quarter points decreased by the Fed immediately before the Governing Board decision, and finally, p_{n-1} is the monthly change on the IGAE index for the previous month.

Each of these regressions can be thought of as Poisson Regression for which is well known, for independent regressors, to be possible to estimate their parameters using a Maximum Likelihood approach, see for instance (Greene, 2003). Moreover, as is well known, the sum of two Poisson distributions is also Poisson. However, for the difference of two such distributions, the analysis is not so simple. The distribution of the difference between two independent Poisson random variables is given by the Skellam distribution and it involves the modified Bessel function of the first kind as explained in (Karlis & Ntzoufras, 2006). Using properties of this function, maximum likelihood estimates of the parameters of the Poisson difference could be derived. However, since from construction we know that both Poisson random variables involved in our estimation are not independent, it is not easy to determine a likelihood function to maximize in our case or to analyze to determine how far are we from best estimates. Given these difficulties, to find the appropriate coefficients we decided to study the Mean Square Error of the actual Target Rate and the Estimated Target Rate.

As it is well known, given a Poisson distribution X with intensity λ, the probability

$$P[X = k] = \frac{\lambda^k e^{-\lambda}}{k!} \tag{19}$$

so the amount of the intensity will determine the jump with the highest probability to occur, which is exactly what one wants to capture to model the decisions of Banxico's Governing Board.

To be precise, if the intensity found determines that a two-fold jump is the one with the highest probability, we would consider that the Board decides to make a two-fold jump, and so on. Normally we will see that the most common situation is that the intensity is very small, so the decision taken is not to change the rate. Note that we are getting as a byproduct the probabilities of any possible change in the rate which can be analyzed further to forecast the actual decisions taken by the Board, as we will see in section 2.4.

To estimate the coefficients of our model for the Target Rate, we considered the maximum probability obtained from each Poisson distribution for each month, as mentioned above. So, we end up having a series of rate values that we compare with the actual Target Rate. The difference of these two rates define the error for each month, so we can get the mean square error obtained by summing up the squares of these differences.

To minimize this MSE, we used a numerical evolutionary minimization process. We had to impose a series of restrictions to this minimization to get sensible results, especially regarding the signs that the coefficients could take. All coefficients for μ_n^D needed to be negative, while those for μ_n^U needed to be all positive, except for the first two that had to be negative. These restrictions allowed us to obtain results that capture the economic reality the Governing Board is analyzing.

After the corresponding minimization, we found that this model proposed captured the general behavior of the Target Rate, as shown in the following graph. However, they failed to be sufficiently timely, so we concluded that some important pieces of information that the Board consider were missing.

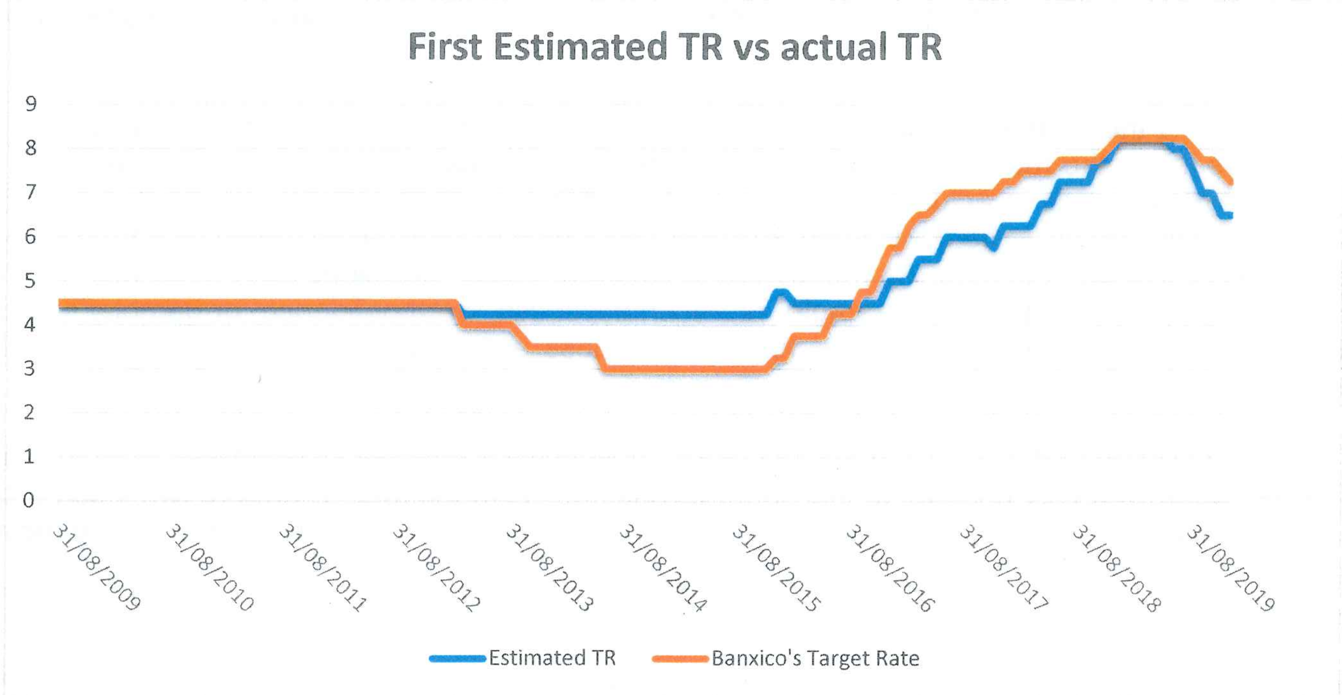

Figure 4: First approach for Estimated Target Rate vs Actual Target Rate

2.2 Model Incorporating Survey Results

Since September 1994, Banxico surveys a group of analysts from Financial Institutions as well as private Consulting Firms about their expectations of several economic variables and publishes every month the main results of the consensual forecasts formed by the average of values given by those surveyed and the time horizon of these forecasts in the "Encuesta sobre las Expectativas de los

Especialistas en Economía del Sector Privado" (Banco de México, Encuestas, 2021). The Banxico survey is the only one that has an official character and is therefore the most extended.

At its beginning the survey included just inflation forecasts, but now it is a detailed report which contains the monthly evolution of inflation expectations, interest rates, exchange rates, as well as economic growth expectations, salary, employment, the behavior of Public Finances and the external sector. Finally, this report analyzes the perception of these specialists about the business environment, investment safety and their confidence about the economy performance. This survey has changed a lot during the time it has been published, starting with 13 participants, and now having about 40. Although the individual answers remain confidential, starting in April 2019, Banxico publishes the list of Institutions which are surveyed.[1]

As shown in (Capistran & López-Moctezuma, 2008) inflation, interest rate, and GDP expectations in the Survey seem to incorporate information in a relatively efficient manner. They show that forecasts appear to be better, in mean squared error terms, than the benchmark forecasts, except for the case of one-year ahead inflation (while exchange rate forecasts do not seem to optimally incorporate available information and do not seem to improve upon forecasts obtained from a random walk model.) After these results, it has been customary to place the survey as a reliable tool for the relevant variables.

One of the surprising results we found in this work is that, in our model, the Governing Board weighs as much the results of these surveys as the actual data, or even more in some cases. Since the surveys are made to actual teams of analysts and then evaluated by the board, the incorporation of these results needs to be considered as the incorporation of information driven by hard data but with some additional components. As mentioned in (Shefrin, 2002), decisions taken this way could show some heuristic-driven bias, for example, representativeness, overconfidence, or gambler's fallacy; or they could show frame dependence, for example, loss aversion/realization, mental accounting, or money illusion.

We noticed that the inflation expectations shown by the monthly survey taken by Banxico to the private sector was very relevant to the Board decisions to hike the rate. Therefore, we incorporate this variable and consider the following model for the natural logarithm of the intensity for the up movement

$$\mu_n^U = \alpha_0 + \alpha_1 i_{n-1} + \alpha_2 \delta_{n-1}^{+} + \alpha_3 r_{n-1}^{3m} + \alpha_4 r_{n-1}^{1m} + \alpha_5 x_{n-1} + \varepsilon_n^U \quad (20)$$

Where all the variables are as given before and we incorporate x_{n-1}, the inflation rate expectation given on the Banxico survey of month $n-1$.

[1] For example, in April 2019 the following list of surveyors was published: Action Economics; Banco Actinver S. A.; Bank of America Merrill Lynch; Banorte Grupo Financiero; Barclays; BBVA-Bancomer; BNP Paribas; Bursamétrica Management S. A. de C. V.; BX+; Capital Economics; Centro de Análisis e Investigación Económica; Centro de Estudios Económicos del Sector Privado A. C.; CIBanco; Citibanamex; Consejería Bursátil; Consultores Económicos Especializados; Consultores Internacionales; Credit Suisse México; Deutsche Bank; Epicurus Investments; Evercore ISI México; Finamex Casa de Bolsa; Grupo de Economistas y Asociados; Harbor Intelligence; HSBC; IHS Markit; Invex Grupo Financiero; Itaú Asset Management; Itaú Unibanco; JP Morgan; Monex Grupo Financiero; Morgan Stanley; Natixis; Raúl A. Feliz y Asociados; Santander Grupo Financiero; Scotiabank Grupo Financiero; Valmex; Vector Casa de Bolsa; y Prognosis Economía, Finanzas e Inversiones, S.C..

For the decision to lower the rate, after analyzing the additional variables that affect the decisions, we consider the following model for the natural logarithm of the intensity

$$\mu_n^D = \gamma_0 + \gamma_1 i_{n-1} + \gamma_2 p_{n-1} + \gamma_3 y_{n-1} + \gamma_4 \delta_{n-1}^- + \varepsilon_n^D \quad (21)$$

Where all variables are as before, but we incorporate y_{n-1}, the GDP growth expectation given on the Banxico survey of month $n-1$.

