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To cite this article: D Bermúdez and F Delgado 2017 *J. Phys.: Conf. Ser.* **839** 012016

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Stability of Quantum Loops and Exchange Operations in the Construction of Quantum Computation Gates

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Abstract. Quantum information and quantum computation is a rapidly emergent field where quantum systems and their applications play a central role. In the gate version of quantum computation, the construction of universal quantum gates to manipulate quantum information is currently an intensive arena for quantum engineering. Specific properties of systems should be able to reproduce such idealized gates imitating the classically inspired computational gates. Recently, for magnetic systems driven by the bipartite Heisenberg-Ising model a universal set of gates has been realized, an alternative easy design for the Boykin set but using the Bell states as grammar. Exact control can be then used to construct specific prescriptions to achieve those gates. Physical parameters impose a challenge in the gate control. This work analyzes, based on the worst case quantum fidelity, the associated instability for the proposed set of gates. A strong performance is found in those gates for the most of quantum states involved.

1. Introduction

Quantum engineering for quantum information and quantum computation applications are currently reaching their technology realizations. In the quantum gate implementation of these theories, the construction of efficient quantum algorithms is being developed to implement and to solve high complexity problems. There, quantum engineering is requiring reproduce, on specific physical systems, the theoretical quantum gates idealized. Control and a performance quantification of those realizations are fundamental for a stable functioning of those elements.

This work presents a stability analysis for a universal set of gates based on the non-local grammar of Bell states for magnetic systems presented in [1]. The system assumes:

$$H_h = \sum_{k=1}^3 J_k \sigma_{1_k} \sigma_{2_k} - B_{1_h} \sigma_{1_h} - B_{2_h} \sigma_{2_h} \quad (1)$$

a near approach to magnetic systems considered as potential implementation. It exhibits a $SU(2)$ block decomposition for its $SU(4)$ dynamics when it is expressed on the Bell states basis in \mathcal{H}^2 [2]. It gives a convenient structure to set alternative quantum gates in those theories. In spite time and external magnetic fields involved are highly controllable, the analysis of stability should be centered on the J_i strength values, the more sensible error source here [3, 4]. Based



on the universal set of gates proposed in [1], this work presents a stability analysis on the worst fidelity transmission on the entire \mathcal{H}^2 states involved. The second section presents a brief review of the evolution blocks involved in the gates analyzed. The third section depicts a convenient projective quantum space for \mathcal{H}^2 to shown the performance outcomes. The fourth section presents the corresponding analysis for each gate based on the fidelity presented in [1] to evaluate the performance. The last section states the conclusions.

2. Heisenberg-Ising universal set of gates

Current analysis considers the set of universal gates $\mathcal{D} = \{(\mathbf{1}_1 \otimes S_{\pi/8_2})_B, (\mathbf{1}_1 \otimes S_{\pi/4_2})_B, (S_{\pi/8_1} \otimes \mathbf{1}_2)_B, (S_{\pi/4_1} \otimes \mathbf{1}_2)_B, (\mathbf{1}_1 \otimes H_2)_B, (H_1 \otimes \mathbf{1}_2)_B, (C^1NOT_2)_B, (C^2NOT_1)_B\}$ as it is presented in [1]. Those gates are equivalent to the Boykin set [5], but expressed for the Bell basis instead of the traditional computational basis. All them are reducible from the block form:

$$s_{h_j} = e^{i\Delta_{h\alpha}^+} \begin{pmatrix} e_{h\alpha}^{\beta*} & -qi^h d_{h\alpha} \\ qi^{*h} d_{h\alpha} & e_{h\alpha}^\beta \end{pmatrix} \quad (2)$$

as part of the evolution operator $U = \bigoplus_{j=1}^2 s_{h_j}$, where the parameters $d_{h\alpha}, b_{h\alpha}$ are extensively presented in [2] in terms of the physical parameters J_i, B_{k_h} and t . Note that the most of prescriptions are based on the settlement of Evolution Loops or Exchange operations on (2) [6]. In the current analysis, a deviation dJ_i from the original prescriptions for the Heisenberg-Ising strengths is considered as error source.

