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## Finding Consistent Conjectural Variations Equilibrium in a Human Migration Model

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### Abstract

In this paper, we develop a human migration model using the concept of conjectural variations equilibrium (CVE). In contrast to previous works we extend the model to the case where the conjectural variations coefficients may be not only constants, but also (continuously differentiable) functions of the total population at the destination and of the group's fraction in it. Moreover, we allow these functions to take distinct values at the abandoned location and at the destination. As an experimental verification of the proposed model, we develop a specific form of the model based upon relevant population data of a three-city agglomeration at the boundary of two Mexican states: Durango (Dgo.) and Coahuila (Coah.). Namely, we consider the 1980-2015 dynamics of population growth in the three cities: Torreón (Coah.), Gómez Palacio (Dgo.) and Lerdo (Dgo.), and propose utility functions of four various kinds for each of the three cities. After having collected necessary information about the average movement and transportation costs for each pair of the cities, we apply the above-mentioned human migration model to this example. Numerical experiments have been conducted revealing interesting results concerning the consistency of probable conjectural variations equilibrium states.

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### 1. Introduction

Human migration models attracted a strong interest on the part of the operations researchers in the early nineties of the last century (*cf.*, [2-6], among others). The majority of the relevant papers and books consider a network of locations and develop conditions guaranteeing the existence and uniqueness of equilibrium in the

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proposed models. For example, the works by the group of A. Nagurney cited above examined various forms of the Nash equilibrium under an assumption of perfect competition, that is, each population group neglected the possible influence of the migration flows on the living standards at the destination.

In the book by Isac, Bulavsky and Kalashnikov [1], a new gamma of conjectural variations equilibria (CVE) was introduced and investigated, in which the influence coefficients of each agent affected the structure of the Nash equilibrium. In particular, constant conjectured influence factors were used in the human migration model examined in [1]. More precisely, the potential migration groups were taking into account not only the current difference between the utility function values at the destination and original locations, but also the possible variations in the utility values implied by the change of population volume due to the migration flow. In other words, we considered not a perfect competition but a Cournot-type model with influence coefficients in general different from 1 (as it is the case in the classical Cournot model).

In the proposed work, we extend the latter model to the case when the conjectural variations coefficients may be not only constants, but also (continuously differentiable) functions of the total population at the destination and of the group's fraction in it. Moreover, we allow these functions to take distinct values at the abandoned location and at the destination. As an experimental verification of the proposed model, we realize computational processing of the relevant population data of the three-city agglomerate at the boundary of two Mexican states: Durango and Coahuila. We consider the 1980 – 2015 dynamics of population growth in the three cities: Torreón (Coah.), Gómez Palacio (Dgo.) and Lerdo (Dgo.), and deduce experimentally utility functions for each of the three cities. After having collected necessary information on the average movement and transportation (i.e., migration) costs for each pair of the cities, we apply the above-mentioned human migration model to this example. Numerical experiments employing modern big data techniques have been conducted with interesting results concerning the probable equilibrium states revealed (*cf.*, [2-3]).

The paper is organized as follows. The next section (Section 2) describes the examined human migration model and introduces the appropriate notation. Section 3 is dedicated to the definition of the conjectural variations equilibrium in the model in question. In Section 4, Theorem 4.1 is obtained which establishes the equivalence of the equilibrium to a solution of an appropriate variational inequality problem. Furthermore, conditions guaranteeing the existence and uniqueness of the human migration equilibrium are listed in the same section. Several numerical examples of equilibria determined for the human migration model with the above-mentioned three locations are considered in Section 5.

## 2. Model Specification

Similar to [1-6], consider a closed economy with:

Nomenclature	
$i$	locations, $i = 1, 2, \dots, n$
$k$	classes of population, $k = 1, 2, \dots, J$
$\overline{Q}_i^k$	initial fixed population of class $k$ in location $i$
$Q_i^k$	variable population of class $k$ in location $i$
$c_{ij}$	marginal cost of migration from location $i$ to location $j$
$s_{ij}^k$	variable migration flow of class $k$ from origin $i$ to destination $j$

Assume that the (marginal) migration cost reflects not only the cost of physical movement but also the personal and psychological cost as perceived by a class when moving between locations.