We also analyze the use of several other variables and combinations of them, such as market volatility, the Central European Bank rate, monetary basis, actual GDP results with several lags, and so on. However, trying to keep as much parsimony as possible for the model, we ended up with these two sets of variables for the intensity of the increase and the decrease on the Target Rate which give excellent results without involving too many variables as we will see in the next Chapter.

2.3 Results for the modeling of the Target Rate

As mentioned in the previous Section, we define an econometric model of the intensities for the Poisson distributions that determine de probabilities of a rise or a low of the Target Rate. To find the coefficients for the given models, we perform a minimization of Mean Square Error using an evolutionary algorithm. Once our model was set incorporating the information from the Banxico Survey's, we found the coefficients shown in Table 1 for the natural logs of the intensities. Since we are estimating maximum probabilities for discrete jumps, note that these values are among a range of values that give us the same results.

Table 2: Coefficients to the natural log of intensities to raise and to lower the rate.

α_0	α_1	α_2	α_3	α_4	α_5
-4.9	-133	8.3	2	19	54
γ_0	γ_1	γ_2	γ_3	γ_4	
-0.25	-4	-0.24	-0.8	-0.39	

Since the evolutionary process is used to determine probabilities which in turn determine the size of the jump, all the process happens to be highly dependent on the initial values chosen (seeds), so we needed to be very careful to start with some reasonable values that capture the behavior we want to model.

With the values shown on Table 1 for our coefficients, we obtained the following graph of our estimate for the Target Rate together with the actual rate:

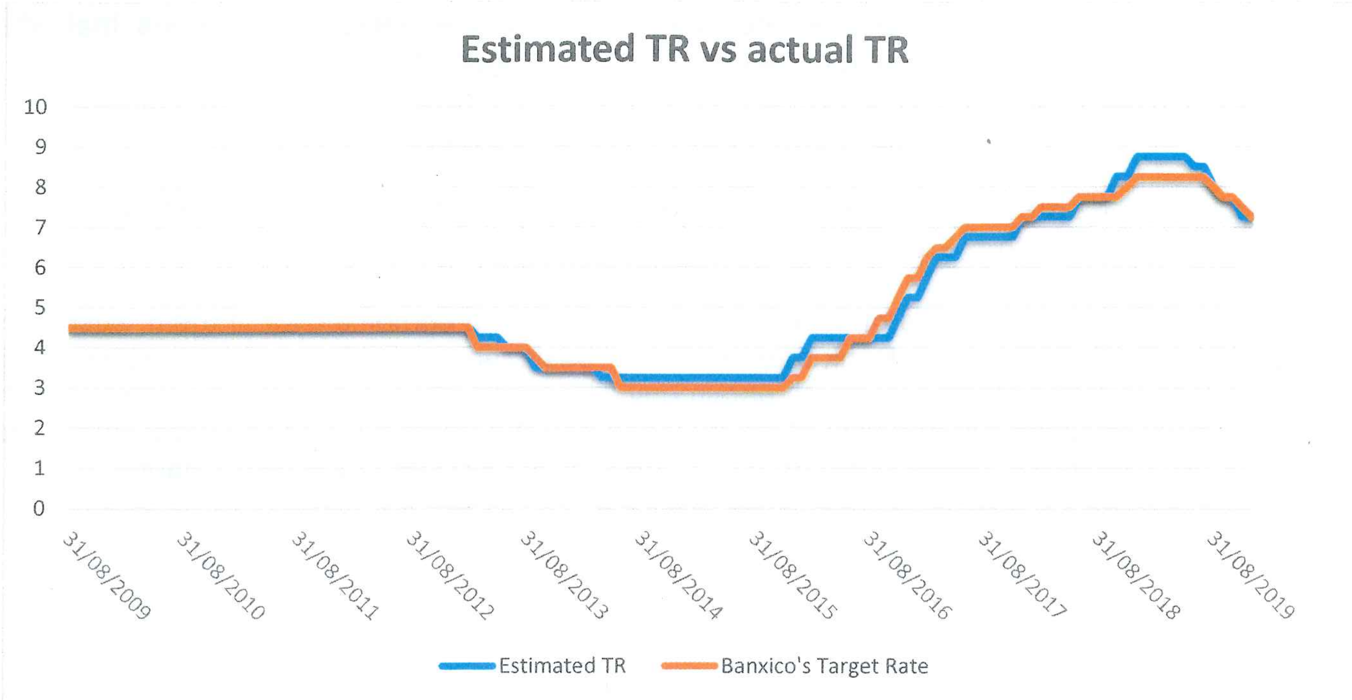

Figure 5: Estimated Target Rate vs. Actual Target Rate

We can see that our estimate captures very well the behavior of the Target Rate. It diminishes when the conditions are given to lower the rate and it increases when it is needed. Unfortunately, we can't precisely measure how well the approximation is following the actual data, given the nature of the estimation. In the following Chapter, we will use our estimated Rate and compare it with actual rates of zero-coupon bonds, to see how well it could be used to estimate them and what extra sources of variability we can consider.

2.4 Determination of the Target Rate using the Model

Once we obtain a model with data up to December 2019, we use this model to forecast the probabilities of movement of the Target Rate for data of January 2020 and then we compare the results and the factors influencing this result with the actual decision taken by the Board and the factors mentioned in the minutes that influence that decision.

Given that the Fed didn't change its rate this month and that exchange rate appreciated with respect to its one month previous value and its three-month previous value as well, together with the fact that inflation results were below target and their expectations changed slightly with respect to previous month, we determine using our model that the probability for not increasing the rate is 99.16%, while the probability for a 25 bps increase was 0.84% and for a bigger increase practically zero.

As mentioned before, once we determine that no hike will come, we analyze the possibility of a decrease. In this case, also the Fed didn't dimmish the rate, so that factor was not relevant in this case, but IGAE continued decreasing by an extra 0.1 of the index and annual growth expectations for 2020 also decreased by 0.1%. Using these data in our model, we find that for a decrease, the probability of not decreasing the rate was 38.52%, but the probability of decreasing in a quarter point was 36.75% and a decrease of half point or more has a probability of 24.73%.

So, we can conclude that the probability of a decrease is 61.48%, significantly larger than the probability of not changing the rate, and among the amount of the decreases, probability of a decrease by 25 bps is 48% higher than probability of a higher decrease. So, the model would predict that using the data up to January, Banxico's Board will decide to decrease the Target Rate, taking into account that the Fed didn't change its rate, that inflation levels and expectations are in range, and that the economic perspectives are low.

Banxico's Board met on February 12th to analyze the recent developments and to determine if the rate needed to be adjusted[2].

Note that the minutes don't mention the specific person which gives the comments recorded, they just mention "one of the members of the Board" or "another member of the Board" so at times it become slightly difficult to see what the position of each individual member was. However, given the relevance of the use of the model for these purposes, following is a reproduction (translated by the author) of all the consideration given in the relevant minutes in this case.

> One member of the Board mentioned that the relevant factors to consider in their decision are the following: a) decrease in risks of inflation; b) low international interest rates; c) good results in internal markets; and d) strength shown by the exchange rate (U.S. Dollar/Mexican Peso) to external shocks.
>
> Another one added that smaller credit risk primes (for Mexico) and a better perception of the country-risk, as well as the stability of public finances and a lower uncertainty on commercial aspects were relevant. This board member expressed that, considering that some risks are still latent, they should consider that there was still room for the Monetary Policy to be used.
>
> Some other members considered the actual Monetary posture too restrictive, given the economic context.
>
> One specified that the real interest rate in Mexico is the highest among a large group of economies, including some with credit rating below the Mexican one. He argued that a restrictive monetary policy for a long time can risk the macroeconomic and financial stability and to affect consumption and investment. He even mentioned that one analysis realized by some broker house estimated the cost associated to a restrictive monetary posture in 0.7% of GDP in 2019 and 1% in 2020. He noted that since it affects growth, it also affects the results of companies and the stock market, as well as public finances through a greater financial cost of debt and smaller tax revenue. He added that this in turn makes less attractive the national financial markets, affecting in turn the risk prime and exchange rate. Because of all these aspects, he considered that, given that inflation

[2] Attendees: Alejandro Díaz de León Carrillo, Governor of the Banco de México. Irene Espinosa Cantellano, Deputy Governor. Gerardo Esquivel Hernández, Deputy Governor. Javier Eduardo Guzmán Calafell, Deputy Governor. Jonathan Ernest Heath Constable, Deputy Governor. Arturo Herrera Gutiérrez, Minister of Treasury and Public Credit. Gabriel Yorio González, Deputy Minister of Treasury and Public Credit. Elías Villanueva Ochoa, Secretary of the Board.

consolidates towards its target, it will be necessary to transit to a monetary posture closer to neutral.

Another one expressed that the Target Rate started 2019 in a very restrictive level of 3.4% in real terms, and with a differential of 575 bp with respect to that in the U.S. He mentioned that even with the cuts in Target Rate applied in 2019, there was no absolute relaxing of policy since real rate closed that year in 4.4%. Regarding the relative position, he commented that the differential with the United States only decreased 25 basis points towards the end of the year, remaining one of the highest spreads. He noted that this is even higher when considering the lower exchange rate risk, the greatest appetite for risk and that there are no changes anticipated in accommodating monetary policies at the global level. He argued that, on the absolute posture side, economic slack demands a lower return rate on equity, so we must look for consistency with the neutral rate. In sum, considering that both positions point to the need for further relaxation in order to achieve the priority objective at the lowest possible cost.

Most believed that economic stagnation is the result of multiple factors and noted that the benefit of using the policy monetary policy to reactivate the economy would be modest, and that the risk of doing so could be high.