3. Hilbert space \mathcal{H}^2 on S^7 and projection on S^3 as representation space

To plot the gate stability analysis, we will construct an adequate projective space as follows. By using the generalized spherical coordinates on S^3 (S^n is the n -dimensional sphere embed in $(n + 1)$ -dimensional euclidean space), a general physical state in the current problem on \mathcal{H}^2 can be written as:

$$|\Psi\rangle = \sin \alpha \sin \beta \cos \gamma |00\rangle + e^{i\iota} \sin \alpha \sin \beta \sin \gamma |01\rangle + e^{i\kappa} \sin \alpha \cos \beta |11\rangle + e^{i\lambda} \cos \alpha |10\rangle \quad (3)$$

with: $\alpha, \beta \in [0, \pi], \iota, \kappa, \lambda \in [0, \pi), \gamma \in [0, 2\pi)$. These states lay on a S^7 sphere, but clearly they do not fill it because we restrict the $|00\rangle$ component to be real. Thus, the space been represented by (3) is isomorphic to S^6 . This space is a two-folded representation of all physical states (with the proper rearrangement of α, β and γ) because a global sign can be still removed. The space could be projected on S^3 in several ways to be partially plotted. One of them, $P : |\Psi\rangle \rightarrow |\Psi\rangle|_{\iota, \kappa, \lambda=0}$, lets understand $S^6 = S^3 \times (H^1)^{\times 3}$ (H^n is a hemisphere of S^n), where H^1 is the fiber bundle corresponding to each phase in (3). This selection sets a S^3 subsphere on S^6 (similarly as the Bloch sphere S^2 for a pure qubit can be projected on its equatorial ring S^1). On this space we can additionally restrict $\gamma \in [0, \pi)$ to avoid the two-folding, settling the component of $|01\rangle$ as positive (signs in other components become still independent). This space will be called the representation space, \mathcal{R} and its motivation is represent the result of the analysis on it. There, Bell states can be identified in \mathcal{R} as precise points (Figure 1a). An issue of \mathcal{R} should be remarked. The concurrence \mathcal{C} of (3) is:

$$\mathcal{C}^2 = 4 \sin^2 \alpha \sin^2 \beta |\cos \alpha \cos \gamma - e^{i(\iota+\kappa-\lambda)} \sin \alpha \sin \gamma \cos \beta|^2. \quad (4)$$

If $\mathcal{C}_{\mathcal{R}}^2 \equiv \mathcal{C}^2|_{\iota, \kappa, \lambda=0}$, then it is possible show $1 \geq \mathcal{C}^2 - \mathcal{C}_{\mathcal{R}}^2 = 8 \sin^3 \alpha \cos \alpha \sin^2 \beta \cos \beta \sin \gamma \cos \gamma (1 - \cos(\iota + \kappa - \lambda)) \geq -1$, then partial entanglement is not injective under the projection. Only some separable states clearly are, those projected on the lateral edges of \mathcal{R} , with $\alpha, \beta = 0, \pi$.

The Figure 1a exhibits level surfaces for the concurrence, but they are applicable only if $\iota + \kappa - \lambda = 2n\pi, n \in \mathbf{Z}$. Inner surfaces are approaching to the maximal entanglement line, $\mathcal{C} = 1$ where Bell states lie in a twisted helix, repeating it vertically. Another inner surface, $\cos \alpha \cos \gamma - \sin \alpha \sin \gamma \cos \beta = 0$ (separating the two tube-like structures), contains the separable states when $\iota + \kappa - \lambda = 2n\pi, n \in \mathbf{Z}$, but other projected states laying there for $\iota + \kappa - \lambda \neq 2n\pi, n \in \mathbf{Z}$ are not separable. Then, the stability analysis could be represented in \mathcal{R} , but there is not an identification with \mathcal{C} there, so this property should be analyzed separately (an alternative projection for S^7 can be achieved in terms of the complex projective map in [7], identifying submanifolds of constant entanglement). Finally, the state (3) should be still expressed in the Bell basis for the analysis.