Unlike the model of human migration described in [1], [5], [6], the utility  $u_i^k$  (attractiveness of location  $i$  as perceived by class  $k$ ), depends on the population at the destination  $Q_i^j$ , that is,  $u = u(Q)$ , as it was introduced in [2] and [3].

The conservation of flow equations, given for each class  $k$  and each location  $i$  and assuming no repeated or chain migration, are given as follows:

$$Q_i^k = \bar{Q}_i^k + \sum_{l \neq i} s_{li}^k - \sum_{l \neq i} s_{il}^k, \quad i = 1, \dots, n, \tag{1}$$

and

$$\sum_{l \neq i} s_{il}^k \leq \bar{Q}_i^k, \quad i = 1, \dots, n, \tag{2}$$

with  $s_{il}^k \geq 0, \forall k = 1, \dots, J; l \neq i$ . Denote the problem's feasible set by

$$K = \{ (Q, s) \mid s_{ij}^k \geq 0, (Q, s) \text{ satisfies (1) and (2)} \}.$$

Equation (1) states that the population at location  $i$  of class  $k$  is determined by the initial population of class  $k$  at location  $i$  plus the migration flow into  $i$  of that class minus the migration flow out of  $i$  for that class. Inequality (2) states that the flow out of  $i$  by class  $k$  cannot exceed the initial population of class  $k$  at  $i$ , since no chain migration is allowed.

Assume that migrants are rational and that migration continues until no individual has any incentive to move, since a unilateral decision will no longer yield a positive net gain (the gain in expected utility minus the migration cost).

As an extension of the human migration model [1], here we introduce the following concepts.

Let  $w_{ij}^{k+} \geq 0$  be an influence coefficient taken in account by an individual of class  $k$  moving from  $i$  to  $j$ . This coefficient is defined by her assumption that after the movement of  $s_{ij}^k$  individuals of class  $k$  from  $i$  to  $j$  the total population of class  $k$  at  $j$  will become equal to  $\bar{Q}_j^k + w_{ij}^{k+} s_{ij}^k$ .

On the other hand, let  $w_{ij}^{k-} \geq 0$  be an influence coefficient conjectured by an individual of class  $k$  moving from  $i$  to  $j$ , determined by the assumption that after the movement of  $s_{ij}^k$  individuals, the total population of class  $k$  in  $i$  will remain equal to  $\bar{Q}_i^k - w_{ij}^{k-} s_{ij}^k$ .

We accept the following assumptions concerning the utility functions and expected variations of the utility values:

**A1.** The utility  $u_i^k = u_i^k(Q^k)$  is a monotone decreasing and continuously differentiable function.

**A2.** Each person of class  $k$ , when considering her possibility of moving from location  $i$  to location  $j$ , takes into account not only the difference in utility values at the initial location and the destination, but also both the expected (negative) increment of the utility function value at  $j$

$$s_{ij}^k w_{ij}^{k+} \partial u_j^k / \partial Q_j^k$$

and the expected (positive) utility value increment in location  $i$

$$-s_{ij}^k w_{ij}^{k-} \partial u_i^k / \partial Q_i^k.$$

### 3. Definition of Equilibrium

Similar to [2-3], a multi-class population and flow pattern  $(Q^*, s^*) \in K$  is equilibrium, if for each class  $k = 1, \dots, J$ , and for each pair of locations  $i, j = 1, \dots, n$ ;  $i \neq j$ , the following relationship holds

$$u_i^k - s_{ij}^{k*} w_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} (Q_i^{k*}) + c_{ij}^k \begin{cases} = u_j^k + s_{ij}^{k*} w_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} (Q_j^{k*}) - \lambda_i^k, & \text{if } s_{ij}^{k*} > 0; \\ \geq u_j^k + s_{ij}^{k*} w_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} (Q_j^{k*}) - \lambda_i^k, & \text{if } s_{ij}^{k*} = 0; \end{cases} \quad (3)$$

and

$$\lambda_i^k \begin{cases} \geq 0, & \text{if } \sum_{l \neq i} s_{il}^{k*} = \bar{Q}_i^k; \\ = 0, & \text{if } \sum_{l \neq i} s_{il}^{k*} < \bar{Q}_i^k. \end{cases} \quad (4)$$