One member indicated that monetary policy has been criticized as the cause, to a large extent, of the stagnation. He considered, however, that its impact has been marginal. He noted that at this point it is important to recognize the limits and scope of monetary policy. He stated that, on the one hand, a too restrictive policy may affect the investment and consumption, but it is not the main factor of its loss of dynamism. He added that the benefits of the monetary policy implemented have been significant, in terms of financial stability and exchange rate, as well as in the preservation of the purchasing power of wages.

Another mentioned that recent research suggests that, given the high proportion of informality in the economy and the low credit penetration, the impact of channels of interest rates and credit to stimulate the activity is limited, while the expectations is the preponderant to control the inflation. He added that while a more pronounced relaxation of monetary policy could improve fiscal balance, it would not correct the structural problems of public finances. What's more, he stressed that the sluggishness of investment is the result of the lack of certainty about public policies and concerns about internal factors, including governance, rule of law and corruption, which continue to configure the greatest obstacles to private investment. In this regard, another pointed out the importance of maintaining an environment of greater certainty, arguing that the Central Bank can contribute to this by preserving macroeconomic and financial stability.

Another pointed out that, given the evolution of underlying inflation, the challenge is that wage increases do not impact inflation and nullify the improvement in purchasing power. He estimated that the inflationary pressures on the supply side are difficult to control with monetary policy, even more so when they are the product of a medium-term policy, like salary. He noted that a salary policy is successful when you can permanently increase the purchasing power and create better paid jobs. He argued that one should proceed with caution and prudence, weighing when it is time to modify the rhythm of relaxation, bearing in mind that the objective is to contribute to the improvement of the purchasing power of wages. He considered that, although there are elements that suggest a reduction in the interest rate, given the persistence of core inflation and the permanent shock of wages, the risks balance is uncertain in the short and medium terms.

Most highlighted the challenges facing monetary policy. Some noted the difficulty to reconcile a wide slack with inflationary pressures and mentioned that there exist some perceptions of delaying the consolidation of inflation around the target. One pointed out that expectations of general and core inflation are converging to 3.5%, which implies a firmer root at that level, while the underlying component in the long-term has increased. If the above is true, that would represent a factor that has to be considered as a second-order effect on prices. Thus, he argued that there are signs of dislodging the expectations of core inflation, in addition that an environment of high uncertainty persists. He believed that in this context, the challenges for monetary policy have been accentuated, so it should continue to be handled wisely. He mentioned that, even under the cautious approach followed, risks of second order effects about prices have increased. He argued that the Central Bank must focus on achieving its goal in the period of influence of monetary policy. He added that, at the same time, it should be ensured that convergence towards the goal is orderly, avoiding potential costs for the economic activity.

Another member considered that although the slack has expanded more than anticipated, the challenges that the economy faces are not only those associated with a cyclical position. He noted that there are other challenges and external and internal risk factors that influence the economy, price formation and inflation. He noted that while under a regime of inflation targets monetary policy stance must be such that general and underlying inflation forecasts remain around its goal on the forecast horizon, other elements should also be considered. Among these he mentioned the effects of the external environment, the expansion of slack and the need to create appropriate conditions in financial markets. In addition to the above, he stressed that monetary policy decisions must also: i) allow an orderly adjustment of the economy; ii) consider the behavior of external and internal yield curves and corresponding risk premia; iii) keep investors interest in the country as investment destination and build trust; iv) consider the effects of monetary relevant relative conditions; and, v) promote that the yield curve incorporates interest rate expectations consistent with the Central Bank's objectives on the horizon in which monetary policy operates. Likewise, he argued that the foregoing suggests that the adjustment of the monetary policy stance must: i) be gradual, to maintain the flexibility needed to face a wide range of scenarios and allow the required adjustments be ordered; ii) incorporate all the information available; iii) respond comprehensively to all factors and characteristics of the economy, expected and in a wide range of scenarios; and, iv) results in the orderly and sustained convergence of inflation overall to our goal within the timeframe in which the monetary policy operates.

Most of the members of the Board highlighted the importance of future adjustments to be consistent with the evolution of the balance of inflation risks and the evolution of the economic situation. One opined that the Board should act prudently and gradually given the environment of uncertainty. Another noted that under these circumstances, future monetary policy actions should be based on information available at the time of decision, because trying to give more precise guidance in an unclear environment can affect the credibility of the Central Bank.

One added that the Board should maintain the message that the monetary stance is consistent with the inflation target achievement in the period in which the monetary policy operates. Another argued that the adjustment speed should be considered period after period, depend on the information available, and maintain a trajectory convergent to the inflationary goal. He highlighted that a further deterioration in the risks balance can limit capability of use of monetary policy room in the relative and absolute positions in the future. He added that since the speed of the adjustment has an effect on expectations, a prudent policy helps a firmer anchoring and maintains the credibility of the Central Bank. He noted that one must be careful not to provoke complacency with relatively low inflation, but higher than the goal.

However, one considered important that this guide recognizes the advisability of adjusting the monetary stance towards a less restrictive one in a relatively short period, leaving said adjustment conditioned to the favorable evolution of the inflationary expectations. He argued that, based on the information available, it is possible to consider a scenario in which we reach the neutral zone of the interest rate towards the end of this year. He further estimated that a scenario like this would not put at risk the outlook for inflation, but it would improve economic prospects.

After all these considerations, Banco de México Board in the presence of all its members decided unanimously to diminish the Target Rate in 25 base points to leave it at 7%. According to the minutes of that session, the different aspects that came into consideration were primarily: general inflation levels, general inflation perspectives in the horizon where monetary policy works, a broader economic amplitude and the recent behavior of internal and external yield curves. Note that the most relevant behavior of internal and external yield curves they consider was that of the U.S. Federal Reserve.

We can see that although the discussions are rich to determine the decision, at the end the relevant variables that they are considering are the ones our Model captures and the weight they give to them is well reflected in our analysis.

Chapter 3
Testing the Goodness of the Model and Determination of the Yield Curve.

3.1 Testing the Goodness of the Model

Given the considerations mentioned in the previous chapter to determine the coefficients in our model for the Target Rate, we were not able to directly analyze the goodness of fit of the model. So, we decided to compare our model's Estimated Target Rate with actual rates seen on short and medium-term bonds to see how well our model could be used to determine these rates.

To compare our estimated rate with the actual short and medium-term rates found on the bond market, we consider that the target rate determines the one-day rate. Then, we transform the values we obtained to bond prices of different maturities considering that the price of a bond of maturity k would be $P = \frac{100}{(1+\frac{ETR}{360})^k}$, where ETR is our Estimated Target Rate. Since the bonds we are considering are CETES, we just have $k = 28, 91, 182, 364, 1092$. Up to one year ($k = 364$) the given rates are considered as short-term rates. For 1092 days, which correspond to 3 years, we considered that we are talking of medium-term rates. As we will show shortly, the behavior of medium-term rates differs consistently with the one that short-term rates follow.

Below we show the linear regressions for different maturities together with the trend line obtained and the value of R^2 for each of the maturities.

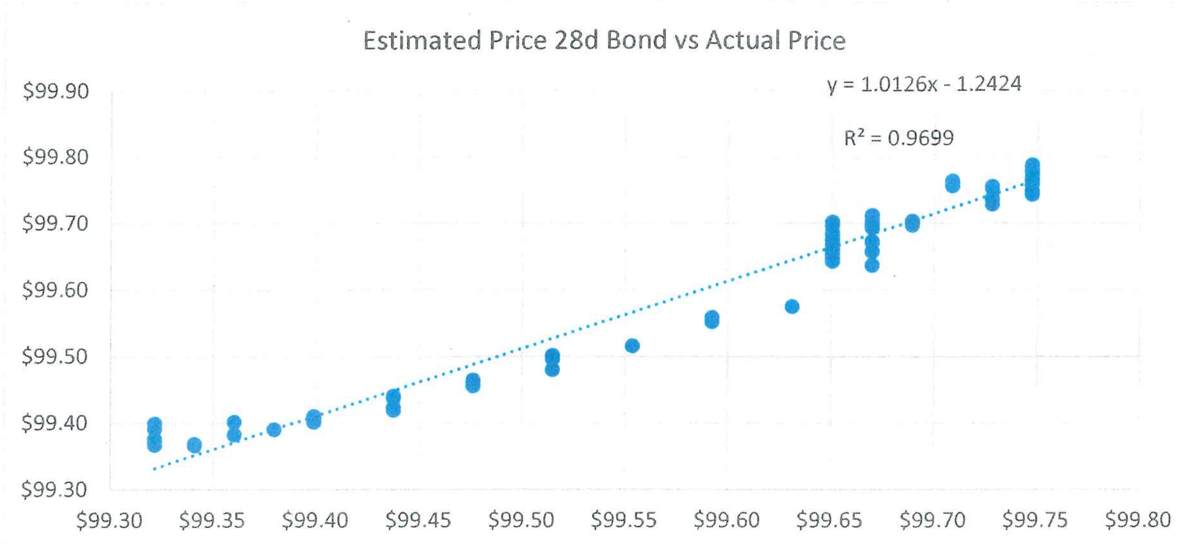

Figure 6: Estimated Cetes 28 vs Actual Price

We can see here that when the actual fit is excellent, coefficient very close to 1 and there is a constant of 1.24 that is undervalued.