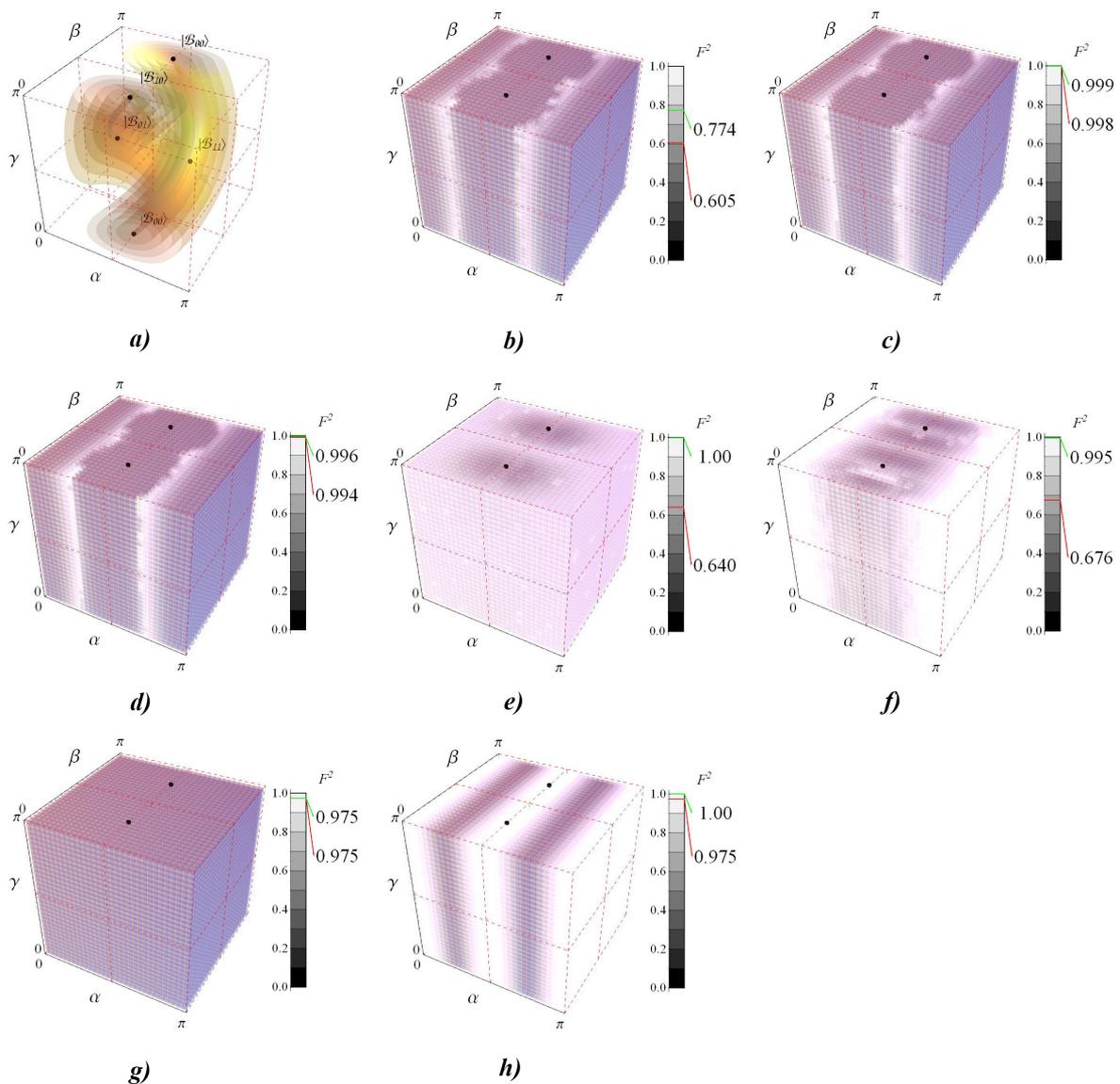


Figure 1. a) Space \mathcal{R} and concurrence level-surfaces $\mathcal{C} = 0.01$ (outer), $0.25, 0.5, 0.75, 0.9$ (inner) and Bell states: 3D-density plots for F^2 and their extremal values for b) $(\mathbf{1}_1 \otimes S_{\phi_2})_B$, c) $(S_{\pi/8_1} \otimes \mathbf{1}_2)_B$, d) $(S_{\pi/4_1} \otimes \mathbf{1}_2)_B$, e) $(\mathbf{1}_1 \otimes H_2)_B$, f) $(H_1 \otimes \mathbf{1}_2)_B$, g) $(C^1 NOT_2)_B$ and h) $(C^2 NOT_1)_B$.

4. Error estimation based on quantum fidelity for the elements in \mathcal{D}

The stability of each gate can be quantified through the quantum fidelity for each state being processed. Fidelity can be analyzed for each block s_{h_j} in $U(t)$ under the prescriptions of each gate following the development in [1]. If $|\Psi_{f_j}\rangle$ and $|\Psi'_{f_j}\rangle$ are the desired and the real states obtained with s_{h_j} and some little variation $s_{h_j} + \delta s_{h_j}$ respectively, departing from any original state $|\Psi_{o_j}\rangle$ (each one in the subspace j of \mathcal{H}^2), then (where $s'_{h_k} \approx s_{h_k} + \mathcal{D}s_{h_k} + \frac{1}{2}\mathcal{D}^2s_{h_k}$, $\mathcal{D} \equiv d\mathbf{J} \cdot \nabla_{\mathbf{J}}$):