**A3.** We assume that both influence coefficients are functions depending upon the resulting population at the location in question and the migration flow from location  $i$  to location  $j$ , satisfying the following conditions:

$$s_{ij}^k w_{ij}^{k+} (Q_j^k, s_{ij}^k) = \alpha_{ij}^{k+} s_{ij}^k + \sigma_{ij}^{k+} Q_j^k,$$

and

$$s_{ij}^k w_{ij}^{k-} (Q_i^k, s_{ij}^k) = \alpha_{ij}^{k-} s_{ij}^k + \sigma_{ij}^{k-} Q_i^k,$$

where

$$\alpha_{ij}^{k\pm} \geq 0, \quad \sigma_{ij}^{k\pm} \geq 0, \quad \alpha_{ij}^{k\pm} + \sigma_{ij}^{k\pm} \leq 1.$$

This turns (3) into:

$$u_i^k - s_{ij}^{k*} \alpha_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} - \sigma_{ij}^{k-} Q_i^k \frac{\partial u_i^k}{\partial Q_i^k} + c_{ij}^k = u_j^k + s_{ij}^{k*} \alpha_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^k \frac{\partial u_j^k}{\partial Q_j^k} - \lambda_i^k, \quad \text{if } s_{ij}^k > 0;$$

or

$$u_i^k - s_{ij}^{k*} \alpha_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} - \sigma_{ij}^{k-} Q_i^k \frac{\partial u_i^k}{\partial Q_i^k} + c_{ij}^k \geq u_j^k + s_{ij}^{k*} \alpha_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^k \frac{\partial u_j^k}{\partial Q_j^k} - \lambda_i^k, \quad \text{if } s_{ij}^k = 0.$$

Now suppose that the utility function associated with a particular location and class can depend upon the population associated with every class and each location, that is, compose a vector-function  $u = u(Q)$ . Assume also that the cost associated with migration between two locations (as perceived by a particular class) can depend, in general, upon the flow of each class between every pair of locations, i.e., consider an aggregate vector-function  $c = c(s)$ . Finally, let us generate an auxiliary vector of the appropriate size as follows:

$$s_{ij}^k \alpha_{ij}^{k-} \frac{\partial u_i^k}{\partial Q_i^k} + \sigma_{ij}^{k-} Q_i^k \frac{\partial u_i^k}{\partial Q_i^k} + s_{ij}^k \alpha_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} + \sigma_{ij}^{k+} Q_j^k \frac{\partial u_j^k}{\partial Q_j^k} = d_{ij}^k(Q, s);$$

and

$$d(Q, s) = (d_{ij}^k(Q, s)).$$

#### 4. Existence and Uniqueness of Equilibrium

Now we are in a position to formulate the following result (see the proof in [2] or [3]).

**Theorem 1.** A population and migration flow pattern  $(Q^*, s^*) \in K$  satisfies the equilibrium conditions (1) and (2) if and only if it solves the variational inequality problem

$$\langle -u(Q^*), Q - Q^* \rangle + \langle c(s^*) - d(Q^*, s^*), s - s^* \rangle \geq 0, \quad \forall (Q, s) \in K. \quad (5)$$

The existence of at least one solution to variational inequality (5) follows from the general theory of variational inequalities, under the sole assumption of continuous differentiability of the utility functions  $u$  and continuity of migration cost functions  $c$ , since the feasible convex set  $K$  is compact.

#### 5. Examples of Human Migration Equilibrium

To realize numerical experiments with the above-mentioned human migration model we consider three distinct locations  $i=1, 2$ , and  $3$ , for a unique class  $k$ , with a population  $Q_i$ . Each inhabitant perceives a utility  $u_i$  in each location  $i$ , and the cost of being transferred of  $i$  to  $j$ , denoted by  $c_{ij}$ . For a base of our research, three real cities have been selected: Torreón, Coah. ( $i=1$ ), Gómez Palacio, Dgo. ( $i=2$ ), and Lerdo, Dgo. ( $i=3$ ). These three cities form the agglomerate with a well-developed transportation and communication networks. We introduce utility functions for each city, making use of the following scheme (for other three schemes, see [2] – [3]).