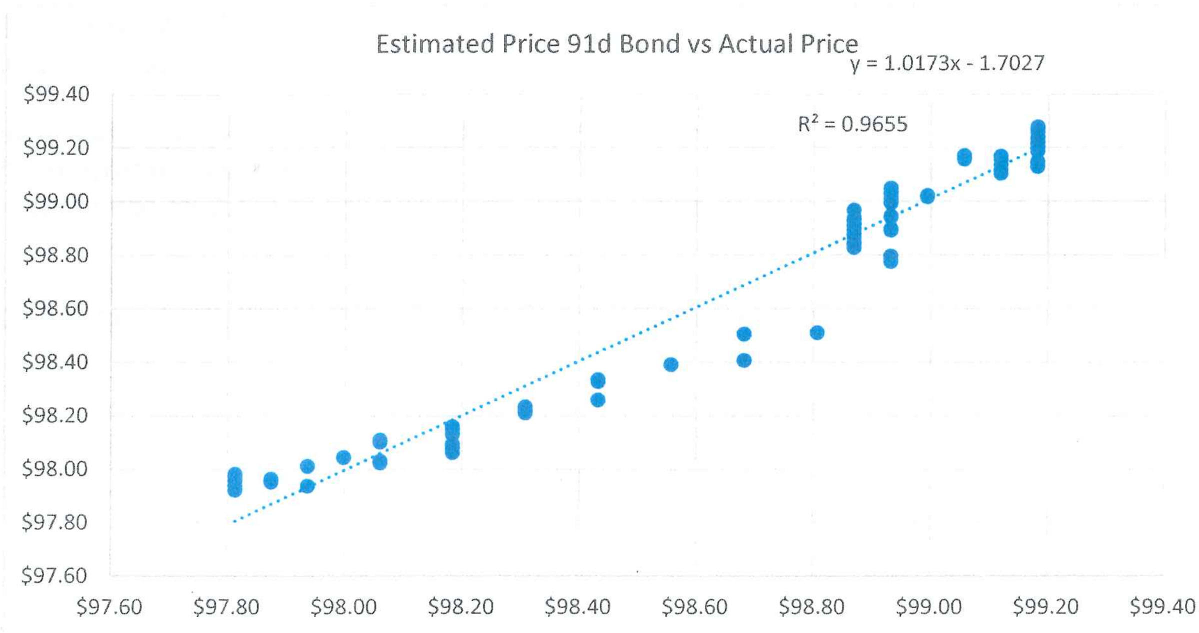

Figure 7: Estimated Cetes 91 vs Actual Price

We can see here that when the actual fit is excellent, coefficient very close to 1 and there is a constant of 1.70 that is undervalued.

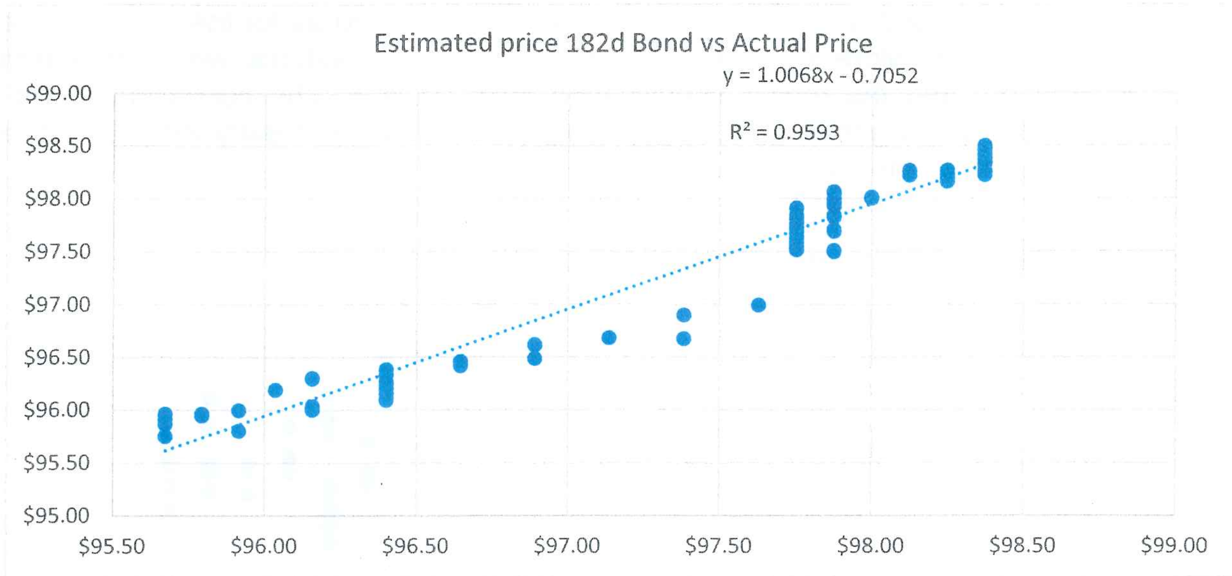

Figure 8: Estimated Cetes 182 vs Actual Price

We can see here that when the actual fit is excellent, coefficient very close to 1 and there is a constant of 0.70 that is undervalued.

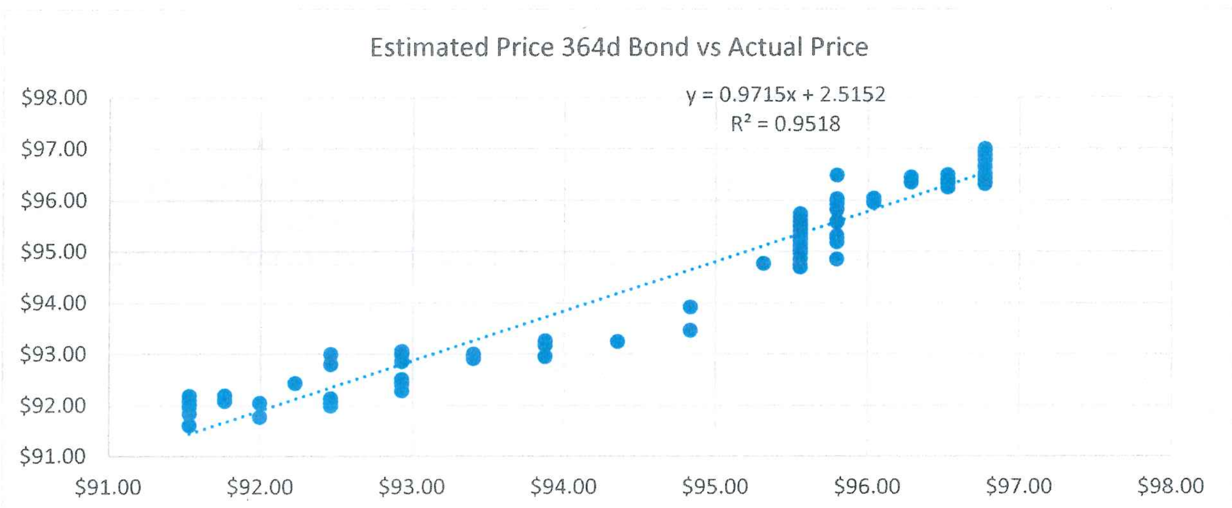

Figure 9: Estimated Cetes 364 vs Actual Price

We can see here that when the actual fit is excellent, coefficient very close to 1 and there is a constant of 2.52 that is overvalued.

We see that for the first four maturities (up to one year), our model captures very accurately the behavior of the Bond prices, it fits very close the given prices (with $R^2 > 95\%$ in all cases). Moreover, note that the coefficient of the independent variable is very close to one reflecting that the price is changing slightly away from the behavior of the estimated Target Rate. After analyzing the residuals, we also found that for those four terms, the Jarque-Bera test deemed the residuals normally distributed and also, the models are shown to be well specified by the Breusch-Pagan test.

However, for the fifth bond analyzed, the model fit is not as good as for the previous ones (still $R^2 = 88\%$ is sufficiently large). More importantly, after analyzing the residuals, we found that they fail the normality assumption, and the model specification tests as well. The coefficient found in the regression being far from one together with the above-mentioned issues clearly indicate that at least an extra variable is missing in this case.

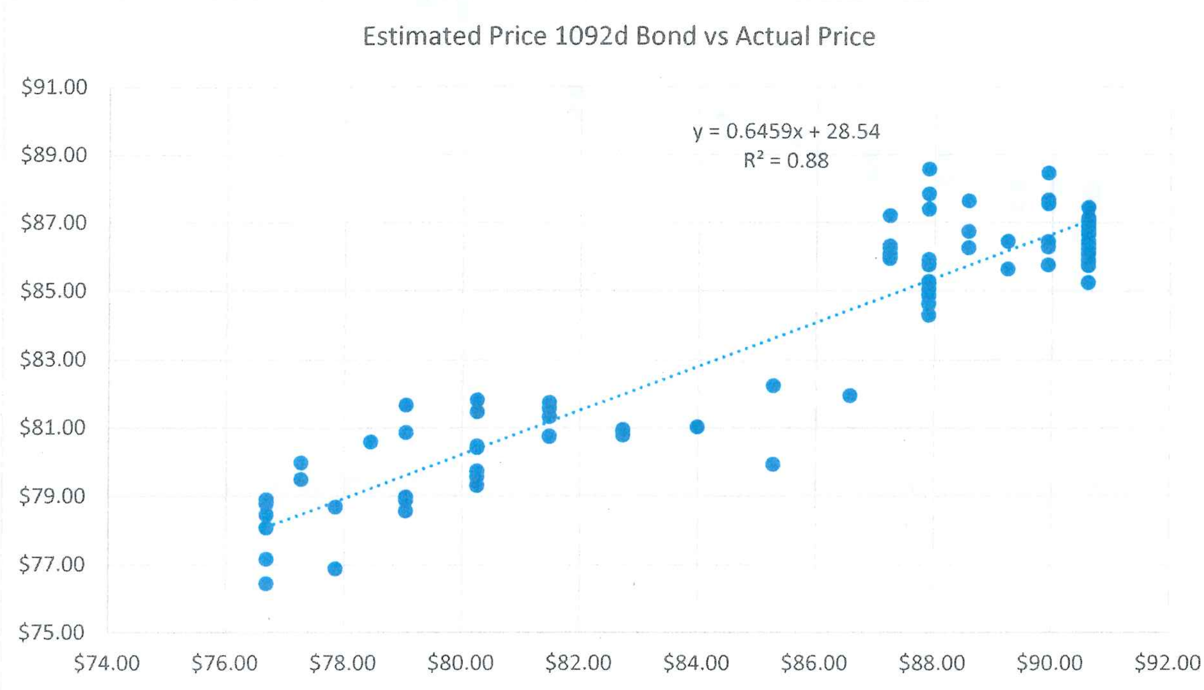

Figure 10: Estimated Cetes 1092 vs Actual Price

Of course, the monetary policy is set to provide incentives for the funding operations between banks being carried out at rates close to that Target Rate. Longer maturity bonds are affected by additional variables related to longer term objectives of institutional investors both domestic and foreign, among others.