$$\mathcal{F}^2 = |\langle \Psi'_f | \Psi_f \rangle|^2 = 1 - \sum_{k,j,j'} \alpha_{k,j'}^* \alpha_{k,j} a_{k,j,j'} + \left| \sum_{k,j,j'} \alpha_{k,j'}^* \alpha_{k,j} b_{k,j,j'} \right|^2 \quad (5)$$

$$\text{with: } a_{k,j,j'} = \left\langle \beta_{f_{k,j'}, f_{j',k}}^h | \mathcal{D}s_{h_k}^\dagger \mathcal{D}s_{h_k} | \beta_{f_{k,j}, f_{j,k}}^h \right\rangle, b_{k,j,j'} = \left\langle \beta_{f_{k,j'}, f_{j',k}}^h | s_{h_k}^\dagger \mathcal{D}s_{h_k} | \beta_{f_{k,j}, f_{j,k}}^h \right\rangle$$

the $\mathcal{D}s_{h_k}^\dagger \mathcal{D}s_{h_k}$ and $s_{h_k}^\dagger \mathcal{D}s_{h_k}$ entries, with: $f_{a,b}^h = \frac{1}{2}((3-h)(a-1) + (h-1)(b-1))$ setting the rules to fix the Bell states in each block. Thus, for a uniform set of 10^6 points in \mathcal{R} , a numerical algorithm was used to find the minimum of \mathcal{F}^2 with respect to $|\delta J_i| = |dJ_i/J_i| \leq 0.05, i = 1, 2, 3; \ell, \kappa, \lambda \in [0, 2\pi)$ around the gate prescriptions for J_i, B_i, t [1]. Minimum is reported in the Figures 1b-h as density plots on \mathcal{R} for each gate in \mathcal{D} . In this analysis, the minimum was calculated on each entire fiber bundle for each projection.