Assume that the initial quantities of construction workers (together with their families) at each location are:  $\bar{Q}_1=105,000$ ,  $\bar{Q}_2=55,000$ ,  $\bar{Q}_3=23,000$ ; the costs to be transferred from a location to another (in thousands of mexican pesos) are as follows:  $c_{12}=1.6$ ,  $c_{13}=1.6$ ,  $c_{21}=1.6$ ,  $c_{23}=1.0$ ,  $c_{31}=1.6$ ,  $c_{32}=1.0$ .

Inequalities (1) and (2) can be re-written as the following complementarity problems:

$$\begin{cases} \psi_{ij}^k \equiv u_i^k + c_{ij}^k - u_j^k - s_{ij}^k w_{ij}^{k-} - \frac{\partial u_i^k}{\partial Q_i^k} - s_{ij}^k w_{ij}^{k+} \frac{\partial u_j^k}{\partial Q_j^k} + \lambda_i^k \geq 0, & s_{ij}^k \geq 0, \text{ and } s_{ij}^k \psi_{ij}^k = 0, \quad \forall i, j; \\ \zeta_i^k \equiv \bar{Q}_i^k - \sum_{\ell \neq i} s_{i\ell}^k \geq 0, & \lambda_i^k \geq 0, \text{ and } \zeta_i^k \lambda_i^k = 0. \end{cases}$$

For example, if we consider the perfect competition case, that is, with  $w_{ij}^- = w_{ij}^+ \equiv 0$ , we find the solution to the problem below making use of the software Maple 9.5<sup>†</sup>:

$$\begin{aligned} s_{12} = s_{13} = s_{23} = 0; & \quad s_{21} = 55000; \\ s_{31} = 18603, & \quad s_{32} = 4397; \\ \lambda_1 = 0, & \quad \lambda_2 = 0, \quad \lambda_3 = 63.136. \end{aligned}$$

The latter means that all the workers of this group together with their families will migrate from Lerdo to Gómez Palacio and to Torreón, and all workers of this group leave Gómez Palacio for Torreón, having enhanced the group population in Torreón up to 178,603 people and decreasing the group population in Gómez Palacio down to 4,397 men and women, i.e.

$$\bar{Q}_1 = 178,603, \quad \bar{Q}_2 = 4,397, \quad \bar{Q}_3 = 0,$$

<sup>†</sup> © Maplesoft, a division of Waterloo Maple Inc. 2003

with the total population in Torreón becoming equal to  $Q_1 = 529,512 + 73,603 = 603,115$ , in Gómez Palacio  $Q_2 = 273,315 - 55,000 + 4,397 = 222,712$ , and in Lerdo  $Q_3 = 112,435 - 23,000 = 89,435$ .

Alternately, if we assume that  $\alpha_{ij}^+ = w^+ = 2.5$ , we come to a different solution of our problem:

$$s_{12} = 6,090, \quad s_{13} = 0, \quad s_{21} = 55,000, \quad s_{23} = 0, \quad s_{31} = 14,114, \\ s_{32} = 8,886; \lambda_1 = \lambda_2 = 0, \quad \lambda_3 = 60.275.$$

We see that the migration flow from Lerdo to Torreón is higher when the influence quotient is lower, since this means that the agents do not expect that their living standards worsen too much with the increase of other groups migration flows. It is also interesting to notice that with the influence coefficient  $\alpha_{ij}^+ = w^+ = 2.5$  the workers start migration from Torreón to Gómez Palacio, which is not the case with the perfect competition  $w_{ij}^- = w_{ij}^+ \equiv 0$ .

## 6. Conclusions and Future Research

We have investigated a human migration model involving conjectures of the migration groups concerning the variations of the affection utility function values both in the abandoned location and in the destination site. To formulate equilibrium conditions in this model, we use the concept of conjectural variation equilibrium (CVE). We establish the existence and uniqueness results for the equilibrium in question.

We also notice that the human migration model with conjectural variations can be further extended and examined in the case when constraint (2) is replaced by a weaker condition, namely,

$$Q_i^k \geq 0, \quad i = 1, \dots, n; \quad k = 1, \dots, J,$$

which allows us to consider the repeated or chain migration. In this case the feasible set  $K$  stops being compact (remaining, however, convex), which makes insufficient the use of the general theory of variational inequality problems to demonstrate the existence of equilibrium. Then subtler results obtained in [1] can be used to that effect. Indeed, the existence of equilibrium will be guaranteed for various classes of utility functions and migration costs that are free of *exceptional families of elements* (EFE).

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