Considering these results, in the next section we will analyze what else could be incorporated to model the Mexican Yield Curve for short and medium-term rates.

3.2 Mexican Yield Curve

As mentioned before, being able to understand the behavior of the interest rate structure, as well as what determines it, is essential to determine capital costs and to be able to value financial assets. The fixed income market in Mexico encompasses a wide range of instruments and is among the most developed in Latin America. The instruments that we have been analyzing are just the zero-coupon nominal rate instruments issued by the federal government (Cetes). To complement the analysis, we could include fixed coupon bonds (M Bonds). Zero coupon bonds are issued by the federal government for terms of 28, 91, 182, 360 and 1082 days, while bonds with a fixed coupon payable every 182 days are issued for 5, 10, 20 and 30 years.

For the next exposition we follow closely that given by (Álvarez del Castillo, 2018) to introduce factor models and in particular the Nelson-Siegel model. It is worth mentioning that we just consider the non-stochastic part of the exposition, since as mention before and shown in the previous sections, our approach follows more closely an econometric one.

The following graphs show the evolution of government bond yield rates at the reference terms from 2008 to 2018. As can be seen, the shape of the curve changes over time both due to the different levels reached by the reference rates, from the change in the slope of the yield curve (that can be approximated by the difference between short-term and long-term rates), and changes in the convexity of the term structure.

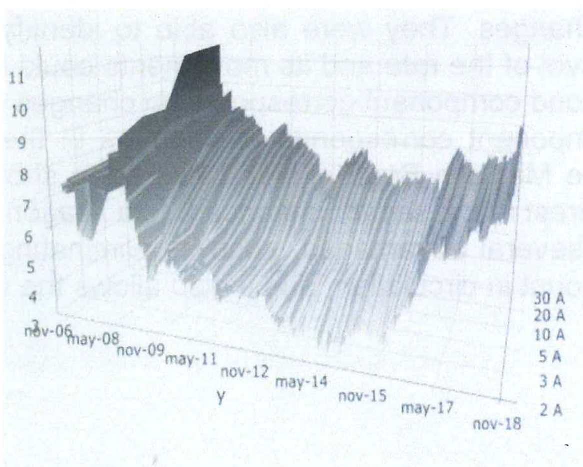

Figure 11: M Bonds at different maturities (Bps)

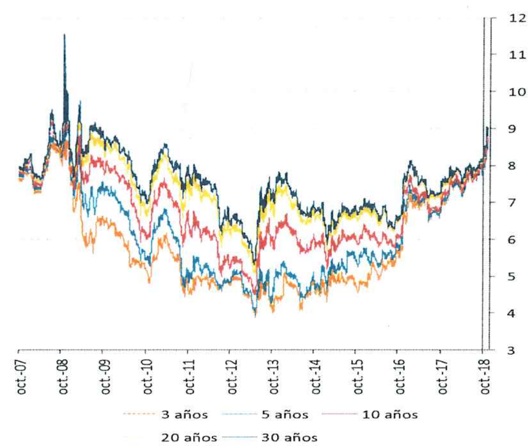

Figure 12: M Bonds Rates (%)

Many models have been developed to explain the behavior of interest rates, but most of them cannot explain the presence of leptokurtic distributions in the changes in interest rates observed in the market. This is because most of these models rule out the possibility of jumps in the dynamics of interest rates. As mentioned before, (Nuñez Mora & Lorenzo Valdés, 2008) show that the inclusion of jumps in the diffusion process improves the precision of the interest rate dynamics. Moreover, we have seen in the previous section that jumps coming from monetary policy directly affect the behavior of short-term rates.

Another phenomenon that encourages the presence of heavy tails in the distribution of changes in interest rates is the change in volatility in the face of stress episodes in financial markets. As in the stock and exchange markets, volatility does not remain constant over time and may increase in some periods. To incorporate this behavior, models with stochastic volatility can be used. In the case of the interest rate curve, the number of instruments represents an additional complication when wanting to carry out an interest rate model and incorporate stochastic volatility since it could involve two stochastic equations for each node of the curve to model the changes in the rate and changes in volatility.

As seen in the previous section, it is already challenging to determine what factors could be incorporated to the model for the longest maturity zero coupon bonds, so in this work we won't analyze fixed coupon bonds. We could incorporate them in our analysis by using cubic splines and a bootstrapping method to transform the rates of the M bonds that are in the market into zero coupon rates for multiples of 182 days up to 30 years. However, in this way, when considering the rates of Cetes and the government funding rate, a total of 63 nodes would be obtained and we would need to analyze each one of them.

From this, it should be clear that one of the main problems we face when modeling the interest rate market is the number of instruments that are traded. For this reason, the use of models with factors that are manageable and whose variables are orthogonal is attractive.

From the above, it is clear how important the principal components methodology is. This methodology allows us to reduce the number of dimensions of the yield curve to only three. (Litterman & Scheinkman, 1991) were among the first to report that the first three principal components of the yield curve explained practically all the observed changes. They were also able to identify that the first component has a direct relationship with the level of the rate and its movements could be interpreted as parallel displacements of the curve. The second component corresponds to changes in the slope of the yield curve (steepness) and the third component corresponds to changes in the curvature or convexity of the yield curve. In the case of the Mexican Bond curve, it has been shown that these components explain most of the changes in interest rates, see for instance (Mota Aragón & Mata Mata, 2018). The use of principal components has several advantages, such as eliminating idiosyncratic effects of each node, such as its liquidity or amount in circulation, and it also allows the volatility of the interest rate curve to be estimated.

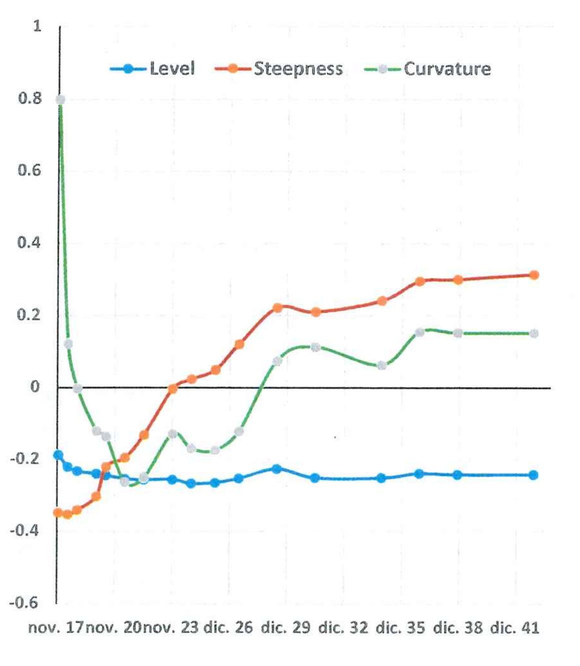

Figure 13: First three factors

With this in mind, we can try to analyze a model with few parameters to represent the yield curve. One of these models is the one developed by (Nelson & Siegel, 1987) which has been widely accepted by market participants including several central banks.

The Nelson-Siegel model adjusts a smooth function to the interest rate curve. They propose this smooth function by considering the solutions to second order linear differential equations with real and unequal roots. They propose this family of curves since with them we can obtain, by adjusting their parameters. the different behavior the curve has shown in different moments of time (S-shaped, humped, or monotonic.) This was in sharp contrast to other curves proposed before their work (basically polynomial fits), all of them curves that explode toward the long part of the curve. However, it is not clear why they considered that the yield curve could be the solution of one of these differential equations. They even mention that experimentation in fitting the curve showed them that the general solution of this kind of equation is overparametrized, so they proposed a more parsimonious function: the solution when roots are equal. The popularity of the model is demonstrated by use among market participants and it is one of the models used to publish zero coupon curves by various central banks such as the Federal Reserve Bank and the European Central Bank (ECB).

After these considerations, they proposed the forward rate to be given by the following function:

$$f(\tau) = \beta_1 + \beta_2 e^{-\lambda\tau} + \beta_3 \lambda \tau e^{-\lambda\tau} \tag{22}$$

which implies that the spot rates are given by the following:

$$y(\tau) = \beta_1 + \beta_2 \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_3 \left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right) \tag{23}$$

Note that for $\lambda\tau$ small, this family of functions have the following expansion:

$$y(\tau) = \beta_1 + \beta_2 + (-\beta_2 + \beta_3)\frac{\lambda\tau}{2} + (\beta_2 - 2\beta_3)\left(\frac{(\lambda\tau)^2}{6}\right) + o((\lambda\tau)^3) \tag{24}$$

Which imply, in particular, that $\lim_{\tau\to 0} y(\tau) = f(0) = \beta_1 + \beta_2$. We also have that $\lim_{\tau\to\infty} y(\tau) = \beta_1$, which will help us find suitable interpretations for the parameters and compare its functional form with our findings from last section.