For $(\mathbf{1}_1 \otimes S_{\phi_2})_B$, the fidelity is independent of ϕ , including the interest cases $\phi = \pi/4, \pi/8$. There, $0.778 \leq \mathcal{F} \leq 0.880$ for all states (3). Density plot (Figure 1b) exhibits lower \mathcal{F} values for the most of separable states in the lateral boundaries of \mathcal{R} . The same analysis is true for $(S_{\phi_1} \otimes \mathbf{1}_2)_B$ obtained for $h = 3$ [1]. For the $(S_{\phi_1} \otimes \mathbf{1}_2)_B$ gates obtained with $h = 1$, \mathcal{F} is ϕ dependent. Despite that for $(S_{\pi/8_1} \otimes \mathbf{1}_2)_B$ and $(S_{\pi/4_1} \otimes \mathbf{1}_2)_B$ the density plots are similar to those for $(\mathbf{1}_1 \otimes S_{\phi_2})_B$, they show improved values of fidelity (Figures 1c-d): $0.999 \leq \mathcal{F} \leq 1.000$ and $0.997 \leq \mathcal{F} \leq 0.998$. Meanwhile, $(\mathbf{1}_1 \otimes H_2)_B$ and $(H_1 \otimes \mathbf{1}_2)_B$ exhibit a wider dispersion in the \mathcal{F} values $0.800 \leq \mathcal{F} \leq 1.000$ and $0.822 \leq \mathcal{F} \leq 0.998$ respectively (Figures 1e-f). An unclear behavior in terms of separability appears, which is analyzed below. $(C^1 NOT_2)_B$ gives an almost uniform value $\mathcal{F} \approx 0.987$ (Figure 1g). Finally, $(C^2 NOT_1)_B$ has $0.987 \leq \mathcal{F} \leq 1.000$ (Figure 1h) with some limited slices of low fidelity involving both separable and entangled states.

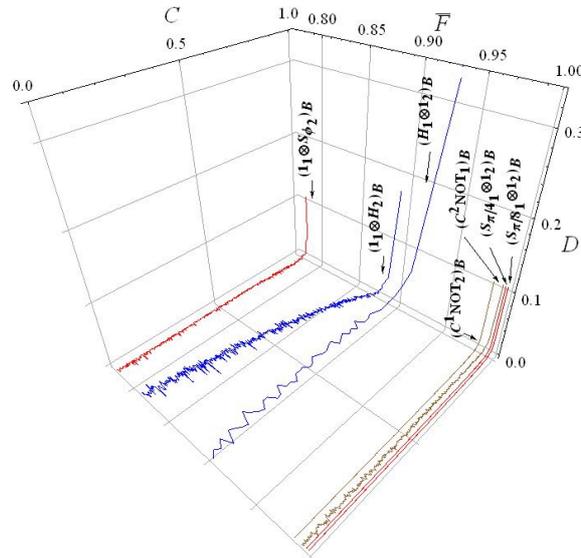


Figure 2. Gates performance summarized showing the mean value of fidelity $\bar{\mathcal{F}}$ for each concurrence C value. Vertical axis shows the density of states D in such situation.

Due to density plots shadow some inner details of \mathcal{F} in \mathcal{R} and the unclear relation with \mathcal{C} , the Figure 2 summarizes some complementary interesting aspects of the analysis. This figure compares the concurrence \mathcal{C} with the average fidelity \overline{F} for the states in such case, together with their density D (the relative proportion with respect to the overall states considered in the analysis). Note the improved fidelity values for entangled states and the growing number of states in that situation. Hadamard-like gates show the lowest fidelity values together with $(\mathbf{1}_1 \otimes S_{\phi_2})_B$ and a notable dispersion in any case for $|\delta J_i| \leq 0.05$. Despite, \mathcal{F} shows values above of 0.77.

5. Conclusions

Quantum engineering in quantum information is a growing development field relating Quantum Mechanics, Control Theory and Computer Science on a common basis of system's physical properties and achievable technologies. Gate design should combine these fields to construct procedures and physical realizations, including quantification of possible failures as was developed here for the set of universal quantum gates proposed for systems interacting via the Heisenberg-Ising model. Last quantification shows considerably stability when sensible and uncontrolled design parameters could be detuned. There, additional error correction procedures can be still applied on the syndromes defined by \mathbf{X}_j and \mathbf{Z}_j [1] in each block.

Acknowledgments

The support from Tecnológico de Monterrey to develop this research is acknowledged.

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