The original Nelson-Siegel model is defined in continuous time where the structure of the instantaneous forward rate is modeled. The forward rate modeling allows consistency between the observed discount rates and the direct interpretation of the expectations of changes in the short-term rate that is dominated by monetary policy as we have seen. Additionally, a parameter λ is included, which determines the exponential decay rate that the level of the short-term rate will have on those rates with longer maturities. In this way, as the rate term is longer, the influence of the first parameter will be greater.

Among the main advantages of the Nelson-Siegel model is that it is a parsimonious model, that is, it requires a small number of parameters to fully characterize the performance curve. This model manages to recover the shape of the yield curve for almost every day, summarizing it in just four parameters. The original motivation of the model was to have a linear structure in the parameters with one non-linear parameter allowing for calibrations.

There are at least three reasons why the Nelson-Siegel model is popular: the first two are its ease of estimation and ease of extension. The third is that it let us obtain rates for all maturities, including terms that are not in the sample, which is very useful when looking to place new bonds.

(Diebold & Li, 2006) mention that considering the Nelson-Siegel factors, a three-factor model is obtained in which the level, slope and curvature are considered. However, the functional structure remains unchanged. There are people that consider a fourth advantage to give an intuitive interpretation of the parameters by the use of the limiting behaviors: the level β_1 that determines the long-term rate, the slope is the parameter β_2 and whose sum with β_1 gives the zero rate ($\beta_1 + \beta_2$), and finally the curvature is estimated by β_3 which in turn allows us to calculate the medium-term rate to be $\beta_1 + \frac{\beta_2+\beta_3}{2}$. As we will see below, we could consider a different interpretation that suits better the results shown in the previous sections.

With risk-neutral investors the rates of return are equal to the expected value of the short-term rate and there are no expectations of excess yields on bonds. However, the evidence that the participants will demand a premium for longer-term investments as shown by (Campbell & Shiller, 1991) and (Cochrane & Piazzesi, 2005) document this phenomenon, which in some way contradicts the theory of expectations.

As we saw before, the correlation that exists between the interest rates is usually high between the rates of adjacent nodes, as shown in the previous section. However, this correlation decreases as the term corresponding to each of the rates moves away. Note for instance the correlation between data corresponding to the one-day repurchase rate and the 30-year Bond rate of return which seems completely independent.

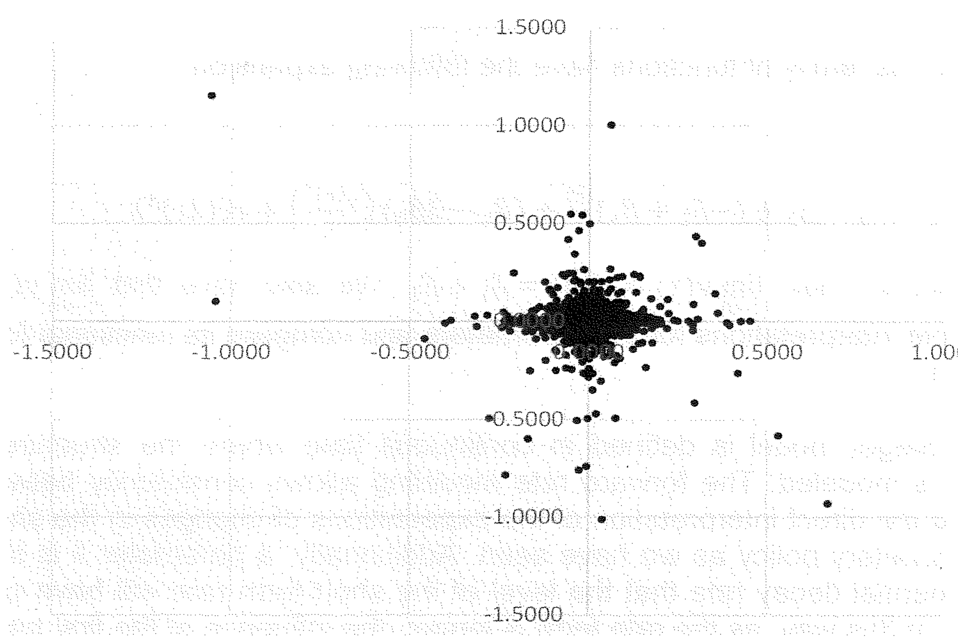

Figure 14: Correlation between repo rates and 30-year M-Bond

Note that this term premium mentioned by various authors and observed in the market can be a function of the additional risk that an investor incurs by lending their money for a longer time and being exposed to greater variations in rate levels. In the case of bondholders, it could represent a loss if they decide to sell part of their bonds before maturity. Therefore, this risk premium can be interpreted as the fair payment for investing for a longer time, it is not constant over time and varies depending on the volatility and risk events present in the market.

The three factors that are regularly considered are the following:

The long-term rate that is thought of to be determined by macroeconomic variables such as long-term inflation expectations, potential growth, and premiums for inflationary risk and for liquidity, for instance. That one is represented by β_1 in the Nelson-Siegel model and is the rate we approach as the term tends to infinity.

The short-term interest rate as shown in the previous section is determined by the central bank to preserve inflation within its range of variability in a stable manner and help boost growth when needed. Recall that from the results given in the previous sections, the central bank collects many different pieces of information to take the decisions for the short-term rate, so is not true that only inflation and growth results are relevant for the short-term rate, market expectations are a very significant source of information as well as, in the Mexican case, the dollar-peso exchange rate.

As mentioned before, the Nelson-Siegel model for short term rates can be well approximated by

$$y(\tau) = \beta_1 + \beta_2 + (-\beta_2 + \beta_3)\frac{\lambda\tau}{2} + (\beta_2 - 2\beta_3)\left(\frac{(\lambda\tau)^2}{6}\right) \quad (25)$$

a quadratic function on $\lambda\tau$ with constant term $\beta_1 + \beta_2$, first order coefficient $\frac{-\beta_2+\beta_3}{2}$ and quadratic term given by $\frac{(\beta_2-2\beta_3)}{6}$. With this, we can consider differently the level, steepness, and curvature terms for small values of $\lambda\tau$ in terms of this parabola. Note that given the magnitudes of β_2 and β_3, our function can be increasing or decreasing. Moreover, also from the relative values of these two coefficients, our function would be concave up or concave down. In both cases, the behavior is independent of β_1.

Given the results of previous section, for the Mexican yield curve, we can consider that, statistically speaking, up to one year, $\lambda\tau$ is sufficiently small so that the yield for all bonds can be well approximated by the yield of the one-day rate. This corresponds to the Target Rate, which in the Nelson-Siegel model would be given by $\beta_1 + \beta_2$. For the three-year bond, we see that our fit is not enough to keep this approximation valid so we will need to incorporate additional factors. As we can see from the approximation, the additional term will depend on β_3 as well. This help us determine that additional terms for medium-rate bonds must be shared by longer-term rates.

Although (Diebold & Li, 2006) consider curvature to be the third component, they also argue that this third component is not correlated to macroeconomic variables. However, we see that even for small values of $\lambda\tau$, all three Nelson-Siegel factors are involved and so, we will consider that additional macroeconomic variables are involved in the development of the yield curve from early on from going from short-term rates to medium-term rates.

3.3 Incorporating Macroeconomic and Fiscal Factors

As we saw in section 3.1, for medium-term rates, the approximation using only the Target Rate is not only slightly bad in terms of goodness of fit, but more importantly, it has some serious problems of model specification and normality of the residuals.

The graph for the residuals is shown below and clearly is far from being random and normally distributed:

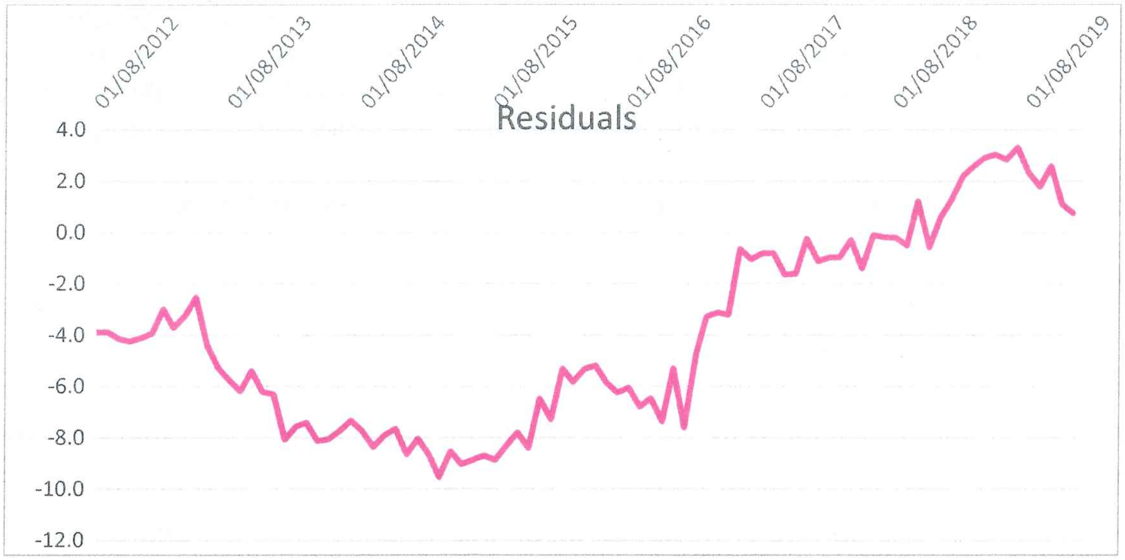

Figure 15: Residuals after first regression (2012-2019)

After the discussion given in previous section, we analyze which additional macroeconomic factors can be considered to obtain better approximations for the medium-term rates. As mentioned above, we see that factors that are considered for longer-term rates are important also for the transition from short to medium rates.

It is well known that for Government bonds, fiscal and macroeconomic factors affect their behavior, see for instance (Baldacci & Kumar, 2010) and (Mota Aragón & Mata Mata, 2018).

It is easier to analyze the relevant factors for longer-term rates. According to the information given by Banco de México in (Banco de México, Portal Interactivo de Información Financiera, 2021), for the year 2020, the main investors in government longer term bonds were foreign investors with 45% of holdings, Pension Funds (Siefores), Banks, Investment Funds, and Insurance Companies with 34% and other national investors with the remaining 11%.

Note that foreign investors and national institutional investors have very different investment strategies to hold positions in Mexican Bonds: the first ones look for yield, particularly in times like the present ones where yield in government bonds in advanced economies is practically zero, while the second group use longer-term bonds more as a strategy to match their long-term liabilities with zero credit risk (for the local market) instruments. For this last group, there are few fiscal factors that affect their decision to invest in these bonds.

The first group, however, invest in bonds from different governments and is very sensitive to possible rating decisions about the issuing government so they are monitoring constantly the relevant fiscal ratios that can trigger rating modifications. The most important ratios that rating agencies analyze and report to determine possible fiscal imbalances that could end up in defaults are those related to government spending, particularly spending without considering financial costs. See for instance the two reports given by Fitch for their decisions to reduce Mexico's credit rating (Fitch Ratings, Baja Calificación de México, 2019) and (Fitch Ratings, Baja Calificación de México, 2020).

With this in mind, we analyzed many ratios involving Government spending to see if some of them could help us determine an additional factor that could explain better the behavior of medium-term rates.

The graph of the 1y moving average of the ratio of Government spending before financial costs to GDP is as follows:

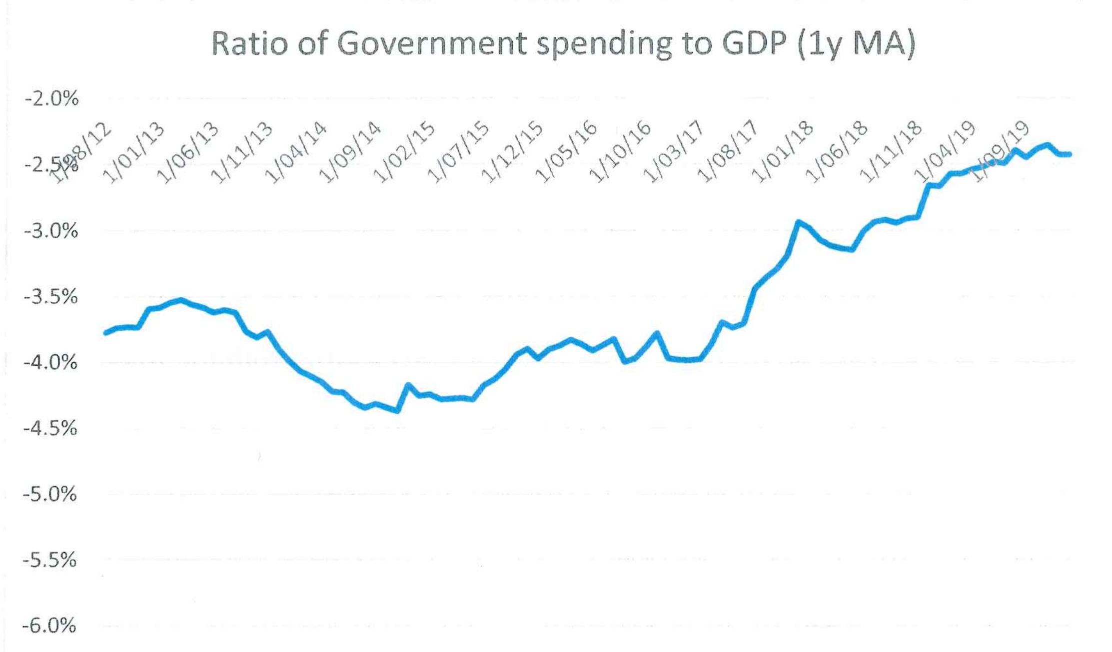

Figure 16: Ratio of Government spending to GDP (2012-2019)

We see that this graph exhibits very similar features as the one for the residuals given above.

After noticing this, we decided to perform a standard least squares multiple regression on the price of medium-term bonds using as regressors the estimated Price obtained from the Estimated Target Rate and this ratio.

The LS multiple regression performed produced the results shown in the following Table:

Table 3: Coefficients multiple regression for the price of the medium-term Bond.

	Coefficients	*Standard Error*	*T-Statistic*	*P-value*
Intercept	11.6045945	2.821399289	4.113063524	8.9059E-05
Estimated Price from Estimated Target Rate	0.96955313	0.047356988	20.4732853	8.1116E-35
(Debt w/o Fin Costs)/GDP	298.831912	39.64835122	7.537057736	4.5247E-11

With the coefficients shown, we found that R^2 and adjusted R^2 were 93%, the regression as a whole had a p-value of 8.7×10^{-50} which deems a very good fit.

More importantly, the residuals were found to be normally distributed with the following Quantile plot:

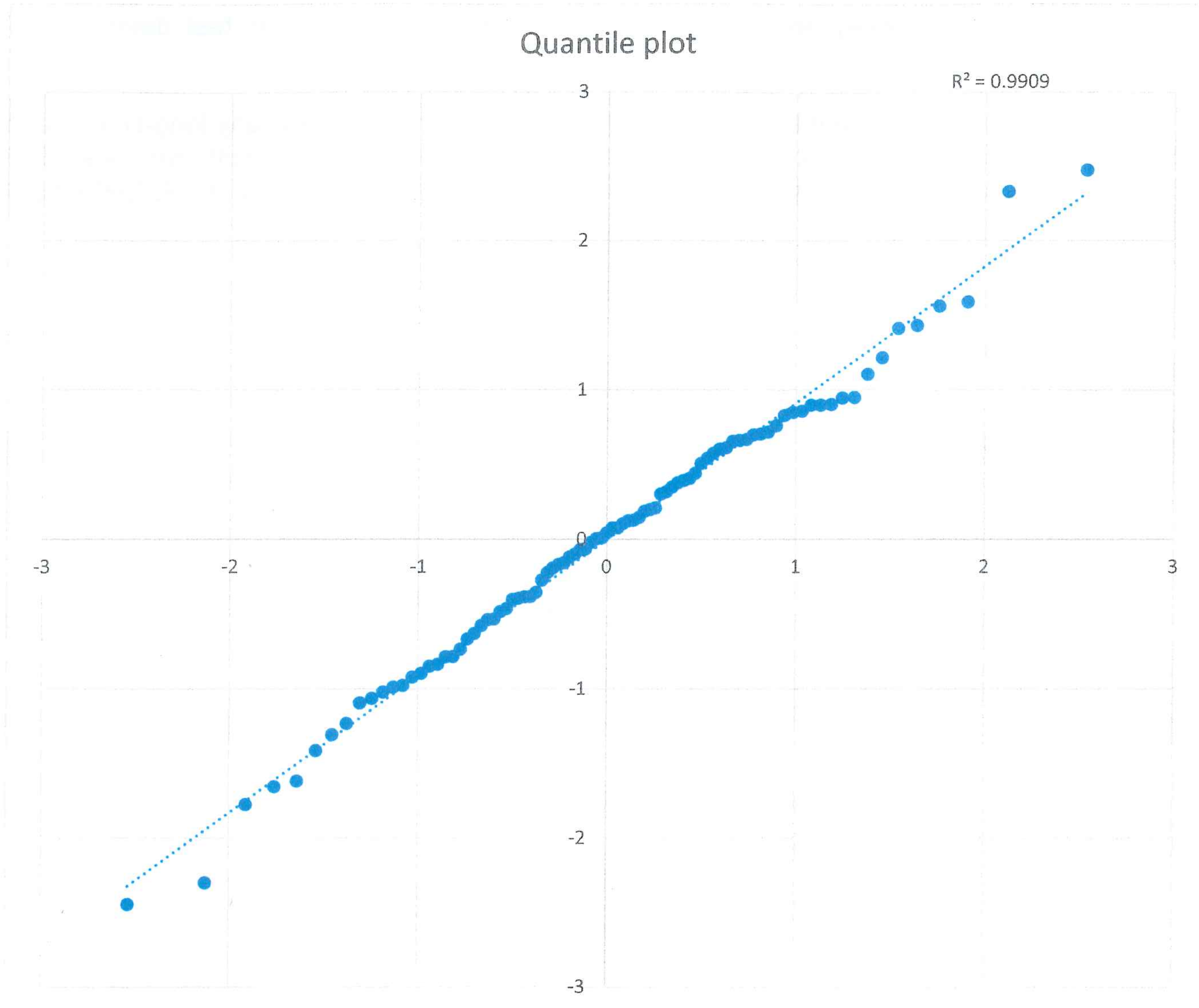

Figure 17: Quantile plot for residuals (multivariate regression)

And a Jarque Bera p-value of 0.70. So, we see that this regression, after incorporating the ratio of Government Spending without financial costs to GDP, has a very good fit and has normally distributed residuals.

We also analyzed if it is well specified. We found that in this case, as oppose to the regression without the ratio, the Breusch-Pagan test gave a p-value of .35, so indicating that there is no evidence to reject the hypothesis of a well specified model.

It is worth notice that the coefficient corresponding to the estimated price coming from the Estimated Target Rate is again very close to one as in the case of the short-term rates indicating that the price follows closely the decisions of monetary policy but adjusting them by the reactions of investors on long-term bonds to fiscal policy.

Since we are dealing with time series, it is important to analyze if the residues of this regression are stationary or if we are still missing some important information. After performing an Augmented Dickey-Fuller test, the results of the test made us strongly reject the Null hypothesis of non-stationarity (at a 0.99 confidence level); even the results of a simple Dickey-Fuller test deemed the same conclusion.

This also suggested that the two independent variables didn't have any long-time relationship. However, to verify this, we test for co-integration of the two variables. The results were also very clear. After performing an Engle-Granger test we conclude (at a 0.01 significance level) that they are not cointegrated.

Chapter 4

Conclusions

The present thesis proposes a novel method that we think could be a useful tool to forecast the Target Rate set by the Board of Governors of Banco de México. We also consider it could be useful to analyze the weights given by the Board to the different variables considered in the determination of a rate movement.

After analyzing the minutes given by the Board, we concluded that we had to consider different variables for a raise and for a lower of the rate. We proposed a discrete econometric model for the Target Rate at the nth month, R_n, given in terms of the Rate at the previous month by (14)

$$R_n = R_{n-1} + .25C_n$$

with C_n being a discrete random variable with values on the integers and which responds to a group of external variables given by $C_n = U_n - D_n$ with both U_n and D_n being non-negative integer valued random variables depending on two different sets of external variables (although some variables could affect both.) The behavior the Target Rate shows is well captured by two Poisson distributions, one corresponding to U_n and another one corresponding to D_n given that the second one is restricted to be non-zero only if the first one is zero. The intensities of these distribution are given as the exponential of some auxiliary function that will be the one affected linearly by financial and economic data, hard and soft.

Testing the best way to incorporate the relevant variables we realized that additional information other than the one obtained from pure economic and financial hard data was needed, so we decided to incorporate Banxico's surveys information. This decision was crucial to obtain good results on our approach.

For a raise, we considered the rate of inflation, the amount the Federal Reserve increased its rate, the exchange rate, and the inflation rate *expectations* published on the Banxico survey. For a lower, we also consider the rate of inflation, the amount the Federal Reserve decreased its rate, the change on the IGAE index and finally, the GDP growth *expectation* given on the survey.

We tried to keep our model as simple as possible, so we restricted ourselves to these variables since a group of additional variables analyzed didn't contribute substantially to a better fit.

After the relevant coefficients were found using an optimization procedure, we realized that the approximation was very consistent with the behavior of the decisions of the Governing Board comparing our estimates with the actual decisions taken from August 2009 to December 2019.

To get an indication of the validity of our model, we used it to forecast the decision of the Board taken on January 2020. Here we found that our model predicts very well the decision taken, as well as revealing the relevant information that was considered for the decision.

It is worth noticing here that the decision analyzed was the last one taken before the Covid-19 pandemic hit the financial markets. As part of further work to do, it will be interesting to analyze if the pre pandemic rational on the Target Rate decisions changed after the economic and financial effects of the pandemic started to be felt worldwide. The decisions taken by Central Banks during 2020 have not been seen before, even in the wake of the 2008 crisis, so we consider an analysis of what happened during this year has to be focused separately to the one given in this work, since on itself it will be revealing of the changes produced.

Since the surveys are made to actual teams of analysts and then evaluated by the board, we see that behavioral economics and finance should be studied in further work on this subject as well. The surveyors are asked for their expectation on many financial and economic variables, and as mentioned by (Ariely, 2008) on page 111, ""Expectations can influence nearly every aspect in one's life."

It is well known that survey results vary vastly during the evolution of crisis. For instance, growth expectation (in annual % change in GDP) shown in (Banco de México, Encuestas, 2021) changed during 2020 from January to December in a monthly basis like this: +1.0, +0.91, -3.99, -7.27, -8.16, -8.97, -10.02, -9.97, -9.82, -9.44, -9.1, -8.99, the last figure being somehow close to the actual figure of -8.5. Of course, the impact of the Covid-19 pandemic was not forecast by anyone at the beginning of the year, but this variability reflects the changes in expectations and the importance given to different data sets.

Furthermore, (Shefrin, 2002) also mentions that decisions taken this way could show, in a first place, some heuristic-driven bias, for example, representativeness, overconfidence, or gambler's fallacy. On the second hand they could show frame dependence, for example, loss aversion/realization, mental accounting, or money illusion. It would be very interesting to analyze if there is evidence that such possible biases arise in the actual survey results or on the Board's decisions taken with such information.

After determining our model for the Estimated Target Rate, we used this rate as our approximation for the one-day rate. We used it to determine the prices of short and medium-term government bonds (CETES of all maturities from 28 days up to three years). For those bonds with maturities up to one year, we showed that our model fits very well the prices of these bonds, in all cases with $R^2 > 95\%$ and good model specification and normality of the residuals. This shows that, statistically speaking, we can consider that the yield is constant for all these bonds and it is given directly by the Target Rate.

However, the power of adjustment and model specification decreases with the maturity and for three-year bonds, the model ceases to be well specified and to show normally distributed residuals signaling the need for additional considerations.

To address these issues, we analyzed some theoretical models for the yield curve (mostly the Nelson-Siegel deterministic model) to understand what additional variables or else what different structures could help us overcome the model specification problem.

The analysis of the N-S model allowed us to conclude that the same factors affecting long term rates could be introduced in our medium-term model to determine the additional elements to consider.

For long term rates, it is well stablished that fiscal elements are very relevant. Here, related as well, to the behavioral finance aspects mentioned above, we read a lot of analyses made by rating agencies, in particular the ones given to Mexico when there was a decision to change the sovereign rate. It was very relevant to notice that, in the same way as the analysts surveyed by Banxico, they put different emphasis on different variables, fiscal results and *expectations*; however, they weigh heavily data related to government spending and GDP.

After these considerations, looking at the residuals obtained after considering only the Estimated Target Rate for the three-year bond, we analyzed several variables related to Government Spending relative to GDP as well as other fiscal and macroeconomic data. We found that a one-year moving average of Government Spending without financial costs as a proportion of GDP showed a very similar behavior as the one given by the residuals. We directly incorporated this variable to our model, making it a multivariate linear model with only two variables. This additional variable helped us correct the model specification problem, allowed us to obtain an $R^2 > 93\%$ and gave us residuals which, statistically speaking, were random and normally distributed. It would also be interesting to compare the given results with the many jobs done on the term structure of interest rates in terms of principal components, to determine if these principal components could be expressed in terms of the variables we found to be relevant to our estimation, for instance it would be interesting to see the possible connection of this job to the one given in (Mota Aragón & Mata Mata, 2018).

Finally, as mentioned before, we decided to stop our analysis with the three-year bond, covering all the maturities of CETES. We now know that longer-term Bonds exhibit different characteristics but also that they should inherit certain ones from shorter-term Bonds. Therefore, also in further study, it would be interesting to analyze those Bonds on the light of the results found in this work to see how the evolution from mid-term bond to longer-term ones is, and to see if there are additional financial, fiscal, or macroeconomic variables that get involved in the specification of their price as their maturities start getting further from three years.

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Curriculum Vitae

Breno Lorenzo Madero Salmerón was born in Mexico City in 1975. He graduated Summa cum Laude in Physics from the Universidad Nacional Autónoma de México in 2000. He then pursued graduate studies at the Courant Institute of Mathematical Sciences of New York University where he received a Master of Science Degree in 2002 and the Candidacy for Ph.D. in 2005 for Applied Mathematics. He was admitted to the Ph.D. program in Financial Sciences at the Instituto Tecnológico y de Estudios Superiores de Monterrey, EGADE Business School (Campus Santa Fe) in 2017.

He has a long history of professional experience in different roles in Risk Management areas in the Energy and Financial sectors (in banking, treasury and insurance).

At P.M.I. Comercio Internacional, S.A. de C.V. (commercial subsidiary in the international market of Petróleos Mexicanos) he was Portfolio Manager from 2009 to 2014 where he was responsible for the valuation of Derivatives (commodities, equity, interest rates and exchange rates) and support for their correct disclosure; responsible for the market risk determination and evaluation of positions obtained from the hydrocarbon trading and the development of market risk hedging strategies using derivative financial instruments. Later on, he was Risks and Financial Planning Manager from 2014 to 2015 where he was responsible in addition for the determination of financial risks associated to financial investment projects and infrastructure projects in the Oil and Gas sector, and responsible for the financial projections for the companies of the PMI group.

At the Financiera Nacional de Desarrollo Agropecuario, Rural, Forestal y Pesquero (Mexican National Development Bank specialized in financing the Rural Sector and Agricultural Projects), he was the Chief Risk Officer from 2015 to 2019. At this position he was in charge of the Risk Management process (identification, measurement, monitoring, delimitation, control, information, and disclosure of risks) for both quantifiable and non-quantifiable risks of the Institution. In particular, the management of discretionary risks (Credit Risk, Market Risk, Liquidity Risk) and non-discretionary risks (Operational Risk, Technological Risk and Legal Risk). He was also responsible for the development of parametric evaluation methodologies for credit applications (Credit Scoring), for achieving the greatest effectiveness of the credit coverage available to the institution (Mutual Guarantees, Liquid Guarantees, Insurance, etc.) and for measuring the hedge-effectiveness of Derivatives.

Finally, from 2019 to date, he is the Vice-president of Risk Analysis and Sector Studies at the *Comisión Nacional de Seguros y Fianzas* (Mexican Regulator and Supervisor of the Insurance and Surety Markets) where he is responsible for the analysis and specialized supervision in risk matters of the Insurance and Surety Institutions, analyzing the Mexican insurance and surety market trends and developments and the possible build-up of systemic risks, undertaking analysis of macroprudential and financial stability issues. He is also in charge of the preparation and updating of the general formula for calculating the solvency capital requirement for the sector, as well as carrying out the studies necessary for that purpose.

He has also done extensive private consulting, teaching and research.

This document was typed in using Microsoft Word by Breno Lorenzo Madero Salmerón.

Published Articles

"AN EMPIRICAL ECONOMETRIC MODEL FOR THE MEXICAN TARGET RATE AND ITS APPLICATION TO DETERMINE THE INTEREST RATE CURVE".

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